

THE ASTRONOMICAL JOURNAL

PUBLISHED BY THE AMERICAN INSTITUTE OF PHYSICS
FOR THE AMERICAN ASTRONOMICAL SOCIETY

VOLUME 66

1961 December ~ No. 1293

NUMBER 10

CONFERENCE ON THE INSTABILITY OF SYSTEMS OF GALAXIES

Santa Barbara, California, August 8-9, 1961

EDITED BY JERZY NEYMAN, THORNTON PAGE, AND ELIZABETH SCOTT

Statistical Laboratory, University of California, Berkeley, and Van Vleck Observatory, Wesleyan University, Middletown, Connecticut

Foreword

THIS Conference, sponsored jointly by the Statistical Laboratory, University of California, Berkeley, and by the Van Vleck Observatory, Wesleyan University, Middletown, Connecticut, with financial support of the National Science Foundation, was organized for the purpose of discussing the evidence for and against the hypothesis of instability of systems of galaxies. The implications of this hypothesis, first proposed in 1954 by V. A. Ambartsumian of the Burakan Observatory, Erevan, Armenian S.S.R., extend from clusters to groups and pairs of galaxies, to studies of the form, structure, and dynamics of individual galaxies, to extragalactic radio sources, interactions of galaxies, and cosmology.

As we see it, the Ambartsumian hypothesis conjectures that at least some of the clusters and groupings of galaxies are unstable, that they have positive total energy, that they are flying apart and cannot be expected to last more than 10 to 1000 million years. It thus bears on the time scale of cosmology and on the evolution of galaxies. Ambartsumian has not attempted to explain the source of excess energy—the enormous “explosions” that provided energy sufficient to fling galaxies of about 10^{44} g mass outward at velocities of about 100 km/sec. But it is clear that they may be identified with the extragalactic radio sources. Moreover, the assumed instability interferes with dynamical calculations of masses of galaxies from their motions in pairs, small groups and clusters, and eliminates the basis of much of the reasoning fundamental to modern astrophysics.

The reason for the hypothesis in the first place was the discrepancy between masses of galaxies determined individually from internal rotations, and the much

larger average mass of a galaxy as determined by the virial theorem applied to the dispersion of velocities in clusters assumed to be stable.

Ambartsumian seeks to explain this discrepancy by assuming excess energy; alternatively, it can be explained by assuming the presence of extra mass that cannot be seen or detected by other means as yet—mass either in an intergalactic medium, or in numerous objects of low luminosity. However, this alternative implies that some 99% of the mass of the universe has remained so far invisible, and it is perhaps distasteful for astronomers and cosmologists to think that their theories are based on observations of less than 1% of the matter that is really there!

The Ambartsumian hypothesis has a similar implication: that very large amounts of energy have been released in the past by processes as yet unobserved—not as yet understood in terms of terrestrial physics. The strength of the hypothesis depends on the possibility of linking together, under one postulate, the interpretation of a variety of phenomena—a unification that may not be possible under alternative assumptions. If large explosions or energy releases account for the major clusters of galaxies, then smaller ones may account for smaller groups and pairs of galaxies, still smaller ones for spiral arms, jets, and filaments associated with individual galaxies, and the extragalactic radio sources are simply early stages in similar cataclysmic explosions.

It must not be overlooked that there are separate alternative explanations in each of the above cases, most of them controversial in themselves: Spiral arms may be explainable in terms of resonances in dynamical orbits, or in terms of stream motions, or magnetic fields. Collisions of galaxies were first proposed as the cause of

extragalactic radio sources, and close passages may account for jets and filaments through tidal action. Clusters and groups of galaxies are generally supposed to have condensed in regions of higher-than-average density in the medium from which galaxies were formed.

Although the Conference was not devoted to this aspect, Ambartsumian's hypothesis also raises interesting problems in physics: The first is the *mechanism* whereby such tremendous amounts of energy can be released—energy equivalent roughly to the total annihilation of a globular cluster, or to the radiation from a million stars over their whole lifetimes.

Secondly, how was the energy *directed*, particularly in the case of a double galaxy that must be flying apart along a single line under this hypothesis. How could so energetic an explosion be anything but isotropic?

Thirdly, how was the energy *imparted* to each galaxy as a whole? The acceleration of a galaxy of stars by an external force other than gravity is difficult to explain, although primordial galaxies of highly ionized gas could be subjected to magnetohydrodynamic forces.

So much for the setting in which the Conference took place. The undersigned three organizers were exposed to the various trends of thought for and against Ambartsumian's hypothesis in the course of their own studies of clustering and of the statistical determination of masses of double galaxies. After a few participants agreed to meet in Santa Barbara just before the IAU General Assembly and Symposium 15 on Problems of Extragalactic Research, they recommended others, and the number of invited papers grew to the list of 17 given below. At this point it was decided to invite all those attending IAU Symposium 15 on Problems of Extra-

galactic Research and the final attendance of the Conference included the 62 persons listed below.

The papers presented at the conference vary in the relation to the Ambartsumian hypothesis. There are several in which new observational data are reported for the first time. Others are given partly to methodology and partly to analysis of previously published data in which new regularities are seen. Finally, there are several theoretical papers based on specific sets of *a priori* assumptions from which conclusions are drawn concerning the stability of models representing systems of galaxies. There is naturally a close connection with the IAU Symposium No. 15, which followed immediately, and in which there were several references to the earlier Conference. Likewise, a number of points in the discussion reported below refer to the later Symposium proceedings of which are to be published separately.

Conferences cannot be expected to solve the problems posed, and it is not surprising that this Conference came to no firm conclusion on the question of Ambartsumian's hypothesis. On the other hand, conferences may be expected to help crystallize ideas through free discussion of various points of view among widely informed participants. The organizers take this opportunity to thank the invited speakers and other participants whose knowledge of the various fields involved made the papers and recorded lively discussions so valuable. Appreciation is also due the National Science Foundation for its financial support.

JERZY NEYMAN,
THORNTON PAGE
ELIZABETH SCOTT

LIST OF PARTICIPANTS

- Abell, G. O., University of California, Los Angeles, California.
van Albada, C. B., Sterrenkundig Instituut, Amsterdam, Holland.
Ambartsumian, V. A., Academy of Sciences, Erevan, U.S.S.R.
Arp, W., Mt. Wilson and Palomar Observatories, Pasadena, California.
van den Bergh, S., David Dunlap Observatory, Ontario, Canada.
Baldwin, J. E., Cavendish Laboratory, Cambridge, England.
Bok, B. J., Mt. Stromlo Observatory, Canberra, Australia.
Burbidge, Mrs. E. M., Yerkes Observatory, Williams Bay, Wisconsin.
Burbidge, G., Yerkes Observatory, Williams Bay, Wisconsin.
Carpenter, E. F., Steward Observatory, Tucson, Arizona.
Christy, R. F., California Institute of Technology, Pasadena, California.
Courtes, G., Observatoire de Marseille, Marseille, France.
Drake, F. D., National Radio Astronomy Observatory, Green Bank, West Virginia.
Fehrenbach, Ch., Observatoire de Marseille, Marseille, France.
Fish, R. A., Lick Observatory, Mt. Hamilton, California.
Gascoigne, S. C. B., Mt. Stromlo Observatory, Canberra, Australia.
Godart, O., 10 Rue Père Damien, Brussels, Belgium.
Heckmann, O., Hamburger Sternwarte, Hamburg-Bergedorf, Germany.
Heeschen, D. S., National Radio Astronomy Observatory, Green Bank, West Virginia.
Hodge, P., Mt. Wilson and Palomar Observatories, Pasadena, California.
von Hoerner, S., Astronomisches Rechen-Institut, Heidelberg, Germany.
Holmberg, E., Observatory, Uppsala, Sweden.
Irwin, J. B., Goethe Link Observatory, Bloomington, Indiana.
Just, K., Institut für theoretische Physik, Berlin-Dahlem, Germany.
Kerr, F. J., C.S.I.R.O. Radiophysics Laboratory, Sydney, Australia.
Lemaître, G., 61 Rue de Namur, Louvain, Belgium.
Lilley, A. E., Harvard College Observatory, Cambridge, Massachusetts.
Limber, D. N., Yerkes Observatory, Williams Bay, Wisconsin.
Lindblad, B., Stockholm Observatory, Saltsjöbaden, Sweden.
Lindblad, P. O., Sterrewacht, Leiden, Holland.
Lust, R., Max Planck Institut für Physik, München, Germany.
Marcus, A., Statistical Laboratory, Berkeley, California.
Markarian, B. E., Burakan Observatory, Erevan, U.S.S.R.
McCrea, W. H., Royal Holloway College, Egham, Surrey, England.
Michie, R. W., University of California, Berkeley, California.
Mills, B. Y., Chatterton Astronomy Department, Sydney, Australia.
Minkowski, R., Radio Astronomy Laboratory, University of California, Berkeley, California.

- nyman, J., Statistical Laboratory, University California, of Berkeley, California.
- ort, J. H., Sterrewacht, Leiden, Holland.
- ge, T. L., Van Vleck Observatory, Middletown, Connecticut.
- shmish, Mrs. P., Observatorio Astronómico Nacional, Mexico, Mexico.
- yveda, A., Observatorio Astronómico Nacional, Mexico, Mexico.
- endergast, K. H., Yerkes Observatory, Williams Bay, Wisconsin.
- oberts, M. S., Harvard College Observatory, Cambridge, Massachusetts.
- abin, Mrs. V., Georgetown College Observatory, Washington, D. C.
- ndage, A. R., Mt. Wilson and Palomar Observatories, Pasadena, California.
- hmids, M., Mt. Wilson and Palomar Observatories, Pasadena, California.
- huecking, E., Hamberger Sternwarte, Hamburg-Bergedorf, Germany.
- Scott, Miss E. L., Statistical Laboratory, University of California, Berkeley, California.
- Seeger, C. L., Sterrewacht, Leiden, Holland.
- Sersic, J. L., Cordoba Observatory, Cordoba, Argentina.
- Smith, Harlan, Yale Observatory, New Haven, Connecticut.
- Spinrad, H., Jet Propulsion Laboratory, Pasadena, California.
- Sreenivasan, S. R., Institute for Space Studies, New York, New York.
- Stibbs, D. W. N., University Observatory, St. Andrews, Scotland.
- Swope, Miss H. H., Mt. Wilson and Palomar Observatories, Pasadena, California.
- Thackeray, A. D., Radcliffe Observatory, Pretoria, South Africa.
- Tift, W. G., Arthur J. Dyer Observatory, Nashville, Tennessee.
- de Vaucouleurs, G., Department of Astronomy, Austin, Texas.
- Vorontsov-Velyaminov, B. A., Sternberg Institute, Moscow, U.S.S.R.
- Weymann, R. J., Steward Observatory, Tucson, Arizona.
- Woltjer, L., Sterrewacht, Leiden, Holland.

TABLE OF CONTENTS

- A. Ambartsumian**: Instability Phenomena in Systems of Galaxies.
- and E. M. Burbidge**: Recent Work on Groups and Clusters of Galaxies.
- Vorontsov-Velyaminov**: On the Possibility of Explosive Formation of Galaxy Patterns.
- Kalloghjan**: On the Dynamical Instability of Several Groups of Galaxies.
- E. Markarian**: A Physical Chain of Galaxies in the Virgo Cluster.
- Minkowski**: NGC 6166 and the Cluster A 2199.
- van den Bergh**: Radio Sources and Clusters of Galaxies.
- van den Bergh**: The Stability of Clusters of Galaxies.
- N. Limber**: The Virial Theorem and the Stability of Clusters of Galaxies.
- Neyman and E. L. Scott**: Magnitude-Redshift Relation for Galaxies in Clusters in the Presence of Instability and Absorption.
- B. van Albada**: Evolution of Clusters of Galaxies under Gravitational Forces.
- Heckmann**: On the Possible Influence of a General Rotation on the Expansion of the Universe.
- Lemaître**: The Exchange of Galaxies between Clusters and Field.
- O. Abell**: Evidence of Second-Order Clustering of Galaxies and Interactions between Clusters of Galaxies.
- L. Page**: Statistics of Double Galaxies.
- Holmberg**: On the Dynamics of the Virgo Cluster.
- de Vaucouleurs**: Recent Studies of Clusters and Superclusters.
- Neyman, T. L. Page, and E. L. Scott**: Summary of the Conference.

Instability Phenomena in Systems of Galaxies

V. A. AMBARTSUMIAN

Academy of Sciences, Erevan, U.S.S.R.

TO consider the problems of the dynamics of clusters and groups of galaxies it is essential to know their masses. The data available on the masses of galaxies are, unfortunately, poor.

Observations of the orbital motions in double stars are used to determine stellar masses. A great number of already determined orbits of visual and spectral doubles forms a sound basis for our knowledge of stellar masses. In contrast to this, we are in no position to determine the orbits in double galaxies, and attempts to make statistical use of the differences of radial velocities in double galaxies meet with considerable difficulties in introducing this or that hypothesis on the nature of motion (elliptical or hyperbolic). That is the reason why we must determine the masses of galaxies from measurements of rotation and of the internal motions within a given galaxy.

Unfortunately, the data obtained in this way accumulate very slowly. Thus the value commonly accepted at present of the mass of our Galaxy might very likely turn to be incorrect by a factor of the order of 2. The value of the mass of the Large Magellanic Cloud is very uncertain. Quite limited in scope is our knowledge of the masses of both giant and dwarf elliptical galaxies.

Nonetheless, the data available have made it possible to draw the following valuable conclusions as to the values of the ratio $f = M/L$:

(a) The ratio $f = M/L$ decreases at least 10 times when passing from the elliptical galaxies of high luminosity to the spirals and further to the irregular galaxies.

(b) The ratio f does not increase but, most probably, decreases when passing from supergiant elliptical galaxies through giant ones to the dwarf systems like those in Fornax and Sculptor.

As a result, the ratio of the masses, say, of the supergiant and the dwarf galaxies, turns out to be larger than the ratio of their luminosities.

Thus, for instance, the supergiant elliptical galaxy NGC 4889 in the Coma cluster surpasses by nearly one million times in luminosity the dwarf system discovered in Capricorn by Zwicky, and the ratio of the masses of systems is probably much larger.

Quite different is the picture in the case of stars. The luminosity there increases in proportion to a rather high power of the mass. This accounts for the fact that although the luminosity of stars can vary by hundreds of millions of times, their masses vary at most by about a thousand times and most of the stars have masses differing from the average by not more than a few times.

In consequence of this, the gravity field in any stellar

cluster is determined almost equally by bright as well as by faint members of the cluster. Such is not the case, however, with clusters and groups of galaxies. Here the dwarf galaxies have almost no influence upon the structure of the gravity field, which is determined chiefly by a small number of supergiants and partly by the giant galaxies.

Zwicky (1957) has rendered great service proving the monotonic increase of the number of galaxies with decrease of luminosity (the monotonic form of the luminosity function). Probably, such an increase takes place in the majority of clusters and groups of galaxies. However, even in the case of a relatively large number of low-luminosity galaxies, their influence upon the structure of the field of gravitation inside and outside the cluster must be negligible. Suffice it to say that in the Local Group the total mass as well as the gravity field are determined mainly by two members—M31 and our Galaxy.

This circumstance makes it possible to achieve a certain simplification by solving a number of the problems regarding the dynamics of a cluster of galaxies at first for the small number of its massive members.

It is known that we observe multiple galaxies in great numbers. The question of the type of configuration of these systems can be raised as it has with multiple stars. The division of all configurations into two types will best suit our aims: *common configurations* and *configurations of the trapezium type*. The latter include multiple systems in which at least three members can be found having mutual distances of the same order of magnitude. These configurations cannot be steady and they disintegrate within a period of the order of several revolutions in the system.

The observations indicate that in *real stellar systems*, of the *trapezium type* one of the components belongs to O or B spectral types. Such stars are of recent formation and the number of revolutions they complete in the system is expected to be small. However, the observations show that a few multiple stars of later spectral classes also possess trapezium-like configurations. Of course, the configurations we observe on the sky are projections of true space configurations. Therefore, even if there are *no* real trapezium configurations of late-type stars, when they are projected on the sky a small percentage ($\sim 8\%$) of *apparent* configurations of the trapezium type will appear. This is almost precisely the percentage of trapezium configurations observed in cases where the components of multiple stars do not belong to the O and B spectral types. In other words, there are *no* or almost *no* *real* configurations of the trapezium type among the late-type multiple stars.

Quite reverse is the case with multiple galaxies. Out of the 132 multiple galaxies, as pointed out in one of our papers (Ambartsumian 1956) in Holmberg's catalogue of the double and multiple galaxies (Holmberg 1937), 7 have configurations which should, doubtless, be classed as trapezium type. Thus, systems of galaxies of the trapezium type are markedly dominant, and most of the multiple galaxies are of recent formation; i.e., their components could have made, from the moment of the formation of the system, *but a few revolutions*.

However, two remarks are to be made in this connection: (1) The periods of revolution in the multiple and double galaxies should be of the order of 10^9 years. Therefore, the multiple systems we have observed could possibly be of the age of 5×10^9 years or more. In the absence of instability of multiple systems, the galaxies are probably young, although their age can be in some cases three or four times more than 5×10^9 years. (2) The instability of trapezium configurations has not yet been given a clear-cut mathematical treatment. However, proceeding from simple considerations it should be evident that the above-mentioned time of dissolution (several periods of revolution) holds true only for cases where the masses of all three components on which the trapezium configuration is based are of the same order of magnitude; otherwise the system can exist considerably longer. Furthermore, the components must be of comparable luminosity. A substantial number of the observed multiple galaxies in fact meets this requirement. In particular, in such systems as Stephan's Quintet and Seyfert's Sextet the differences in stellar magnitudes are comparatively small. By contrast, trapezium configurations where one of the components is much brighter than the others (for example, the system M31, M32, and NGC 205) are, presumably, much more stable.

On the other hand, there are cases where a cluster of galaxies contains a considerable number of members, three or four of which are noticeably brighter than the rest (and therefore contain the greater part of the mass) and together form a trapezium configuration. Considering only the interaction of these brighter galaxies, it can be asserted that such systems should be unstable. For example, the four galaxies NGC 3681, 3684, 3686, and 3691 form a typical multiple system of the trapezium type; at least a dozen other much fainter galaxies are included in this system, but the system is evidently unstable. The galaxies NGC 7383-7390 form part of a small cluster containing six bright members and more than a dozen faint components. The bright members constitute a trapezium-type system. Finally, the three galaxies, NGC 3613, 3619, and 3625, form a small group containing at least eight fainter objects. In this case there are fainter objects of considerable angular diameters and low surface luminosity. Again we have an unstable group, although this group is not, apparently, a cluster of its own, but forms a condensation in the Ursa Major Cloud.

The arguments put forth in our previous papers speak in favor of the *joint formation* of the members of each cluster or physical group of galaxies. We refrain from repeating these arguments, but because we still encounter published assertions as to the possibility of forming groups and clusters of galaxies from independent members of the general metagalactic field, a new argument will be advanced. This is based on the existence of systems of a few very bright galaxies and a larger number of faint ones. In principle, it is possible to understand the dynamical formation of one physical pair by the accidental encounter of three galaxies. Generally speaking, this pair in the course of time can capture other galaxies too. However, the exchange of large amounts of energy must take place between the interacting galaxies, and to achieve this end the interacting galaxies must have masses of about the same order. Let us assume that a multiple system of three or more massive galaxies has come into being in this manner (although this can be proved to be highly improbable); no galaxy of an essentially small mass (say by two orders less) can then ever be captured by such a group, because the exchange of kinetic energies in the case of large mass ratio is always negligibly small. Thus, the mechanism of capture meets with new difficulties in any attempt to account for the existence of galaxies of small mass in groups and clusters. This difficulty applies to all three of the above examples of multiple systems of bright galaxies with an additional number of fainter members, and also to the case of a pair of bright galaxies NGC 521, 533, which has a number of very faint companions.

The great difference in distribution of bright and faint members is most apparent in the large spherical clusters of galaxies. The bright members are densely concentrated, while the faint ones occur relatively more frequently on the periphery. This phenomenon was given special consideration by Zwicky, who showed that the Coma cluster is of very large dimensions if we judge from the distribution of galaxies of low luminosity. However, the case is completely different with irregular clusters. According to Reaves (1956), faint galaxies of low surface luminosity in the Virgo cluster reveal approximately the same distribution and, consequently, same degree of concentration as bright galaxies.

Such a picture occurs also in the case of the objects of low surface brightness and small density gradient in the well-known cluster in Fornax. As Hodge points out (1959), the search for similar objects in regions neighboring the cluster has led to negative results. Finally, in the above-mentioned case of NGC 3613, 3619, and 3625, galaxies of low surface brightness and low luminosity do not spread far beyond the bounds determined by the group of bright galaxies.

These examples bear testimony to the fact that equipartition of energies between the bright and the faint members of the irregular clusters is out of the question, and that phenomena of instability are expressed in

irregular clusters much more sharply than in the spherical ones.

The existence of a great number of trapezium configurations signifies that many of the multiple galaxies are unstable formations. If this is the case, we have no right to assume *a priori* that the multiple galaxies should be negative-energy systems. In the case of simple double stars (we exclude O and B stars) it could be asserted without any knowledge of their orbits that, for the most part, they have negative total energies. In fact, if the majority of the multiple stars had positive total energy then the time of disintegration would be only a few tens of thousands of years, and within such a period most of the multiple stars would be replaced by stars of a new generation. In other words, positive total energy would lead to an erroneous conclusion about the rate at which stars are formed in the Galaxy.

Yet in the case of multiple galaxies the assumption of positive total energy for most of them does *not* lead to similar erroneous deduction. The age of the component galaxies derived in this way is but a few times less than that accepted for our Galaxy. Therefore we infer that the sign of the energy of the multiple galaxies, groups and clusters of galaxies should be determined in each case, relying upon observational data.

The reasons put forth above support the view that the *a priori* assumption of positive total energy in a number of systems of galaxies cannot be regarded as more audacious than, say, the assumption that almost all such systems possess negative energy. Nevertheless, let us consider the facts: The data show that, if negative total energy is assumed for a number of multiple systems, it has to be admitted that the ratio $f=M/L$ must be about one order of magnitude larger than follows from other data. Thus it is pointed out in the following paper by Kalloghjan that the multiple system comprising NGC 68, 69, 71, 72 and one anonymous galaxy, if it has negative total energy, would lead to a value of f greater than 300. For the double galaxy NGC 7385-7386, he finds f greater than 600.

The sign of the total energy of Stephan's Quintet has been determined by us and by G. and E. M. Burbidge (1959) in detail, resulting in positive total energy. Later, Limber and Mathews (1960) indicated that, under certain assumptions, when the mass of the components is supposed to be very high, the Quintet can have a negative total energy.

The sign of the total energy in a number of clusters of galaxies is elucidated in several recent investigations in detail. A number of difficulties are caused by uncertainty in the exact value of f for giant elliptical galaxies. This value is believed to lie in the range $30 < f < 70$. However, greater values ($f \sim 100$), although rare, are not excluded, particularly for the brightest supergiants ($M \sim -21.5$). No straightforward data exist that would enable us to estimate the value of f for these brightest supergiants. It is natural, therefore, to believe that the sign of the total energy is determined with greater

reliability in those clusters and systems in which there are no supergiant elliptical galaxies. The case will be even better for systems with no giant elliptical galaxies either, and that is why the positive total energy of the nearby system of galaxies in Sculptor, as established by de Vaucouleurs (1959) is of paramount importance.

Of no lesser value is the result obtained by van den Bergh (1960) in respect to the cluster of galaxies in Canis Venatici, although studies toward determining the borders and identifying the members of this cluster should continue.

The Hercules cluster, investigated by G. and E. M. Burbidge (1959), contains but a small percentage of bright elliptical galaxies. To admit negative total energy of this cluster we have to ascribe the value of f_E of the order of 300, which seems to be improbable. The contrast becomes still sharper in the case of the cluster in Virgo. Assuming the stability of this system we should have to acknowledge that $f_E > 1000$, as shown by de Vaucouleurs (1960).

It might be conjectured that perhaps the Coma cluster could have negative total energy if the modern distance scale of Sandage could be further changed by further diminution of the redshift constant. On the other hand, many members of this cluster are elliptical galaxies of moderate luminosity. The value f cannot be very high for them, so that a particularly high value of f must be ascribed to the remaining supergiant galaxies if the cluster has negative total energy.

Naturally, the sharp discrepancy between the summed luminosities of the clusters of galaxies and the masses found by applying the virial theorem has compelled some authors to favor the hypothesis of supplementary masses in the clusters which do not form part of member galaxies, i.e., intergalactic matter. Yet the data available on the upper limit of opacity in the clusters of galaxies, as well as the data on the 21-cm radiation, are not favorable.

There remains the assumption of a comparatively rich intergalactic stellar population in the clusters. Such a possibility has been contemplated in detail by de Vaucouleurs with reference to the Coma cluster. The result is negative if we refrain from an improbably large value of f for this intergalactic stellar population. This result apparently refers to other clusters as well.

Thus there is only one natural assumption left relating to the clusters cited above—they have positive total energies. It should be stressed that no *a priori* arguments can be advanced against this assumption.

A study of the structure of the irregular clusters of galaxies leads one to the conclusion that often they are made up of several superimposed groupings. An interesting example of such a grouping was pointed out by Markarian a few years ago; the chain of bright galaxies in the Virgo cluster containing NGC 4374, 4406, 4438 and others. This wonderful arc of eight bright galaxies is presumed to represent a physical grouping within the Virgo cluster. On the other hand,

acts about the radial velocities of the members of this group undoubtedly establish its positive total energy.

Recently I looked through the abstract of van den Bergh's latest paper, in which the assumption of irregular clusters being made up of separate subsystems and subclusters is made in the most general form. It is difficult to overestimate the importance of this phenomenon in comprehending the evolution of clusters of galaxies. In this case we apparently have consecutive formation of relatively independent subsystems (subclusters), the superposition of which brings about irregular clusters. It is probable that many of these subsystems have a positive total internal energy.

Of considerable interest are the results of determination of the average value of f from the differences of radial velocities in double galaxies as obtained by Page (1961), who obtained for spiral and irregular galaxies $f = \frac{1}{3}$; for ellipticals and lenticulars $f = 94$. These values are derived on the assumption that in double galaxies the motion takes place in circular orbits. That the value of f for spiral and irregular galaxies is even less than the one derived from rotations of single galaxies means that all or almost all of the observable narrow pairs of such galaxies constitute negative-energy systems. Let us compare this with the unusually large values of f obtained on the basis of the virial theorem for clusters composed of spiral and irregular galaxies. Such a comparison leads to two inevitable conclusions:

(a) All explanations allowing for the negative total energy of clusters and groups made up of spirals and irregular galaxies become still more highly improbable, since the arguments adduced in similar cases are likewise applicable to the double galaxies.

(b) There are almost no systems with positive energy among the isolated double galaxies, for such systems can represent a *narrow* pair only for a very short duration of time (of the order of 10^8 years).

If this is so, then the double elliptical systems are also to be regarded as systems possessing, for the most part, negative energies, and Page's value $f = 94$ can be taken as close to the real value. This brings us nearer to the conclusion according to which the Coma cluster can have negative total energy.

It is interesting to note that when we pass from double galaxies to multiple configurations of the trapezium type the differences of the velocities of the components become much greater. Assuming negative total energies of these configurations leads to the too large values of f_E .

Speaking of instability in systems of galaxies, we should also touch upon the *radio galaxies* which, as a rule, occur in clusters of galaxies. Apparently the radio galaxies are always among the few brightest members of the corresponding clusters. The best example is furnished by the source Perseus A (NGC 1275), which is the brightest member of the cluster in Perseus.

In the radio galaxy NGC 4486, a jet is being ejected from the central nucleus with separate condensations, the luminosities of which resemble those of dwarf

galaxies. Apparently these condensations contain an enormous amount of relativistic electrons. However, it is difficult to refute the suggestion that these condensations include also a substantial amount of common matter in addition to the relativistic plasma. In particular they are likely to contain, especially, *sources* of relativistic electrons.

A strong argument in support of this view is provided by the two galaxies (NGC 3651 and IC 1182) out of the nuclei of which jets are ejected containing blue condensations. These galaxies with blue jets are also among the brightest members in the corresponding clusters. Finally, there are cases when blue components occur in the vicinity of other giant elliptical galaxies, which evidently represent a later stage in the evolution of the above blue condensations.

In all probability, the condensations in NGC 4486 represent an earlier stage of evolution of the same objects. In such cases the intensity of radio emission of the blue condensations and the blue companions is believed to have already weakened.

From this point of view it is interesting to note that, as revealed in Burakan, a blue object of photographic magnitude $18^m.5$ is situated very close to the remote radio galaxy in Hydra. The color index of this object is about $-0^m.5$. It is starlike on our plates (as expected at such a distance, assuming that its diameter is less than 2000 parsecs.) If it can be shown that this object is in fact a physical companion of Hydra A, then a close connection is indicated between the two types of eruptive activities of the nuclei of supergiant galaxies: the ejection of plasma condensations and that of blue condensations. One way or another, all the data indicate that this activity is of great import in the origin of galaxies.

Thus we conclude that there exist clusters which are in a particularly active phase of evolution when new galaxies originate within them. The existence of a radio galaxy is an indicator of this phase. It is possible that even in such a phase the radio emission erupts only now and then, and with varying intensity at that.

In the radio galaxy Perseus A it is well known that large relative velocities are observed, up to 3000 km/sec. Such velocities exceed the velocity of escape from the cluster and thus speak for themselves of instability.

It seems, therefore, that a study of the radio galaxies as systems from the nuclei of which large masses are ejected or which are in the process of division must throw a new light upon the phenomena of instability in the clusters of galaxies.

REFERENCES

- Ambartsumian, V. A. 1956, *Izvest. Akad. Nauk Armyan Fiz. Mat., Estestven. i Tekh. Nauki S.S.R.* **9**, 23.
 Burbidge, E. M. and Burbidge, G. 1959, *Astrophys. J.* **130**, 15.
 ———, 1959, *ibid.* **130**, 629.
 de Vaucouleurs, G. 1959, *ibid.* **130**, 718.
 ———, 1960, *ibid.* **131**, 585.

- Hodge, P. 1959, *Publs. Astron. Soc. Pacific* **71**, 28.
 Holmberg, E. 1937, *Ann. Obs. Lund* No. 6.
 Limber, D. and Mathews, W. 1960, *Astrophys. J.* **132**, 286.
 Page, T. 1961, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (to be published).
 Reaves, G. 1956, *Astrophys. J.* **61**, 69.
 van den Bergh, S. 1960, *ibid.* **131**, 558.
 Zwicky, F. 1957, *Morphological Astronomy* (Springer-Verlag, Berlin), p. 220.

DISCUSSION

LEMAITRE asked whether "positive total energy" necessarily means that clusters are flying apart, or whether it might include the case (covered in his paper below) of a continuous exchange of galaxies between clusters and the field. AMBARTSUMIAN answered that positive total energy is a matter of observation, and could include both possibilities. However, if the galaxies in a cluster had a *common origin*, as most astronomers agree, then the concept of exchange is ruled out, and clusters with positive total energy must simply be expanding, or losing members to the general field. PAGE added that it would seem improbable that field galaxies could gather into a cluster with positive energy, but LEMAITRE argued that such a concept of probability is based on preconceived notions of the processes and earlier states of the galaxies.

POVEDA referred to the Local Group of galaxies, where the center of mass may be taken as that of M31 and the

Galaxy (neglecting the masses of other members). Six of the eight members for which radial velocities are available, omitting the close companions of M31 and the Magellanic Clouds, are found to be approaching the center of mass. Only NGC 598 and 6822 are receding at 14 and 110 km/sec, respectively. The stability of the Local Group is covered more completely by LIMBER in another paper, below.

In answer to JUST, who pointed out that "expansion of a cluster" could mean merely that its dimensions increase relative to the sizes of galaxies in it, AMBARTSUMIAN said that the dimensions of an expanding cluster are generally increasing faster than the Hubble expansion, $V = HD$. He also emphasized that he does not propose that *all* clusters and groups of galaxies are unstable.

HECKMANN referred to numerical studies of von HOERNER showing that a system of mass points can eject members even though it has negative total energy, and PAGE made the point that, unlike the quasi-stable groups of negative total energy, the groups and clusters of postulated *positive* total energy require further explanation of the source of the positive energy. AMBARTSUMIAN replied that he wished first to establish the existence of cases of positive total energy, and to investigate the circumstances where this occurs, before seeking a physical explanation of the original source.

Recent Investigations of Groups and Clusters of Galaxies

E. M. BURBIDGE AND G. R. BURBIDGE
University of Chicago

I. INTRODUCTION

EXCEPT for early work by Lundmark (1926, 1927, 1928) and an extensive investigation by Holmberg, (1937), who provided a catalogue of multiple galaxies, little work had been done in this field before about 1950. In 1952 Page published data from spectra of a number of double systems. This and later work is discussed by him elsewhere in this volume; we shall refer only to systems containing more than two galaxies in this paper. More recently some important work on multiple systems has been carried out by Zwicky; this has been summarized in a number of places, particularly Zwicky (1956, 1959). In addition, recession velocities for a number of members of groups have been obtained by Humason and Mayall and are published in the fundamental list by Humason, Mayall, and Sandage (1956). New impetus was given to such investigations by Ambartsumian (1957, 1958), who pointed out that multiple galaxies with separations of the same order of magnitude between the members would, like multiple stars of the Orion Trapezium type, be unstable configurations, even if the total energy of the system was negative. The time scale for instability in such configurations would be of the order of a few revolutions of the system, where one revolution is $\sim 10^9$ years. Ambartsumian further drew attention to the evidence for some multiple systems, such as the M81 group and Stephan's Quintet, having positive total energy, in which case the time for disintegration would be shorter and would be given by the time taken for the fast-moving members to traverse the dimensions of the system. Thus such multiples would be in a state of rapid evolution.

Since Ambartsumian's suggestion, Vorontsov-Velyaminov has published his Atlas of multiple galaxies (1960), obtained from his investigation of the Palomar Sky Atlas, which is a valuable source of information and from which we have chosen a number of systems for investigation with a view to testing Ambartsumian's hypothesis.

The observations that can be made in this field are necessarily limited. Information that one would like to have for any given group consists of:

(a) good photographs with a large reflector or a large Schmidt telescope from which the galaxies can be classified, their separations measured, and any luminous bridges or other connections between them or extensions from them detected;

(b) spectra of each member of the group from which radial velocities can be measured and spectral types ascertained;

(c) measures of colors and magnitudes of all members;

(d) study of the surrounding field to determine whether the group forms an isolated physical system and whether there are possible outlying members.

For most of the groups which we shall discuss, some data is available under (a), (b), and (d). In a number of groups we lack information in category (c). In Table I we list the groups that we shall discuss. These include some groups investigated by us, together with groups studied by other workers. Weak clusters and the Virgo cluster are discussed by others at this Symposium, consequently, apart from the Hercules and Coma clusters, we shall consider only small groups.

II. APPLICATION OF THE VIRIAL THEOREM

In the literature discussing the properties of these groups, some attempts have been made to test their short-term stability by applying the virial theorem (it being always remembered that the configurations will not be stable on a time scale longer than a few revolutions of the systems even if the virial theorem is satisfied). The virial theorem states that for the system to be stationary

$$2E + \Omega = 0, \quad (1)$$

where E and Ω are time averages of the kinetic and potential energies, respectively.

In applying this theorem, the observations necessarily restrict us to the instantaneous values of E and Ω . Also, only one component of the velocity of each galaxy can be measured, and this gives only one component of the kinetic energy after the velocity of the center of mass of the group has been determined by making some assumptions about the mean velocity of the group as a whole. Moreover, only the separations of the members projected on to the plane of the sky can be measured.

TABLE I. List of groups discussed.

	α (1950)	δ (1950)
Local Group	...	...
NGC 55	0 ^h 12 ^m 5	-39° 30'
NGC 383	1 04.6	+32 09
VV 116	9 36.3	-4 37
M81 (NGC 3031)	9 51.5	+69 18
NGC 3561	11 08.5	+28 58
VV 172	11 29.2	+71 05
VV 150	11 29.8	+53 12
Wild's system	11 44.6	-3 34
IC 3481	12 30.3	+11 40
NGC 6027	15 57.0	+20 54
Hercules	16 02.9	+17 54
Zwicky (Anon)	16 48.0	+45 35
NGC 6769	19 13.9	-60 35
Stephan's group	22 33.7	+33 42
NGC 7619	23 17.8	+7 56

Since in small groups one is dealing with 3–12 members, it is clear that we cannot hope to obtain estimates of the kinetic and potential energies which are not subject to considerable uncertainties.

There is the additional difficulty that in all but very few cases we cannot make any independent estimates of the masses of any of the galaxies in the group by, for example, measuring the rotations. In fact, this has been done in only two cases. Thus the method of analyzing groups has been either to assume that the virial theorem is satisfied and to use it to calculate the average masses of the galaxies, or to put in average values for the galactic masses based on other data and then to calculate the potential and kinetic energies and test whether Eq. (1) is satisfied.

Clearly no very definite conclusions can be drawn without reasonably good average values for the masses of galaxies being available. Individual masses have been determined now for some 20–30 galaxies with varying degrees of precision. The majority of these are spirals and irregulars, but there are data for a few of the brightest ellipticals and one or two S0 galaxies. The results indicate that the majority of spirals and irregulars have masses between 10^{10} and $4 \times 10^{11} M_{\odot}$, and that the much smaller sample of elliptical and S0 galaxies, excluding the dwarfs like M32 and the Fornax system, range from 10^{10} to $5 \times 10^{11} M_{\odot}$. The most massive galaxies known at present are M31 among the spirals, which has a mass of the order $4 \times 10^{11} M_{\odot}$, and M87 among the ellipticals, which has a mass of the order $10^{12} M_{\odot}$. The corresponding mass-to-light ratios range from about 1 to 20 for spirals and irregulars and about 10 to 50 for ellipticals. A complete list of these values has been given by one of us (Burbidge 1961).

Statistical analysis of double galaxies has also given information on average masses (Page 1952, 1959, 1960). Comparison of these results with data for single galaxies is discussed by Page elsewhere in this volume. However, we may just mention that the results given by Page (1960) for the masses of ellipticals and S0 galaxies ($4 \times 10^{11} M_{\odot}$) and spirals together with irregulars ($5 \times 10^{10} M_{\odot}$) tend towards the upper limits of the values from individual determinations. Average mass-to-light values published there are very different from the values obtained from individual galaxies in the sense that these ratios for the elliptical and S0 galaxies are far higher, and the values for the spiral together with the irregular galaxies are far lower.

With these data on mass determinations in mind we shall discuss in Sec. III each of the groups listed in Table I separately. Throughout this paper, when the recession velocity has been used to estimate distance, $H = 75$ km/sec per Mpc has been taken. All velocities that we give will be corrected for rotation of our Galaxy by the same formula as that used by Humason, Mayall, and Sandage (1956).

III. DESCRIPTION AND DISCUSSION OF GROUPS

We consider these not in the order of right ascension as listed in Table I, but in two divisions, first the compact small groups and groups whose members are connected with luminous bridges etc., and second, certain looser groups with more members. In the first division we arrange the objects roughly in decreasing order of the amount of information available.

A. Compact Groups and Groups with Interconnected Members

1. *Stephan's Quintet*

Ambartsumian (1958) pointed out that the velocity dispersion of the four members of this quintet which were measured by Humason (Humason *et al.* 1956) was large enough to suggest strongly that the system was unstable, in particular that NGC 7318b was escaping from the group. The four velocities for NGC 7317, 7318a,b, and 7319 are given in Table II with the apparent magnitudes. On the assumption that the masses of NGC 7317 and 7318a, which resemble ellipticals on the direct photographs, could be about $10 \times$ the masses of the remaining members, an assumption which helps to minimize the masses necessary for stability, the virial theorem was found by Burbidge and Burbidge (1959a) to be satisfied only by the very large masses and mass-to-light ratios given in Table II. In this analysis data on the fifth member of the group, NGC 7320, was lacking and it was taken into account by multiplying the kinetic energy by $5/4$. We concluded that the virial theorem was not satisfied.

Another analysis of the same data was carried out by Limber and Mathews (1960). By making special assumptions, and by taking into account (and probably overestimating) the galactic absorption, they concluded that the virial theorem could be satisfied with the masses and mass-to-light ratios also given in Table II. Though the latter are somewhat reduced, the masses are still very large.

Recently two spectra of the fifth member, NGC 7320, have been obtained and they give the remarkably low recession velocity of +1073 km/sec (Burbidge and Burbidge 1961a; a photograph is published in that paper). This is a startling result and we must conclude that either (i) NGC 7320 is a foreground galaxy, or (ii) NGC 7320 is literally exploding away from the other members.

We estimate that the chance of (i) being true is less than 1 in 1500. Case (ii) is a conclusion that falls outside conventional ideas on the dynamics of clusters and groups.

The tilt of the $H\alpha$ emission lines was measurable in NGC 7320 and from it we can estimate the mass to be roughly $10^{11} M_{\odot}$ if it is of the distance of the rest of the group and $1.7 \times 10^{10} M_{\odot}$ if it is a foreground galaxy at a distance corresponding to a redshift of +1073 km/sec.

TABLE II. Stephan's Quintet.

NGC	Velocity (km/sec)	m_{pg}	For stability			
			B^2	L and M	$M/M_\odot$	M/L
			$10^{12} \times$		$10^{12} \times$	
7317	+7014	15.3	2.8	300	1.0	62
7318a	+6916	14.8	4.5	300	1.7	62
7318b	+5916	14.9	0.3	21	0.5	20
7319	+6935	13.7	0.8	21	1.5	20
			From rotation:			
			$10^{11} \times$			
7320	+1073		0.2-1.0			

In the latter case the radius out as far as measures of the $H\alpha$ line extend would be only 1.6 kpc, and the galaxy would be very dwarfish with a very high mean density. If NGC 7320 is a foreground galaxy, the position regarding stability of the group remains as discussed above, with four members and not five (Stephan's Quartet). It appears to us that the masses necessary for stability are much too large and such a total mass is not likely to be provided by the intergalactic matter within the group.

2. IC 3481, Anon, and IC 3483

This triple system has been discussed by Zwicky (1956) and photographs are published there. Velocities of the three galaxies are given in Humason *et al.* (1956); they are, respectively, +7011, +7229, and +33 km/sec. Zwicky's photographic magnitudes for IC 3481, Anon, and IC 3483 are, respectively, +15.0, +16.0, and +15.6. A luminous bridge connects IC 3481 and the smaller, fainter, anonymous galaxy, and another long luminous structure extends from the latter towards IC 3483; whether it actually joins onto IC 3483, or whether it is one of the long extensions sometimes seen to terminate apparently in free space, is the important question here. The very large difference between the velocities of IC 3483 and of the other two galaxies leads, as in Stephan's group, to the situation that either IC 3483 is a foreground galaxy or it is exploding away from the other two. The probability of its being a foreground galaxy is greater than in the case of Stephan's group since it is in the region of the Virgo cluster. However, if it belongs to the Virgo cluster it is a dwarf galaxy of low absolute magnitude.

3. VV 116 Group

This quintet was discovered by Vorontsov-Velyaminov (1960) and it has been investigated by us (Burbidge and Burbidge 1959b, 1961b; photographs are published in both references). It is a compact group of two ellipticals and three spirals, and a narrow fairly bright filament that runs between the fainter of the two of the spirals. This filament might alternatively be a faint

sixth member of the group. Magnitudes are not available for these galaxies, but we have named them A-E in order of decreasing apparent brightness on our plates. Velocities for four of the five members have been obtained, and they are: A (elliptical), +6356 $\pm$ 40 km/sec; B (elliptical), +6576 $\pm$ 40 km/sec; C (spiral), +6631 $\pm$ 30 km/sec; D (spiral), +6230 $\pm$ 30 km/sec. Spiral C has very strong emission lines; spiral D has less strong emission. For D it was possible to obtain an estimate of the rotation from the tilted $H\alpha$ emission line.

By assuming that the masses of the ellipticals were $10 \times$ those of the spirals, and by allowing for the unmeasured galaxy E by multiplying the kinetic energy by 5/4, we found that the virial theorem could be satisfied by masses of $5.5 \times 10^{11} M_\odot$ for the ellipticals and $5.5 \times 10^{10} M_\odot$ for the spirals, i.e., a total mass of $1.3 \times 10^{12} M_\odot$. Here we can make a direct comparison with the mass of D independently estimated from its rotation; the value thus obtained is $3.3 \times 10^{10} M_\odot$. This value is in sufficiently good agreement with the average value of $5.5 \times 10^{10} M_\odot$ obtained from the virial theorem for us to conclude that the system is stable, although ultimately, of course, it will suffer the rearrangement of kinetic energy that occurs in trapezium-like systems resulting in disruption of the compact configuration.

Two final comments must be made: (i) if galaxy E is found to have a high relative velocity, then the group may still prove to be unstable. (ii) The reason why it appears to be stable on the evidence of the velocities given above is that the galaxies are very close together—their average three-dimensional separation is only about 37 kpc. The outer regions of at least four of them appear to be interconnected.

4. Wild's Triple System

This remarkable group of three irregular barred spirals with luminous connecting bridges was discovered by Wild (1953), and the photograph is published by Zwicky (1956). It has been discussed by Zwicky and Humason (1961), who give velocities, magnitudes, and discuss the total mass and mass-to-light ratio. The velocities are +5108, +5008, and +5396 km/sec. Rotations of all three galaxies were detected, and periods of rotation were given by Zwicky and Humason. If measures of the extent of the tilted spectrum lines could also be made, the masses could be estimated individually, but this has not yet been done. The apparent photographic magnitudes are, in the order given above, +14.5, +14.1, and +15.0. By application of the virial theorem, the total mass is found to be very approximately $3 \times 10^{12} M_\odot$ and the average mass-to-light ratio for the three galaxies is of the order of 100. If the luminosity of the intergalactic bridges, which were also measured by Zwicky and Humason, are included, the total average mass-to-light ratio is 70.

Although the application of the virial theorem becomes much more uncertain when there are only three

members and one has to derive true separations and space velocities by putting in factors which only apply statistically when one has a large number of galaxies; it is clear that the average masses and the mean mass-to-light ratios are much higher than would be reasonable. Unless about 90% of the mass of the system is in the form of dark unseen material, we conclude that this group is not stable but is rapidly disintegrating.

5. Anon Triple ($16^h 48^m 0, +45^\circ 35'$)

Zwicky (1939) published a photograph of this system, and it was studied by Zwicky and Humason (1961), who found velocities and types of +9418 km/sec (S0), +9405 km/sec (Sa) and +9449 km/sec (S0 or SB0). This remarkably small line-of-sight velocity dispersion means that the virial theorem can be satisfied (to within the uncertainties due to projection) by quite small masses. The total mass is only $9 \times 10^{10} M_\odot$ and the corresponding mass-to-light ratio is about 2. Thus in this case there is reason to believe that the group is stable without the addition of intergalactic matter. The mass of the Sa galaxy could be independently estimated from the rotation that Zwicky and Humason detected.

6. NGC 6769, 6770, 6771

This system has been photographed at the Radcliffe Observatory and reproduced in the Cape Atlas of Nebulae (plate 21). Radial velocities have been obtained for all three members by Evans and Wayman (1958). They are NGC 6769 (Sb), +3480 km/sec; NGC 6770 (SBb), +3580 km/sec; NGC 6771 (Sa or S0), +4070 km/sec. The virial theorem gives a total mass of approximately $4 \times 10^{12} M_\odot$, which is larger than the probable sum of the masses of the three galaxies by a factor of about 30. Individual masses could probably be obtained for these galaxies by measuring their rotations.

7. NGC 6027 Sextet

This group of galaxies in Serpens was discovered by Seyfert (1951). Three members are spirals, the two brightest are S0 or Sa, and one is a peculiar diffuse irregular galaxy. The velocities of the two brightest members were measured by both Humason and Mayall (Humason *et al.* 1956); they are well determined and are +4568 and +4160 km/sec. Magnitudes were measured by Seyfert; they range from 14.7 to 16.9. No analysis can be carried out until more measures of velocity are available. We note that the difference of about 400 km/sec between the two measures is of the same order as the extreme range of velocities measured in VV 116. The distance of the system is about 58×10^6 pc, and the largest dimension of the group is only about 27 kpc (not corrected for projection) so that the galaxies are very small.

8. VV 150 Group

This is a compact isolated group discovered by Vorontsov-Velyaminov, close to NGC 3718, which is a foreground galaxy. We have obtained spectra for two of the galaxies; these give velocities of +7900 km/sec and +6700 km/sec. The distance is thus approximately 10^8 pc, so that the diameter of the group is about 60 kpc. The brightest member, a distorted elongated spiral, shows in its spectrum the broad emission features H, [NII] $\lambda\lambda 6583, 6548$, [OIII] $\lambda\lambda 5007, 4959$, and [OI] $\lambda 6300$. If the broadening is due to the Doppler effect, produced by random velocities of gas in the galaxy, the width of H α gives a value of about 800 km/sec in the nucleus. Thus this is a galaxy in the rare category described by Seyfert (1943) of which only about 12 examples have been discovered. A third member of the VV 150 group, whose continuous spectrum was recorded but in which no spectral features were seen, has a light distribution indicating a very blue color. No colors or magnitudes have been measured.

9. VV 172 Group

This remarkable chain of five galaxies, discovered by Vorontsov-Velyaminov, has been photographed (Burbidge and Burbidge 1959b) and spectra of two of its members have now been obtained, but velocities have not yet been measured. The geometrical configurations strongly suggests that the system is not stable.

B. Loose Groups Containing More Members

1. Local Group

Membership and velocities in the local group of galaxies have been discussed by Humason and Wahlquist (1955). Clearly the bulk of the mass of the group is contributed by NGC 224 and our Galaxy. Humason and Wahlquist pointed out that in using the virial theorem the numerical values are very sensitive to the value of the velocity of the center of the galaxy relative to the local group, which depends on the velocity of rotation of the sun relative to the center. Kahn and Woltjer (1959) used a value of 216 km/sec for the circular velocity of the sun and concluded that unless the volume of the group was filled with intergalactic matter to provide the large part of the total mass, the group would be unstable. Godfredson (1961) reanalyzed the same data and showed that for a circular velocity of 260 km/sec at the sun, the local group could be stable without intergalactic matter being present. Thus the result is inconclusive.

2. NGC 3031 Group

The four bright galaxies in this group are NGC 2976, 3031, 3034, and 3077, and there are fainter members also. Holmberg (1952) discussed a way of obtaining the mass of NGC 3031 from motions of its satellites; this is

in reasonable agreement with the value of about $1.5 \times 10^{11} M_{\odot}$ obtained from its rotation. A mass of about $1.5 \times 10^{10} M_{\odot}$ is estimated for the mass of NGC 3034 from its rotation. Ambartsumian (1958) argued that the system as a whole could not be stable because of the large velocity (338 km/sec) of NGC 3034 relative to the rest of the group.

3. NGC 55 Group

De Vaucouleurs (1959) has argued that NGC 45, 55, 247, 253, 300, and 7793 form a physical group about 8×10^5 pc in diameter. The group is only about 2.5×10^6 pc distant and the galaxies are all spirals. The argument for their forming a physical group is based on their distribution among the other bright galaxies in the southern hemisphere. From the radial velocities, de Vaucouleurs showed that the virial theorem would not be satisfied unless the average masses of the galaxies were in excess of $10^{12} M_{\odot}$, with a mass-to-light ratio greater than 500. Thus if the galaxies form a physical group, it must be expanding, unless there is a large concentration of intergalactic matter.

4. NGC 7619 Group

This is a group of five galaxies which form part of the Pegasus cluster (Hubble and Humason 1931; Edson and Zwicky 1941); they are the brightest members of it. The group has been studied by Hodge (1961). The types and velocities are NGC 7611 (S0), +3579 km/sec; NGC 7617 (S0), +5068 km/sec; NGC 7619 (E3), +3953 km/sec; NGC 7623 (E4), +3659 km/sec; and NGC 7626 (E1), +3553 km/sec. The group has an approximate diameter of 3.5×10^5 pc and the separations between the galaxies are on the average greater than 220 kpc. Consequently, Hodge finds that the virial theorem is satisfied only for galactic masses in the range 10^{12} – $10^{13} M_{\odot}$ and mass-to-light ratios of the order of 300. There is some difficulty in estimating the total population of the cluster containing the brightest galaxies listed by Hodge. Probably the estimate by Zwicky (1959, p. 63) of about 60 members is the best. If the five that have been measured form a good sample of the whole, and if the cluster has little central concentration, as seems to be the case, then we may conclude that probably the whole group is unstable, unless a considerable amount of dark material is present.

5. NGC 383 Group

NGC 375, 379, 380, 382, 383, 384, 385, 386, and 388 are a group of nine galaxies for which velocities are given by Humason *et al.* (1956). They form part of what Hubble and Humason (1931) called the Pisces group, which, they suggested, consisted of about 25 elliptical galaxies. They are the brightest members. Zwicky (1937) has suggested that the group is contained in a

large and elongated cluster extending over 15 square degrees or more. The nine galaxies listed above have rather a large velocity dispersion, and application of the virial theorem gives galactic masses in the range 10^{12} – $10^{13} M_{\odot}$ and mass-to-light ratios of about 260 (Burbidge and Burbidge 1961c). Thus if these nine galaxies, together with the other 16, form a separate physical group, as was suggested by Hubble and Humason, then the group is probably unstable unless there is much dark material present.

6. Hercules Cluster

This is a loose irregular cluster containing about 75 bright members and a large number of faint galaxies. Of the bright members, about 70% are spirals and irregulars and, there are a number of multiples. From a sample of 15 measured radial velocities, the total kinetic energy of luminous matter was estimated and the potential energy was obtained from measures of the separations of the 75 bright objects. Application of the virial theorem then showed that it could be only satisfied for an average galactic mass of about $10^{12} M_{\odot}$ (Burbidge and Burbidge 1959d). This fact, together with the irregular shape, lack of concentration, and proportion of irregular and peculiar spirals, led us to conclude that the cluster was probably unstable on a time scale of $\sim 2 \times 10^9$ years. Of course, the presence of a large amount of uncondensed material in a cluster like this cannot be ruled out; luminous intergalactic matter in the form of the curious structure IC 1182–84 was discovered in this cluster by Ambartsumian and Schachbazian (1957). Finally, this area of the sky has a number of concentrations of galaxies and it is possible that Hercules cluster discussed here may be part of a large aggregate.

7. Cluster around NGC 3561

An interesting multiple galaxy at $11^h 8^m 1, +29^{\circ} 2'$, that was discussed by Zwicky and Humason (1960), lies in a cluster that contains a remarkable number of mutually interacting galaxies and peculiar systems. The redshift is about +8700 km/sec. This multiple has not been included among the objects in the first part of this section because of its apparent membership of this cluster. The cluster is extremely interesting and should be investigated further; the types of object in it resemble the types found in the Hercules cluster.

IV. DISCUSSION AND CONCLUSIONS

These results, showing the apparent instability of many groups of galaxies, are not easy to understand. We must accept first that not all groups and clusters exhibit the same dynamical behavior. Then, it must be noted that for each group there are the following uncertainties:

(a) We have no arguments independent of the position and recession velocity of a galaxy to establish with complete certainty that it is a member of a given group.

(b) If a system appears to be unstable, we can still invoke the presence of unseen matter to stabilize it. For some clusters it is also possible, although in general not very probable, that we may confuse two systems lying one behind the other and analyze them as a single physical group. In the case of the Virgo cluster it is being argued by some that this indeed consists of a number of separate physical groups of galaxies.

(c) It is possible that the galactic masses determined for comparatively nearby systems are not applicable to the galaxies in groups and clusters at greater distances. This is hardly acceptable, however, if one continues to accept the cosmological principle.

It appears that arguments concerned with the uncertainty in measurement of radial velocity and in the assumptions made in applying the virial theorem are somewhat downgraded by the number of systems for which data are now available. At least some investigation has now been made for four categories:

(i) Small groups such as Stephan's Quintet, VV 116, and some triples.

(ii) Comparatively weak clusters, such as the group around NGC 383 (Pisces group), up to richer clusters like the Hercules cluster.

(iii) The rich, irregular, and noncondensed Virgo cluster.

(iv) Highly condensed symmetrical clusters like the Coma and Corona Borealis clusters.

Among the small groups, some may be stable, while the visible members of others do not satisfy the virial theorem. In the latter, the discrepancy is so large as to make it highly improbable that enough dark matter can be present to stabilize them, for the total volume of such groups is small and the density of uncondensed matter would have to be comparable to the density of interstellar material in a spiral galaxy. With such a density, the whole volume of the group might then be expected to be filled with condensations of stars. If systems such as this are indeed unstable, then the time scale for disruption lies in the range 10^8 – 10^9 years.

There is no doubt that most astronomers have felt this short time scale to be the greatest difficulty in the way of accepting the conclusion that some groups and clusters may be unstable. This objection is based on the widely accepted idea that in evolutionary cosmologies all galaxies must have ages comparable to the age of the universe. Because of the trend of the results discussed in this paper, this viewpoint needs to be critically examined.

No age determinations are available for any galaxy other than our own, where a minimum age near 2×10^{10} years is the present value from the ages of globular clusters. If all galaxies have to be at least as old as

$\sim H^{-1}$, where H is the Hubble constant, then probably the only way to explain the occurrence of expanding systems is by supposing that they are the result of multiple collisions between field galaxies. As was shown by Holmberg (1937) and Zwicky (1939), this demands very long time scales $\gg H^{-1}$. Such time scales are unacceptable in any of the cosmological models usually discussed (cf. Bondi 1952), including Lemaitre's, where expansion sets in as soon as condensation into galaxies begins, and the steady-state model, where the average age of galaxies is $\sim (3H)^{-1}$.

Even if one leaves the cosmological question open, such very large ages for galaxies are not supported by the observational evidence. The spectral characteristics and colors of galaxies so far observed indicate that the time scale for evolution of the stars which form the bulk of the mass of a galaxy is not greater than about 5×10^{10} years, unless star formation has gone on continuously without much diminution. If it had, the large numbers of white dwarfs would lead to high mass-to-light ratios and the velocity dispersion of stars in the centers of elliptical galaxies so far measured do not support this. It is also unlikely that the time which elapses between the separation of a protogalaxy from the intergalactic medium and the formation of the first stars is exceedingly long ($\gg H^{-1}$). This time is determined by the decay rates of quantities such as the primordial magnetic field or turbulence, both of which may inhibit star formation. Thus we conclude that the formation of groups and clusters by capture is very improbable.

Let us accept therefore, that groups of galaxies formed as such out of the intergalactic medium, and that expanding groups do indeed have short time scales, and let us examine the evidence for galactic ages from the opposite viewpoint: Can the members of such groups have evolutionary ages as small as the expansion ages?

Apparently all types or combinations of types of galaxy can occur in small groups—irregulars, spirals, or, as in VV 172, ellipticals. The rate at which a galaxy evolves should depend predominantly on the mass function or luminosity function of the stellar population. There is observational evidence that some galaxies are young in evolutionary terms, e.g., some of the systems in the atlas of Vorontsov-Velyaminov such as the irregular agglomerate of HII regions in NGC 2444-5 (Burbidge and Burbidge 1959c) or single irregulars such as NGC 4449 (Burbidge 1961). That these are young is based on two arguments—their irregularity and the high proportion of high-luminosity stars and uncondensed gas in the form of HII regions. Some of these may be as young as 10^8 years or less. (It must be remembered, however, that what are apparently very old stars are present in the Magellanic Clouds. Such stars might also be present in these systems, in which case we would have to conclude that at least part of the material condensed long ago.) Presumably a rather long time, $\sim 5 \times 10^8$ years, is required for such objects to

each a regular configuration (the flattening depending on the angular momentum). It seems to us, therefore, not unreasonable that the spiral and irregular galaxies in some groups may have evolutionary ages which are compatible with the time scales for local expansion. Can this also be true for the ellipticals?

Some indication of the stellar population of an elliptical can be obtained from its integrated spectrum or colors, if these are measured on some more detailed system than the U, B, V system and correlated with spectra for a good sample of galaxies. No such information is available in, for example, the galaxies in VV 172. In practice, any faint galaxy which has a comparatively bright amorphous nucleus without dust will be classified as an elliptical. For example, NGC 7317 and NGC 7318a in Stephan's Quintet were classified in this way though the structure of the material around them and the connections to the other galaxies make this somewhat questionable. Ellipticals are thought to contain little gas, so that star formation is rather complete. It is unlikely on general grounds that all galaxies have the same stellar luminosity function. Thus, although there is a common *belief* that all ellipticals contain old disk population and population II stars in an advanced state of evolution, the only observational test of age appears to be the following.

If a galaxy of yellow or reddish color and late spectral type contains main-sequence stars up to about solar type but none of the more massive stars, together with a population of K- and M-type giants, then it must be old, since a star of about a solar mass takes $\sim 15 \times 10^9$ years to evolve into a giant. If, on the other hand, there is no spectral evidence for giants of small mass, then such a galaxy might contain only main-sequence stars together with stars that are still contracting onto the main sequence, and be no older than a few hundred million years. This would require that the formation of whatever heavy elements are detected in the spectra can be accounted for by a very rapid rate of star formation and turnover of material from stars to gas in the very early history of the galaxy, and that the mass function for the formation of stars is now such that very few massive stars are produced (e.g., conditions might be right for rapid fragmentation of any condensation).

The contribution of giants to the luminosity of a stellar population with a reddish integrated color and late-type integrated spectrum is indicated by the relative strength of C/V and has been discussed by Morgan and Mayall (1957). The existence of a large dwarf population may be indicated by the spectra of some galaxies that have very strong $N\alpha$ D lines, as shown by Spinrad (1961). If giants are present, then it is important to determine from the integrated spectrum the upper extent of the main sequence, since giants that have evolved from A-type stars and those from further down the main sequence have similar properties. Galaxies with populations of giants and main-sequence stars not earlier than late F or early G must have ages

at least $\sim H^{-1}$, so that the presence of these in groups that are unstable on a short time scale would be hard to explain. According to present evidence, the nuclei of M81 and M31, and M32, together with some ellipticals in the Virgo cluster, satisfy such criteria. This suggests that the local group, the M81 group, and the Virgo cluster should not be unstable on a short time scale. M81 and its satellites appear to form a stable system, but M82 appears to be escaping from the group as a whole. The stability of the local group is indeterminate. In the case of the Virgo cluster, a number of conflicting opinions have been expressed. Possibly the cluster is evolving dynamically but is not unstable, or else it may consist of a number of physically separate subclusters, as suggested by Morgan and de Vaucouleurs, so that the stability of each group should be investigated separately.

For the small groups that apparently do not satisfy the virial theorem, the existence of stellar populations indicating ages far in excess of the expansion ages is not established. To get spectra of sufficient dispersion in distant groups would be difficult, but detailed color measures should be easier to obtain, and if these could be correlated for the nearer galaxies with spectral criteria such as CN strength, $N\alpha$ strength, and indicators of the brightest main-sequence stars, they might enable lower limits to be set on the ages. Until this is done, we conclude that the expansion phenomenon cannot be ruled out on the basis of the short time scale, whatever type of galaxy is contained in the group.

Before going on to discuss categories (ii), (iii), and (iv), it should be mentioned that if explosive expansion is found to occur in small groups (cf. the question raised in the case of NGC 7320 in Stephan's Quintet and IC 3483 in Zwicky's triple system), this would be very difficult to explain according to current concepts and all the time-scale problems would be greatly enhanced. Only Ambartsumian (1958) has made any attempt at discussion of this problem. A fundamental problem is that of the origin of the energy which would have had to be present.

The discussion of groups in class (i) applies equally to those in (ii) so we shall add only one or two further points. Class (ii) contains groups for which errors in carrying out the analysis are most likely to occur because of the difficulty in deciding which are field and which are cluster galaxies. The Hercules cluster is a particularly interesting example because it contains a high proportion of galaxies which may reasonably be considered to have evolutionary ages compatible with the expansion age. The geometrical configuration suggests that the group is unstable. The cluster may be physically associated with a complex of subclusters in its neighborhood, and large-scale investigation of the whole area should be made.

The Virgo cluster [category (iii)] is discussed by others at this conference. We therefore turn finally to the rich, centrally condensed clusters (iv). The method

of the virial theorem has been applied to the Coma cluster by a number of authors (e.g., Zwicky 1956; Oort 1958; Burbidge and Burbidge 1959d). The radial velocities of about 50 members have been measured (Humason *et al.* 1956; Mayall 1960). For the cluster to be stable the average masses of the bright galaxies must be $\sim 10^{12} M_{\odot}$. If the average masses of the elliptical and S0 galaxies are about $4 \times 10^{11} M_{\odot}$, as suggested by Page, then the discrepancy is only by a factor of order 2. If the masses are rather lower, as is suggested by the masses obtained for a few individual ellipticals, the discrepancy may be rather worse (nearer to 10). Since the symmetrical distribution of the galaxies suggests that this cluster may be stable, dark material in the form of intergalactic matter or evolved galaxies, or large numbers of dwarf galaxies, may comprise from 50 to 90% of the mass. We run into considerable difficulties associated with the time scales in clusters of this type, but in the opposite direction to that encountered in the apparently expanding groups.

The problem has been stated by Zwicky (1939, 1960). He finds that clusters like the Coma cluster have a radial distribution of bright galaxies corresponding to the density distribution in an isothermal gravitational gas sphere. He believes that the velocity dispersion of galaxies of equal brightness is constant throughout the cluster, that bright and faint members are partly segregated, with the bright ones concentrated to the center, and that such clusters at all distances, and hence clusters with quite a spread in ages, are remarkably alike. Is sufficient time available for this state to have been achieved? The relaxation time for such a cluster can be calculated from the formulas given by Chandrasekhar (1942). The relaxation time depends critically on the parameters which are assumed for a cluster of the type of the Coma cluster. It appears that this relaxation time lies in the range 5×10^{11} – 10^{13} years. In making this estimate, the galaxies have been treated like stars, in that inelastic collisions are not considered. Despite the uncertainties and the effect of inelastic collisions, it seems that time scales for the achievement of the stationary state of affairs described by Zwicky must be very long, $\gg H^{-1}$. This is so even without going so far as to say that such a cluster must have been formed by capture of individual galaxies on a very long time scale.

The observational basis of Zwicky's belief that clusters of this type have reached a stationary state must be discussed. Zwicky has established that the density distribution in the Perseus and Coma clusters correspond to Emden spheres. However, the uniformity of velocity dispersion needs further testing, as only for the Coma cluster have a moderate number (50) of radial velocities been published. Only five are published for the Perseus cluster and only eight for the Corona Borealis cluster. Even 50 velocities is not many for establishing the constancy of the velocity dispersion

throughout the cluster. There is clearly some segregation between the brighter and fainter members of the nearer clusters of this type, but it is not clear what proportion of the fainter galaxies are background objects. That clusters at all distances are remarkably alike appears to be at variance with Abell's conclusion (1961) that the linear extents of different clusters vary far less than their total populations.

Thus we conclude that the argument for a very long time scale is far from conclusive on the general scale but for nearby clusters the formation and evolution to their present form is not easy to explain and may well demand time scales $\gg H^{-1}$.

The problems which are encountered in discussing the forms and stability of clusters and groups are predominantly those of the time scales involved. If, as has been suggested by Abell (1961), all galaxies are members of clusters, then the investigations that have been made along the lines described here are only a beginning. For example, many of the weak clusters may well form part of larger physical aggregates whose membership is very difficult to determine. Even here the comparatively large velocity dispersions encountered and the lack of central concentration indicate that the argument of the virial theorem may lead to the same type of result. It may well be that the large extended aggregates are dynamically stable because there is a sufficient concentration of intergalactic matter with density maxima in such regions. On the other hand, we cannot rule out, on the basis of the evidence at hand at present, that local expansion is occurring in many regions. In most cosmological investigations it has been assumed that the clusters of galaxies partake of the general expansion but do not expand themselves.

It is of interest, finally, to mention one cosmological model which may allow of what we interpret to be local shears and expansions. This is based on the solution of Einstein's field equations discovered by Gödel [(1949); see also Chandrasekhar and Wright (1961)]. Gödel's universe, which is rotating, differs from the Friedmann-Lemaître solutions of relativistic cosmology and the steady-state solution in that it does not allow the possibility of defining a universal time. [The concept of universal time is contained in Weyl's postulate (Robertson 1933), which is an assumption based on the observed fact that there exists in any region of space-time a mean motion that represents the actual motions to within relatively small and unsystematic deviations.] The concept of universal time is by now deeply imbedded in our thinking on cosmological questions. However, Gödel's models should be investigated further. In such a universe, ages derived in our Galaxy would necessarily have no relation to H^{-1} . The cases given by Gödel (1949) are rotating universes which are static and spatially homogeneous and have a cosmical constant < 0 . Such solutions are not applicable to the real universe which is expanding. However, in a later paper

Gödel 1950) he has stated that solutions exist that satisfy the condition of homogeneity in space, with space finite, and density not constant, i.e., models that allow expansion.

ACKNOWLEDGMENT

This research has been supported in part by a grant from the National Science Foundation.

REFERENCES

- Abell, G. 1961, *Proc. Santa Barbara Conf.*
- Ambartsumian, V. A. 1957, *Izvest. Akad. Nauk Armyan Fiz. Mat., Estestven. i Tekh. Nauki S.S.R.* **11**, 1-20.
- . 1958, Solvay Conference Report, Brussels, Belgium, p. 241.
- Ambartsumian, V. A. and Shachbazian, R. K. 1957, *Doklady Akad. Nauk Armyan S.S.R.* **25**, 185.
- Bondi, H. 1952, *Cosmology* (Cambridge University Press, New York).
- Burbidge, E. M. 1960, *Proc. La Plata Conf. Stellar Evolution.*
- Burbidge, G. R. 1961, *Proc. Princeton Conf. Interstellar Matter in Galaxies.*
- Burbidge, E. M. and Burbidge, G. R. 1959a, *Astrophys. J.* **130**, 15.
- . 1959b, *ibid.* **130**, 23.
- . 1959c, *ibid.* **130**, 12.
- . 1959d, *ibid.* **130**, 629.
- . 1961a, *ibid.* **134**, 244.
- . 1961b, *ibid.* **134**, 248.
- Chandrasekhar, S. 1942, *Stellar Dynamics* (University of Chicago Press, Chicago, Illinois), p. 202.
- Chandrasekhar, S. and Wright, J. P. 1961, *Proc. Natl. Acad. Sci.* **47**, 341.
- Edson, J. B. and Zwicky, F. 1941, *ibid.* **27**, 366.
- Evans, D. S. and Wayman, P. A. 1958, *Monthly Notices Astron. Soc. South Africa* **17**, 137.
- Gödel, K. 1949, *Revs. Modern Phys.* **21**, 447.
- . 1950, *Proc. Intern. Congress Math.* (Cambridge, Massachusetts), Vol. 1, p. 175.
- Godfredson, E. A. 1961, *Astrophys. J.* **134**, 257.
- Jodge, P. 1961, *ibid.* **134**, 262.
- Johmberg, E. 1937, *Lund. Ann.* **6**.
- Jubbe, E. and Humason, M. L. 1931, *Astrophys. J.* **74**, 131.
- Humason, M. L. and Wahlquist, H. D. 1955, *Astron. J.* **60**, 254.
- Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *ibid.* **61**, 97.
- Lahn, F. D. and Woltjer, L. 1959, *Astrophys. J.* **130**, 705.
- Limber, D. N. and Mathews, W. G. 1960, *ibid.* **132**, 286.
- Lundmark, K. 1926, *Arkiv. för Math. Astron. Och Fysik* **19B**, No. 9.
- . 1927, *Uppsala Medd.* No. 30.
- . 1928, *ibid.* No. 41.
- Mayall, N. U. 1960, *Ann. d'Astrophys.* **23**, 344.
- Morgan, W. W. and Mayall, N. U. 1957, *Publs. Astron. Soc. Pacific* **69**, 291.
- Port, J. H. 1958, Solvay Conference Report, Brussels, Belgium.
- Page, T. L. 1952, *Astrophys. J.* **116**, 63.
- . 1959, *Astron. J.* **64**, 53.
- . 1960, *Astrophys. J.* **132**, 910.
- Peeyfert, C. K. 1943, *ibid.* **97**, 28.
- . 1951, *Publs. Astron. Soc. Pacific* **63**, 72.
- Spinrad, H. 1961, thesis, University of California, Berkeley, California.
- de Vaucouleurs, G. 1959, *Astrophys. J.* **130**, 718.
- Vorontsov-Velyaminov, B. A. 1960, *Atlas of Peculiar Galaxies* (Moscow).
- Wild, P. 1953, *Publs. Astron. Soc. Pacific* **65**, 202.
- Zwicky, F. 1937, *Proc. Natl. Acad. Sci.* **23**, 251.
- . 1939, *ibid.* **25**, 604.
- . 1956, *Naturwissenschaften.*
- . 1957, *Morphological Astronomy* (Springer-Verlag, Berlin).
- . 1959, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin), Vol. 53, p. 373.
- . 1960, *Publs. Astron. Soc. Pacific* **72**, 365.
- Zwicky, F. and Humason, M. L. 1961, *Astrophys. J.* **133**, 794.

DISCUSSION

VAN DEN BERGH noted that the radial velocity of NGC 7320 is close to that of the nearby supergiant spiral NGC 7331, which is known to have at least one other companion. Therefore, NGC 7320 may well be a member of the NGC 7331 group (or a larger cluster, as preferred by DE VAUCOULEURS) rather than of Stephan's Quintet. MRS. BURBIDGE remarked that it would still have an abnormally large mean density if it is a foreground object at a distance corresponding to its velocity. POVEDA countered that if it is considered to be at the distance of Stephan's Quintet, NGC 7320 has M/L less than 1, whereas if it is treated as a foreground object $M/L=4.5$, which is more in agreement with the M/L ratio to be expected from its morphological type, Sc. Further discussion revealed that the magnitude had not been measured accurately enough for these figures to be reliable, but that the lack of tidal distortion in NGC 7320 argues strongly for its being far from the rest of Stephan's Quintet.

VORONTSOV-VELYAMINOV felt that this reason is insufficient because many close pairs and groups, undoubtedly close in space, show no "tidal" distortion. As explained in his paper below, he believes that when distortions occur, they are due to some nongravitational influence—not to tidal action—because the observed distortions are not related to separations, masses or luminosities.

G. BURBIDGE noted that if NGC 7320 is a foreground object, the mass discrepancy from applying the virial theorem still remains in "Stephan's Quartet." SPINRAD mentioned an empirical relation he has discovered between absolute magnitude and strength of the D lines in the spectra of galaxies, which might be used to distinguish foreground and background objects. However, the spectra of NGC 7320 obtained to date show no absorption lines.

LIMBER agreed with the BURBIDGES that it did appear improbable *a priori* that NGC 7320 was a foreground object, but he argued that it seemed at least as improbable that this object was a physical member of the group since, according to the measured velocity difference, NGC 7320 can have had its presently observed excess velocity as a member of the group for only a few million years—an interval that is certainly very much less than the ages of the members of the group. This very short characteristic time scale—within which this one galaxy had to be accelerated to a velocity of about 5500 km/sec with respect to the rest of the quintet—is also only a few percent of the period of galactic rotation that would follow for this object from the BURBIDGES' observations if it is at the distance of the other members. Thus, if it is a member of the quintet, one might well expect NGC 7320 to show marked distortions in form. Further, under these conditions it appears questionable whether any attempt to derive the mass of NGC 7320 on the assumption of circular internal motions is meaningful.

In answer to McCREA, MRS. BURBIDGE described her method of computing the gravitational potential energy of a group from projected separations assuming random orientations, and admitted that it is more uncertain for a small group than for a large cluster. G. BURBIDGE added that the mass discrepancy from applying the virial theorem still remains, in several cases, when projection factors are all taken in the most favorable extremes.

NEYMAN noted a bias and infinite variance in this method of estimating the potential energy due to chance lineups, with one galaxy directly behind another or nearly so. He described a better estimate obtained by simply measuring the projected positions of all galaxies in the cluster, and stressed that assumptions of symmetry and of some statistical law of space separations of the galaxies in a group or cluster must be made in

either case. More serious is the *selection* of galaxies for velocity measurement. Since the luminosity is correlated with the mass of a galaxy, velocities have been systematically measured for the more massive galaxies of each type, and this may strongly influence conclusions from a statistical law like the virial theorem.

McCREA asked how the virial theorem is applied when there are subsystems such as close binaries within a cluster. LIMBER replied that, if the total energy is negative—if the whole system is stable—twice the time average of the total kinetic energy is equal to the time average of the potential energy, whether there are subsystems or not.

SCHUECKING commented on the possible use of Gödel's rotating cosmological model, pointing out that it can only change the time scale for objects very far from the observer, at least 10^9 light years.

On the Possibility of Explosive Formation of Galaxy Patterns

B. VORONTSOV-VELYAMINOV

Sternberg Institute, Moscow, U.S.S.R.

AMBARTSUMIAN'S conception is this (1958): (1) Components of multiple galaxies and of clusters were formed together. Only an insignificant minority of them could have been formed by capture in triple encounters.

(2) The configurations of groups are predominantly of the unstable type. The measured velocities in some groups and clusters seem to give evidence of the disintegration of these systems.

(3) Some radio galaxies are systems where the division of the nuclei is taking place, but the separation of galaxies is not yet complete.

(4) The appendages of NGC 4486 and 3561 are considered as ejections of smaller masses transforming into clouds of blue stars and possibly containing the sources of nonthermal blue emission.

(5) Connecting filaments arise during the process of fission of a galaxy, and spiral arms are formed by the same process (cf. systems of the M51 type.)

(6) Galaxies and their arms form at the expense of matter contained in the nuclei, which are of high density because the genesis of galaxies cannot occur at the expense of the stellar population of the nuclei. Globular clusters are expelled from the galactic nuclei.

(7) Stars and diffuse matter both originate from extra dense matter.

It is evident that the Ambartsumian hypothesis is much broader than the hypothesis of mechanical instability of groups of galaxies. Its essential features are that nuclei are superdense formations of great activity, that they can divide giving rise to multiple galaxies, and that they expel matter which is transformed into spiral arms, globular clusters, and diffuse matter.

The common origin of galaxies in physical doubles and multiples now seems to be unquestionable. This was sufficiently proved by Ambartsumian and others. In particular, the interacting pairs of galaxies, whose physical partnership is unquestioned, are so frequent that they could not be produced by chance encounters. The theory of capture was thoroughly discussed in the U.S.S.R. relative to Schmidt's cosmogonic theory. It was shown that a very specific encounter of three bodies is necessary in order to make the capture possible. It follows that the probability of capture among isolated field galaxies (where multiple systems are so frequent) is also much less probable than believed heretofore.

The difference of velocities in physical pairs is on the average much smaller than the mean peculiar velocity of galaxies—also a strong argument in favor of the common origin of the components.

The instability of clusters and of groups—of their positive energy—is not definite. The majority of contributions tend to confirm instability, as far as data on radial velocities go. However, the calculations involve various assumptions which are debatable. For instance, Limber and Mathews (1960) object to the instability of Stephan's Quintet, which is the most representative of groups of the trapezium type, and which is now commonly considered as being unstable. Though no definite data exist on the presence or absence of large intergalactic masses, it seems that the disintegration of many groups is probable.

The disintegration of large clusters seems less evident. The membership of galaxies ascribed to a cluster is always doubtful. The dimensions of a cluster are always much smaller than its distance. A background galaxy distant from the center, say five times the radius of the cluster along the line of vision, cannot be distinguished from its members.

The systems of trapezium type are regarded as unstable by Ambartsumian and by the Burbidges, who consider their configuration. The same conclusion was reached by the Burbidges (1961) concerning the chains of galaxies that I discovered recently. Among these groups we find elliptical galaxies, which are believed to be very old formations. The very long connecting filaments between elliptical galaxies, and very long tails, as I have shown (1957), indicate short existence, and hence instability. These formations must be of very recent origin. Ambartsumian concludes that, in the case of rapid disintegration of recently formed filaments, the components of the groups must be young. He believes the elliptical galaxies in such systems to be young as well. We must choose between two alternatives: (1) all interacting galaxies and probably the ellipticals in trapezium systems are young; (2) all such configurations are stable in spite of the mechanical considerations. The latter conclusion appeals to the law of gravitation, which is postulated as the sole law governing the interaction of galaxies.

I am convinced that although the law of gravitation between galaxies is valid, some other mutual influences are at play. In particular, I share the idea of Ambartsumian that the nuclei of galaxies present some properties yet quite unknown to us. It must be admitted that some nuclei are capable of dividing into two or more. Probably the enormous energy necessary to repel the expelled part of the nucleus with large velocity can be generated in some way. But it is very difficult to imagine the mechanism of formation and separation of galaxies. If a stellar system is formed continuously at the expense of the matter contained in the nucleus, the

latter must be very dense. I do not know whether observations allow such large densities for nuclei so far observed. Apparently the densities so far found are compatible with the stellar densities. It may be expected that if there exist systems whose separation is not yet complete (some radio galaxies according to Ambartsumian), their nuclei must be younger—probably larger or more active than those in systems whose separation is complete. This can be checked by observation.

It is quite possible that the appendage projected near the nucleus of the radio galaxy NGC 4486 is really a jet. This has not been proved, though it is always mentioned as a jet. There is some possibility that it is a kind of a spiral arm seen in projection. The nature of the "jet" is still very obscure. I was the first to stress the common nature and probably the similar origin of connecting filaments and of spiral arms. If Ambartsumian is right in ascribing the connecting filaments to the separation of two parts of dividing galaxies, then the spiral arms are formed also at the expense of the matter expelled from the nucleus.

However, it is difficult to accept the formation of spiral arms by the process of ejection of dense matter from the nucleus. The motion of a projectile from the nucleus is expected to be rectilinear. No spiral trajectory seems possible. Moreover, in order to form a thin, continuous, spiral arm, the expelled dense body must continuously leave matter along its path. It is difficult also to explain the preponderance of cases where two spiral arms start from the opposite sides of the nuclear region.

It also seems that it is not the disk system that develops at the expense of matter of the spiral arms—as a consequence of their disintegration. I have studied thousands of spiral galaxies on the Palomar Sky Atlas and it seems that, on the contrary, it is the spiral system that develops inside the disk. Very often the arms are independent of the nucleus and of the bar. For instance, we observe the following facts:

- (1) The arms start often at the rim of the disk and not from the nucleus.
- (2) The outer arms start often in the disk far from the inner spiral regions and are not connected to the inner spirals.
- (3) The ring structures, quite similar to the spirals, are detached from the nucleus, as are the arcs in the "early type" galaxies.
- (4) In many cases the spiral arms start not from the nucleus or from the ends of a bar, but from a ring, at a great distance from the nucleus.

On the other hand there are facts which can be considered as confirming the Ambartsumian's idea that the spiral arms form from the matter expelled from the nucleus.

It is true that in some cases the major axes of the

inner and of the outer structures are in different directions. This can be interpreted as the result of ejections of matter in two different planes. It can also be caused by unsymmetrical structures in a common plane, but with different orientation of axes of symmetry in this plane. But there are systems, the irregular M82 for instance, where streams of matter seem to be at right angles to the main plane of symmetry. The existence of several arms of different strength is also in favor of accidental differences of ejection. Small, thin, dusty lanes emerging directly from the minute inner nucleus of M31 also favor Ambartsumian's hypothesis, though the bright stellar arms have no connection with these inner dark spirals.

Personally, I prefer the conception that double and multiple galaxies are of common but independent origin. They originate in the immediate vicinity of each other. Some galaxies possess some power of repelling certain stars or other matter belonging to the neighboring galaxy. This leads to mutual recession of the galaxies despite their mutual gravitation. The matter which is more subject to the repulsion forms a tail. Owing to the great viscosity that must be postulated, this tail persists in the same way as the spiral arms do. At smaller distances, or when a galaxy possesses a greater power of hindering the formation of the spiral structure inside its companion, this spiral structure cannot develop properly. In other cases, or when the receding galaxies become sufficiently far apart, the spiral structure develops normally. In some cases of separation of already developed galaxies a connecting filament forms between them. Owing to its great viscosity it persists for a very long time and is not disrupted by tidal effects and by the rotation of galaxies. Probably an exchange of matter along these filaments is possible.

One must remember that the metagalaxy can be considered as a medium to some degree, and that the galaxies and their appendages are condensations in this medium rather than isolated bodies. Physical experience has had nothing to compare with the low density and peculiar structure of the metagalaxy as a medium. Probably many enigmatic phenomena in the realm of galaxies are related to this fact in somewhat the same way as the dualistic nature of light and of electrons presents the well-known apparent contradictions.

In studying interacting galaxies I have discovered many phenomena that point to a great viscosity of large stellar systems and to a kind of partial repulsion of matter. It is possible that this observable repulsion has something in common with the repulsion of galaxies themselves—repulsion in doubles and in groups postulated by Ambartsumian and to some degree confirmed by the observations.

The other great contribution made by Ambartsumian is that he draws attention to the important role of the nuclei and to the possibility of their exceptional properties. After he stated this, many new facts were added that support the idea of exceptional properties of the

nuclei: the discovery of the large spins of the nuclei, of the discontinuity of rotation in their vicinity, and of the high density. The discovery of gases in the nuclei of our Galaxy and of M31, and their flow outward with high velocity that is so difficult to explain, adds to the picture. Still more enigmatic is the case of Seyfert's galaxies with very broad emissions in their spectra. They may be considered as supporting the idea of outflow of matter from a superdense nucleus.

Some time ago we started a study of galactic nuclei at the Sternberg Institute. For the remote galaxies, the innermost nuclei (as observed in M31) are inaccessible. We can observe only the great inner bulge, but it is possible that the structure of the inner regions with no spiral structure is related to the structure of the innermost nuclei. In fact, though at present I can give no exact data, I can state that there is a great variety of nuclear structures among the galaxies ascribed to the same Hubble type. For instance, among the Sb spirals some have a central lens, others have small stellarlike nuclei. Some nuclei in Sb spirals are rather faint, some very bright. Besides smooth central lenses there are nuclei with very sharp edges, and so on.

Probably the size and the structure of these central regions reflects the evolutionary status of the galaxy. The study of the nuclei in binary systems might be of greatest importance.

Finally I draw attention once more to many facts which show that among galaxies the influence on the other systems is not confined to gravitation alone. I appeal to the enormous difference in dimensions between the galaxies and the solar system, where the law of gravitation is sufficient to explain its behavior. Double galaxies may have properties different from the properties of two systems of similar masses and dimensions. (We know that a molecule formed of two atoms possesses quite new properties.) I doubt that astronomers treating phenomena in an infinite universe only by means of gravitation will be able to understand the endless variety of the true laws of nature.

REFERENCES

- Ambartsumian, V. A. 1958, *On the Evolution of Galaxies*, Erevan, U.S.S.R.
 Burbidge, M. and Burbidge, G. 1961, *Sci. Am.* **204**, 50.
 Limber, D. N. and Mathews, W. 1960, *Astrophys. J.* **132**, 286.
 Vorontsov-Velyaminov, B. 1957, *Astron. J. U.S.S.R.* **34**, 8.

DISCUSSION

DE VAUCOULEURS noted that forms suggestive of repulsive forces are evident in the barred spiral NGC 6872 with a close companion IC 4970, photographs of which appear in *Mem. Commonwealth Obs.* **III**, No. 13, Fig. 26, and in the Cape Atlas.

FISH suggested that instability may account for spiral arms starting from the disk rather than from the nucleus of a galaxy, and mentioned the case of NGC 5055. In this Sb galaxy the luminosity measured in three colors is symmetrical along the major axis beyond 30" from the nucleus, but asymmetrical within 30", where the velocity curve of Burbidge, Burbidge, and Prendergast (*Astrophys. J.* **131**, 282, 1960) indicates solid-body rotation. These observations suggest that solid-body rotation of a compressible fluid may be unstable—a matter worthy of theoretical study.

MRS. PISHMISH recalled the tendency of recession of OB associations as related to the outward streaming of neutral hydrogen from the center of our galaxy, and the probable origin of the 3-kpc arm as an ejection from a limited region of the nucleus. In a recent article (*Bol. Obs. Tonantzintla y Tacubaya* No. 21, 1961) she attempts to explain the symmetrical emergence of spiral arms as a consequence of a poloidal magnetic field at the nucleus with dipole axis perpendicular to the axis of rotation.

B. LINDBLAD allowed that strange appearances in spiral structure may easily suggest forces of repulsion, but warned against assuming such nongravitational forces until resonance phenomena have been carefully investigated. VORONTSOV-VELYAMINOV concurred.

In answer to G. BURBIDGE, VORONTSOV-VELYAMINOV agreed that magnetic forces could not account for the thin filaments and tails he has observed, he feels that there must be some other influence at work to form these structures, and emphasized that they must have high viscosity to be maintained for any length of time. WOLTJER pointed out that strong magnetic fields could not provide such viscosity, even if the filaments were formed of a conducting plasma; such fields would rather cause the filaments to expand. VORONTSOV-VELYAMINOV added that the filaments are in any case now formed of stars.

On the Dynamical Instability of Several Groups of Galaxies

A. KALLOGHJAN

Burakan Observatory, Erevan, U.S.S.R.

AMBARTSUMIAN'S idea (1958) of the dynamical instability of several groups and clusters of galaxies has been supported by studies by the Burbidges (1959, 1959) and de Vaucouleurs (1959). The high value of the mass-luminosity ratio ($f=97$) for the elliptical components of double galaxies, recently obtained by Page (1960), leads to the conclusion that some of such systems may also have positive energy. Limber's and Mathews' (1960) objections against the dynamical instability of Stephan's Quintet seem to us to be somewhat artificial, but at any rate they may concern only the Quintet.

In the present report four groups of galaxies are examined. On the basis of present data about the radial velocities of the members of these systems they must have positive total energies. The radial velocities used have been taken from Humason, Mayall, and Sandage (1956).

NGC 68, 69, 71, 72, AND ANONS

These five galaxies form a dense group at the distance of 7×10^7 pc, if the value of Hubble's constant is 75 km/sec per Mpc. If the system has negative total energy, then the virial theorem may be applied. The following values for the projection factors are assumed: $p=1.57$ for the separations of the galaxies, $K=\sqrt{3}$ for the conversion of radial velocities to space velocities. Pettit's values of apparent magnitudes have been used (1954). The luminosities relative to the sun have been calculated using the photographic absolute magnitude of the sun given by Stebbins and Kron (1957) and taking into account the galactic absorption by the equation $A_{pg}=0.25 \text{ csc } b$. Under these conditions, when the virial theorem is applied, we obtain for the mass-luminosity ratio $f=385$, which is too high even for elliptical galaxies and, especially, for one of the members of the group which is of SBa type.

GROUP CONTAINING NGC 80 AND 83

NGC 80 and 83 are very bright compared with other members of the group. Their radial velocities differ from each other by more than 1000 km/sec. Since no radial velocity is known for the other members of the group, we have calculated the values of gravitational potential at the positions of NGC 80 and 83. The kinetic energy of each galaxy has been obtained from the radial velocities relative to their common center of mass

(500 km/sec). The apparent magnitudes of remaining galaxies have been estimated on the charts of Palomar's atlas. All the brightnesses have been purposely overestimated.

The mass-luminosity ratio for spiral galaxies has been taken to be the half that of the ellipticals. On the assumption that the total energy of NGC 80 and 83 is negative, and applying the same values of the projection factors as in the preceding case, the lower limit to the mass-luminosity ratio for the elliptical galaxies is found to be $f_E=330$, in the case of NGC 80, and $f_E=270$ in the case of NGC 83.

DOUBLE SYSTEM NGC 7385-7386

These two galaxies are much brighter than the neighboring ones. So we have neglected the masses of the other galaxies of the group and considered NGC 7385 and 7386 as a double system in order to calculate the lower limit of the sum of the masses. The projection factors $p=1.5$, $K=\sqrt{2}$ were assumed, and we obtain $m_1+m_2 \geq 4.8 \times 10^{13} m_\odot$. (Using $f_E=60$, this same sum is equal to $4.2 \times 10^{12} m_\odot$.) That is, we obtain $f_E=685$.

CHAIN OF GALAXIES AT $\alpha_{1950}=1^h 4^m 6^s$, $\delta_{1950}=+32^\circ 8'$

This chain of elliptical and S0-type galaxies resembles that in the Virgo cluster and is composed of nine members. Radial velocities of all nine are known. All other galaxies in the region are very faint and probably do not belong to the same system. Application of the virial theorem, using $p=1.5$ and $K=\sqrt{3}$, leads to the value for the mass-luminosity ratio, $f_E=260$.

Unless we accept mass-luminosity ratios of several hundred in the ellipticals, we conclude that, even with all the uncertainties in the projection factors and in reasonable densities of intergalactic matter, the above-considered systems have positive total energies; i.e., they are unstable.

REFERENCES

- Ambartsumian, V. A. 1958, Solvay Conference Report, Brussels, Belgium.
- Burbidge, G. R. and Burbidge, E. M. 1959, *Astrophys. J.* **130**, 15.
- , 1959, *ibid.* **130**, 629.
- Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* **61**, 97.
- Limber, D. N. and Mathews, W. G. 1960, *Astrophys. J.* **132**, 286.
- Page, T. L. 1960, *ibid.* **132**, 910.
- Pettit, E. 1954, *ibid.* **120**, 413.
- Stebbins, J. and Kron, G. E. 1957, *ibid.* **126**, 266.

Physical Chain of Galaxies in the Virgo Cluster and Its Dynamic Instability

B. E. MARKARIAN

Burakan Observatory, Erevan, U.S.S.R.

GALAXIES with NGC numbers 4374 (M84), 4406 (M86), 4435, 4438, 4458, 4461, 4473, and 4477, all in Virgo cluster, form a slightly bent chain extending over about $1\frac{1}{2}^\circ$ (see Fig. 1). These galaxies are located in a ring bounded by circles centered at $\alpha=12^h 22^m 9$, $\delta=+14^\circ 28'$, with radii of $74'$ and $80'$, respectively. The chain occupies only one-seventh of this ring, with an area of about one square degree. The question arises whether the formation of this chain in the Virgo cluster is the result of chance projection of galaxies.

In order to clarify this question, consider a circular region, say R , about the chain, with radius of 7° , representing the densest part of the Virgo cluster. According to the Shapley-Ames catalogue, this region contains 88 galaxies brighter than 13 mag. Table I gives the dis-

TABLE I. Distribution of galaxies in the region R according to types.

Type	E	S0	Sa	Sb	Sc		S		Total
Number	31	8	9	8	20	2	7	3	88

tribution of these galaxies according to types. As indicated in Table II, almost all the galaxies forming the chain are of types E or S0, the exception being NGC 4438, which is of type Sap. For this reason, in contemplating the question posed, it may be appropriate to consider 39 galaxies belong to types E and S0. However, in order to be cautious, we are adding to this number the galaxies of types Sa and one-third of galaxies which are not assigned to any type. As a result, the number of galaxies in the region R which could participate in the formation of the chain becomes 52.

The contemplated chain could be placed in a square of one radius on the side. The ring which formed about the chain admits the presence in the square of three galaxies because through any three points it is always possible to draw a circle. For this reason, in order to calculate the probability of a chance formation of the chain, it is necessary to multiply the possible number

of chance formation of triplets (located in area of 1 square degree) by the probability that some other five galaxies will be located, by chance, in the same domain. The probability of a chance formation of such triplet of galaxies out of N galaxies available in a domain of area of S square degrees is

$$P_1 = NC_{N-1}^2 (1/S)^3. \quad (1)$$

The number of such groups will be obtained by multiplying P_1 by S , that is, $M = SP_1$. Further, the probability that, by chance, there will be five more galaxies located in the domain of any such group, the domain of area equal to one-seventh square degree, will be

$$P_2 = C_{N-3}^5 (1/7S)^5 (1 - 1/7S)^{N-8}. \quad (2)$$

For this reason the probability of chance formation of the chain is determined by

$$P = MP_1 P_2. \quad (3)$$

The order of magnitude of this probability is 6×10^{-9} .

It is obvious that this small probability practically excludes the possibility that the chain of galaxies considered was formed by chance.

In this way I arrived at the conclusion that the chain of galaxies in the Virgo cluster is not a chance grouping but a real physical system. If this is true, in which there will hardly be any doubt, then this system must be a linear system. If this conclusion corresponds to reality, then the chain must be an exceptionally young system. The reason is that, in spite of considerable dispersion of radial velocities of particular members, the observed deviations from the linear structure are exceedingly small.

The corrected radial velocities of components of the

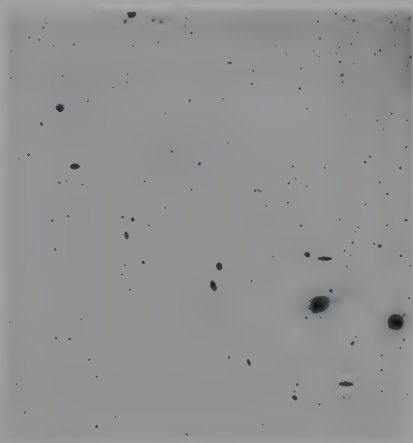


FIG. 1.

TABLE II. Eight galaxies forming the chain in the Virgo cluster.

NGC	α_{1960}	δ_{1960}	Type	m_{pg}	V_r'	V_r	k
4374	$12^h 22^m 6$	$+13^\circ 10'$	S0	10.2	+880	+54	0.91
4406	23.7	13	E3	10.1	-452	-1278	1.00
4435	25.1	21	SB0	11.8	+796	-30	0.21
4438	25.2	17	Sap	11.0	-105	-931	0.36
4458	26.4	31	E0	13.0 ^a	+309	-517	0.07
4461	26.5	28	S0	12.0	+1813	+987	0.17
4473	27.3	42	E5	11.3	+2173	+1347	0.33
4477	27.5	55	SB0	11.8	+1195	+369	0.21

^a The magnitude of the galaxy NGC 4458 was determined by the author.

chain given in the 6th column of Table I are taken from Humason *et al.* (1956). They vary over a substantial range from -452 to 2173 km/sec, and the radial velocities with respect to the center of gravity of the system vary from -1278 to 1347 km/sec (see column 7 of Table II). The dispersion of the radial velocities is 902 km/sec.

Calculations show that, with this dispersion, the linear structure of a system would be totally destroyed during 10^8 years.

It is clear that a chain, as such, cannot be a stable system to any reasonable degree, quite irrespective of the sign of its total energy. Independently from this circumstance, the sign of the total energy of systems is of considerable interest for many reasons. Therefore, we attempted to determine this sign from the data given in Table II.

Assume that the total energy is equal to 0, that is,

$$T + U = 0, \quad (4)$$

where T and U represent, respectively, the kinetic and the potential energy of the system. Assuming that the components of this system and their space velocities are randomly distributed, we may put $V_i^2 \cong 3V_{ri}^2$ and $r_{ij} \cong \frac{1}{2} \pi \rho_{ij}$, where the V_{ri} are the radial velocities of the components and ρ_{ij} the projections of the linear distances between the components on the plane perpendicular to the line of sight. Then we shall have

$$\frac{3}{2} \sum \mathfrak{M}_i V_i^2 - \frac{2}{\pi} \sum \frac{\mathfrak{M}_i \mathfrak{M}_j}{\rho_{ij}} = 0. \quad (5)$$

As already noticed, the galaxies forming the chain belong to the types E and S0, with the exception of one which is of type Sap. For this reason, the mass-luminosity ratio may be assumed constant. Then the relative values of the masses may be obtained from the luminosities of the galaxies.

On the assumption that the distance of the Virgo cluster is 15 Mpc (on the new distance scale) and using the apparent magnitudes of Table II [these are averages of the data of Holmberg (1958) and Pettit (1954)], the luminosities of the galaxies were determined and then their masses. The latter are expressed in terms of the mass of the brightest galaxy of the system, namely NGC 4406. Denoting the latter mass by $\mathfrak{M}$ and substituting in (5) $\mathfrak{M}_i = k_i \mathfrak{M}$ and $\mathfrak{M}_j = k_j \mathfrak{M}$, we obtain

$$\mathfrak{M} = \frac{3\pi}{4G} \sum_i k_i V_{ri}^2 \left(\sum_{i,j} \frac{k_i k_j}{\rho_{ij}} \right)^{-1}. \quad (6)$$

The values of the quantities V_{ri} and k_i are given in the two last columns of Table II and the values of the ρ_{ij} were obtained using the above estimate of the distance of the Virgo cluster and the angular separations of the members of the chain. These were measured on a photograph taken with the 21-inch Schmidt telescope of the Burakan observatory.

Using formula (6) the mass of NGC 4406 was calculated to be $4 \times 10^{13} \mathfrak{M}_\odot$. In turn, this figure determines the luminosity ratio of the galaxy, equal to 10^3 .

The values of the mass and of the mass-luminosity ratio so obtained appear inordinately large: They exceed the known maximal values of these quantities by $1\frac{1}{2}$ to 2 orders of magnitude. It is true that the values obtained depend to a certain extent on the adopted Hubble constant and on the projection factor used in the calculation of the space velocity from the radial velocity. However, if we replace the adopted values by 100 km/sec per megaparsec and by 2, respectively, then the values obtained for the mass and for the mass-luminosity ratio will be diminished only in the ratio of two to one.

It may be appropriate to mention that, if galaxy NGC 4473, with the greatest relative radial velocity of the system, is excluded from the calculations, on the assumption that it is not a physical member of the chain, this does not alter the general situation.

While the assumption that the total energy of the system is equal to zero leads to very unusual conclusions, the alternative assumption that the total energy is negative would lead to even greater estimates of the mass and of the mass-luminosity ratio. As a result, we arrive at an inescapable conclusion that the total energy of the system must be positive.

The indicated chain of galaxies may be taken as an outstanding example confirming the Ambartsumian hypothesis (1958) that among multiple galaxies and among groups of galaxies there are unstable systems having positive energy.

REFERENCES

- Ambartsumian, V. A. 1958, *Izvest. Acad. Sci. Armenian S.S.R.* 9, 9.
Holmberg, E. 1958, *Medd. Lund. Obs. Ser. II*, No. 136.
Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* 61, 97.
Pettit, E. 1954, *Astrophys. J.* 120, 413.

DISCUSSION

SCOTT asked whether a search had been made for chains in other parts of the Virgo cluster or elsewhere. MARKARIAN replied that he had found no others in the Virgo cluster, but that there are several other examples of chains in Vorontsov-Velyaminov's catalogue, although there is doubt that some of them are dynamical systems. Of course, projection can lose some of the chainlike character. SCOTT then asked whether one should not compute the probability of at least one chance chain in the larger area (with many more galaxies involved and thus a much larger value of N). On the other hand, DE VAUCOLEURS pointed out that the 3° core of the Virgo cluster, where this chain is located, is much more densely populated with E and S0 galaxies than the larger area of 7° square over which Markarian took, in effect, an average density to compute the probability of a chance chain. MARKARIAN answered that any smaller

area leaves out some of the denser parts of the cluster. As a result, if one narrows the area S containing the chain, the increase in the probability in question will be negligible.

SCOTT went on to say that in her study with Shane and Swanson (*Astrophys. J.* **119**, 91, 1954) a synthetic $6^\circ \times 6^\circ$ field was constructed by distributing clusters at random and galaxies within each cluster according to a trivariate normal distribution as in the model of spatial distribution derived from Lick plates (Neyman, Scott, and Shane, *Astrophys. J.* **117**, 92, 1953). Many curious configurations, including straight and circular chains of galaxies of nearly the same magnitude, were found on this plot, the arrangement of almost all of these galaxies being due to fortuitous lining up of clusters one behind the other. This is empirical evidence of the strong effect of projection, and casts doubt on any conclusion that such configurations are physical systems.

MARKARIAN replied that the chain considered by him is entirely composed of bright galaxies. Because the Scott-Shane-Swanson experiment referred essentially to faint galaxies, the conclusions drawn from this experiment cannot be applied to the chain he studied. SCOTT answered that this is not the case—each galaxy in each cluster was treated individually and its exact position and magnitude determined synthetically. We can fix our attention on bright galaxies or on nearby galaxies or on what we please. The reader can visualize the superimposing *five* plots typified by Figs. 2(a) and 2(b) in the publication. This distance is reduced to 20 million parsecs by ignoring the smallest dots, etc. In any one plot there are already some chance configurations; with five their number is multiplied.

NEYMAN admitted an intuitive feeling that the 8 galaxies in Fig. 1, possibly combined with a few others, are the debris of some larger structure akin to a spiral arm. Nevertheless, he found the probability argument used to support the asserted physical connection among the eight galaxies not very convincing. The argument is as follows: We observe a configuration of objects on a photograph; arbitrarily, we select a feature of this configuration, and then compute the probability that such a feature will be created by chance. We find that this probability is small and conclude that the particular configuration is not due to chance.

Of course, this kind of reasoning is very common and has been used for ages; he does not mean to single out Dr. Markarian. In fact, the weakness of the reasoning was noticed some 50 years ago when it was the subject of a discussion between Joseph Bertrand and Emile Borel. They agreed that in general the conclusion is unfounded for at least one reason: Whatever the observed configuration of objects, it is always possible

to invent a characteristic of this configuration such that the probability of obtaining it by chance will be as small as desired. In other words, among points distributed by chance on a plane one inevitably obtains configurations some features of which are extremely improbable. Thus, probability of a particular observed feature does not constitute convincing ground for denying the chance mechanism underlying the distribution of points.

Bertrand's pessimistic conclusion was that no probability argument could be useful in testing hypotheses. Borel disagreed; according to his intuition, probability theory could be used for testing hypotheses provided that certain precautions are observed. Two of these precautions are (1) that the test criterion be chosen *before* observations are started and (2) that the test criterion be a function of the observable variables "en quelque sorte remarquable." Borel did not indicate the exact meaning of this phrase, but in the late 1920's it served as a starting point for the modern theory of testing statistical hypotheses.

The phrase "fonction en quelque sorte remarquable" can be given a very precise meaning. Heuristically it is that function of the observable random variables for which: (i) the probability of rejecting the hypothesis tested (say H) when it is true be small and (ii) that the probability be greater of rejecting the hypothesis tested when it is false. In order to determine a test criterion of this sort it is necessary to know the hypotheses, $h_1, h_2, \dots$, that may be true when H is false. In other words, it is necessary to know all the ways in which H might be false.

With reference to the present case, H is that the 52 galaxies are distributed over the area S with "statistical uniformity," that is, with a probability density which is constant over S , each galaxy being independent from the others. The hypotheses $h_1, h_2, \dots$ (alternatives to H) are covered by the statement "the eight galaxies constitute a physical system." At first sight this appears meaningful, but not under closer examination. To be entirely meaningful, the description of the alternative(s) must be sufficient to calculate the probability of any preassigned configuration of galaxies considered. If we merely say that eight of them form a physical system, this is not enough for the calculation of probabilities.

MARKARIAN agreed that, by itself, small probability of the chance occurrence of a given chain of galaxies cannot be regarded as a proof that it is a physical system. However, many other groups of galaxies (pairs, clusters) are being treated as physical systems without adequate proof. In the present case, the small probability of the chain being formed by chance speaks strongly in favor of the hypothesis that the chain is a physical system.

NGC 6166 and the Cluster Abell 2199

R. MINKOWSKI*

Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology

THE quadruple galaxy NGC 6166, shown in Fig. 1, differs by its extreme compactness from other multiple systems that have been investigated recently. The three fainter components appear projected onto the brightest component. NGC 6166 is the brightest member of the cluster Abell 2199 (Abell 1958). Its faintest component conceivably could be a faint member of the cluster projected onto the system, but it is highly improbable that either the third or the second brightest component is not a physical member of the system. NGC 6166 is much brighter than any of the other members of the cluster. It seemed desirable to establish its membership in the cluster by comparing its radial velocity with that of the cluster. The radial velocities of 19 cluster members were observed and will be used for a discussion of the stability of the cluster.

Radial velocities were obtained at the prime focus of the 200-inch telescope with a dispersion of 400 Å/mm.

Only one plate of each galaxy was taken, the velocities may therefore have a mean error of 50 to 100 km/sec, but the velocity differences are so large that the possible errors are irrelevant.

Photometric data were obtained with the 36-inch telescope at the Pine Bluff Observatory in cooperation with C. R. O'Dell. To obtain magnitudes of the individual components of NGC 6166, the object was scanned in right ascension with a slit 0'.1 wide and 1' long. The brightest component was assumed to be symmetrical and the contribution of the fainter components was obtained by subtracting the reflected intensities of the preceding side from the intensities of the following side. Disregarding the faintest component, the magnitudes of the second and third components were derived by assuming them to be symmetrical. The procedure is illustrated in Fig. 2. Three scans in *B* only were obtained; unfavorable observing conditions pro-

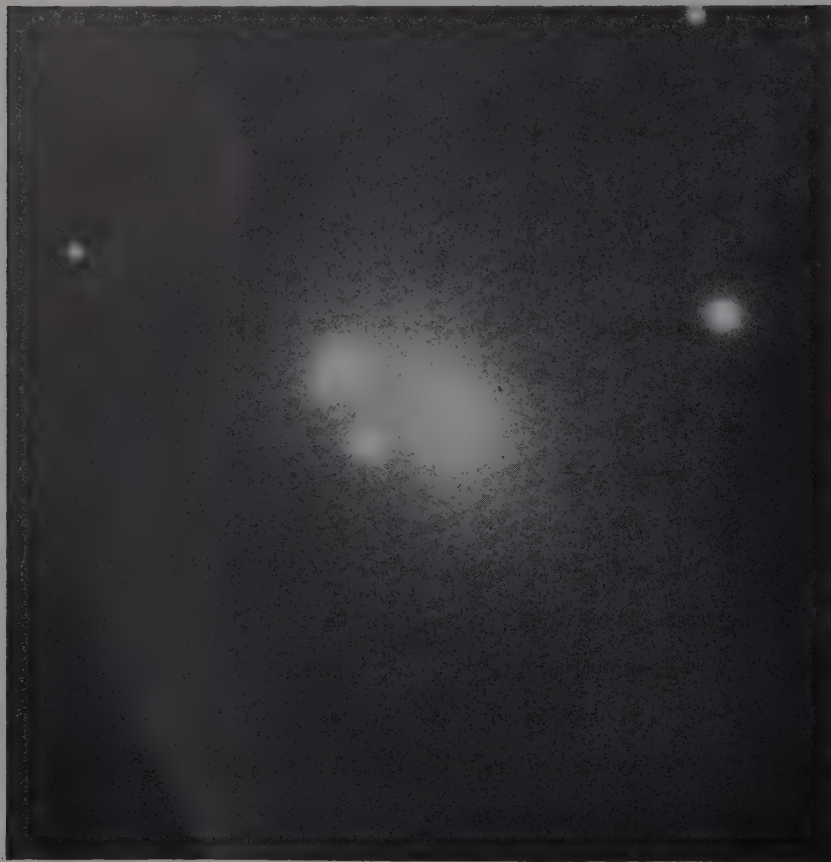


FIG. 1. NGC 6166. 200-inch telescope, Eastman 103 aO, 1 mm = x'' .

* Present address: Radio Astronomy Laboratory, University of California, Berkeley, California.

hibited scans in V and determination of individual $B-V$ colors. The fourth component is embedded in nebulosity from which it cannot easily be separated; only a crude estimate for its magnitude can be given.

The data for NGC 6166 and its components are given in Table I. Column 3 lists the observed B magnitudes. To judge from the appearance of the object on the 48-inch Schmidt plates and from the photoelectric scans mentioned below, the 2.5 aperture gives the magnitude to the standard isophote 25 mag./sq sec of arc. Observed $B-V$ colors are in column 4. Magnitudes B_0 , corrected for obscuration in our own galaxy and for the redshift, are in column 5. The obscuration was assumed to be $0.25 \csc b$, where b is the galactic latitude. The correction for redshift is 0.14 mag. (Humason, Mayall, and Sandage 1956). Column 6 gives radial velocities corrected for solar motion. For the whole object, the velocity of the center of gravity is given, obtained under the assumption that all components have the same mass-luminosity ratio. If the Hubble constant is $100 h$ km/sec per Mpc, the distance is $90.8 h^{-1}$ Mpc, and the distance modulus $m-M=34.79-5 \log h$. Absolute B magnitudes are given in column 7.

TABLE I. Observational data for NGC 6166.

Object	Aperture	B	$B-V$	B_0	v_0 km/sec	$M_B-5 \log h$
NGC 6166	1'	13.75	+0.85	13.24	...	...
	2.5	13.30	+0.83	12.79	+9082	-22.01
Comp. A	...	13.6	...	13.1	+9480	-21.68
B	...	15.0	...	14.5	+7960	-20.3
C	...	16.5	...	16.0	+10 050	-18.8
D	...	18:	...	17.5	...	-17.3

The brightest component of NGC 6166 is a very remarkable system. Its luminosity is very high, exceeding by 0.4 mag. that of NGC 4889, the brightest member of the Coma cluster. It is very large; the diameter of the major axis to the isophote 25 mag./sq sec of arc is about 2.5 or 68 000 h^{-1} psc. On the south side of the central mass structure appears to be present; this has been confirmed by E. M. Burbidge. The structure is most likely caused by absorbing clouds. It is not quite impossible, however, that optically superposed very faint members of the cluster contribute to the structure. The degree of central condensation is remarkably low. The bright nuclear region typical for galaxies of types E and S0 is absent. Only a very faint semistellar nucleus has been found by E. M. Burbidge; it is blue and appears most distinctly in the ultraviolet. The spectrum of the galaxy is quite normal; a moderately strong emission line of [OII] $\lambda 3727$ is present in the center of the galaxy only. It seems probable that the emission is restricted to the semistellar nucleus. It may contribute much to the light of the nucleus and may be the reason for its blue color. It is too faint, however, to affect the color of the object as a whole.

NGC 6166 provides the identification for a radio source (Minkowski 1958). The source 3C338 (Edge *et al.*

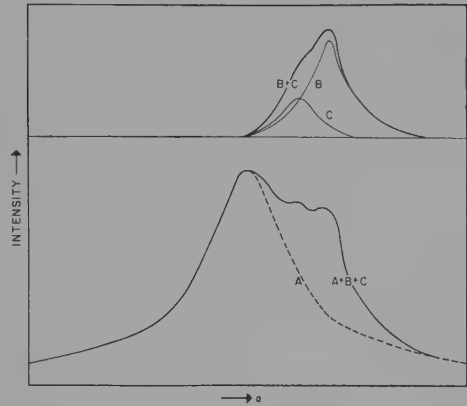


FIG. 2. Analysis of the photoelectric scan of NGC 6166.

1959) has an intrinsic strength comparable to that of such sources as NGC 1316 (Fornax A) and NGC 4486 (Virgo A). The suggestion that the radio emission might be connected with the fact that the object is quadruple has lost much of its original attraction since it can be shown (Minkowski 1961a, 1961b) that most radio sources identified with double galaxies of normal appearance are sources not because the galaxies interact, but because one of them is a galaxy with strong radio emission. In the case of NGC 6166, where the brightest component is an unusual object and actually somewhat similar to NGC 1316 by the presence of absorbing clouds and possibly similar to NGC 4486 by the presence of $\lambda 3727$ in emission restricted to the nucleus, it seems very probable that the brightest component is the radio source and that the presence of the fainter components is irrelevant.

To test the stability of NGC 6166, the virial theorem must be used. The minimum mass needed to make the system stable is obtained from the condition

$$2T = -\Omega \quad (1)$$

if we take for the kinetic energy T its minimum value

$$T \geq \sum_i \frac{1}{2} \mathcal{M} \mu_i v_{ri}^2 \quad (2)$$

($\mathcal{M}$ total mass of the system, μ_i relative mass, and v_{ri} radial velocity of the i th galaxy), and for the potential energy, its maximum value

$$\Omega \leq - \sum_{\text{pairs}} G \mathcal{M}^2 \frac{\mu_i \mu_j}{r'_{ij}} \quad (3)$$

(G gravitational constant, r'_{ij} projected linear separation of the i th and j th galaxies). The sums are over all galaxies and over all pairs of galaxies, respectively. The relative masses were obtained from the luminosities in Table I with the assumption that all four components have the same mass-luminosity ratio. Since the radial velocity and therefore the kinetic energy of the faintest component is unknown, the total kinetic energy was assumed to be $\frac{4}{3}$ times that of the sum of the three

TABLE II. Separations of the members of NGC 6166.

Pair	Projected separation of centers		Pair	Projected separation of centers	
	sec of arc	pc		sec of arc	pc
1-2	12.5	5500	2-3	8.0	3500
1-3	8.5	3700	2-4	3.2	1400
1-4	13.4	5900	3-4	6.7	2900

brighter components. With the projected separations in Table II, the minimum mass of the system becomes $3.8 \times 10^{12} h^{-1}$ solar masses, and the minimum mass-luminosity ratio 42 h .

We may also test by using the average projection factors for spherical symmetry (Limber and Matthews 1960), with

$$2T = 3 \sum_i \mathcal{M}_i v_{ri}^2, \quad (4)$$

$$-\Omega = - \sum_{\pi \text{ pairs}} \frac{G \mathcal{M}_i^2 \mu_i \mu_j}{r'_{ij}}. \quad (5)$$

We then obtain for the mass of the system the value $1.4 \times 10^{13} h^{-1}$ solar masses and the mass-luminosity ratio 149 h .

The mass-luminosity ratio needed to make the system stable is high, but consistent with the mass-luminosity ratio 94 h with a dispersion of 38 h found by Page (1960) for double galaxies of types E and S0. No definite conclusion seems possible. If the double galaxies in Page's sample are stable, NGC 6166 also may be stable. A high mass-luminosity ratio for NGC 6166 with its very high luminosity would be quite likely if the mass-luminosity ratio increases with luminosity as Poveda (unpublished) has suggested. On the other hand, it is obviously not possible to exclude the possibility that NGC 6166 is dynamically unstable.

Cluster Abell 2199, whose brightest member is NGC 6166, is, according to unpublished counts by Abell, similar to the Coma cluster in richness, but has less central condensation. It is not the type of cluster that one might select for a study of its stability. There is reason to doubt that it is an isolated cluster since cluster Abell 2197, which is in the same distance class, is only $81' N$ of Abell 2199. Radial velocities of cluster members were observed primarily to test whether NGC 6166 is really a member of the cluster.

TABLE III. Radial velocities in the cluster Abell 2199.

No.	Dist. from NGC 6166 in min. of arc		v_0 km/sec	No.	Dist. from NGC 6166 in min. of arc		v_0 km/sec
	α	δ			α	δ	
1	-2.9	+1.2	+9070	11	+1.2	+5.3	+8780
2	-2.8	+3.3	+9690	12	+1.4	-5.4	+7790
3	-1.4	-1.8	+10 400	12	+1.4	-1.4	+8460
4	-0.9	-0.1	+10 090	14	+1.5	+3.5	+8300
5	-0.5	-1.0	+8610	15	+1.5	+6.3	+9220
6	+0.1	-2.0	+7850	16	+1.8	+5.9	+8210
7	+1.1	-4.7	+9360	17	+2.2	+3.9	+8410
8	+1.1	-4.0	+10 270	18	+2.8	+0.5	+8939
9	+1.1	-2.2	+8510	19	+5.5	-3.4	+8840
10	+1.2	-7.0	+10 740				

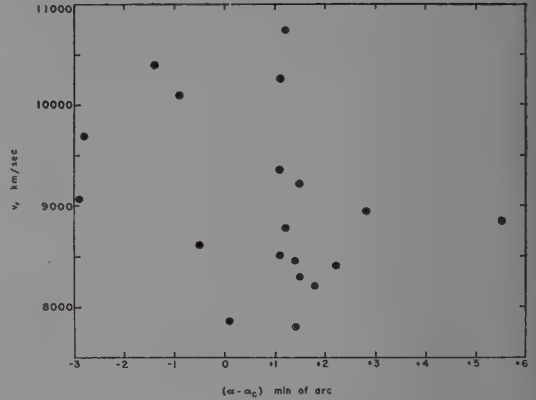


FIG. 3. Radial velocity of galaxies in the cluster Abell 2199 against the distance from NGC 6166 in right ascension.

The radial velocities for 19 members of the cluster Abell 2199 are given in Table III. They are plotted in Figs. 3 and 4 against the distance from NGC 6166 in right ascension and in declination, respectively. There is no dependence of the radial velocities on declination, but a systematic decrease of the radial velocities with increasing right ascension shows in Fig. 3. This correlation is essentially the result of the presence of four galaxies—Nos. 3, 4, 8, and 10 in Table III—with radial velocities larger than +10 000 km/sec; all four are in the southern part of the cluster. It seems justified to question whether these galaxies really belong to the cluster. If they are members of the cluster, the average velocity for the cluster is +9028 km/sec with a sample variance of 864 km/sec; if they are not members, the average velocity is +8736 km/sec with a sample variance of 541 km/sec. In any case, the velocity of the cluster agrees well with the center of gravity velocity +9082 km/sec of NGC 6166. There is no reason to doubt that NGC 6166 is a member of the cluster.

If the four galaxies with velocities above +10 000 km/sec are members of the cluster, the cluster seems to rotate around an axis approximately in the $E-W$ direction. A least-squares solution, assuming a linear dependence of the velocity on right ascension, i.e., solid body rotation, gives an increase of the radial velocity of 137 km/sec per minute of arc, or 51 km/sec per 10 000 pc. Such a rotation is not necessarily unacceptable. The Keplerian mass

$$M = v^2 r / G \quad (6)$$

(v radial velocity at the distance r from the center) to a distance of $5'$ from the center is $1.5 \times 10^{13} h$ solar masses, a not unacceptable value since the mass of NGC 6166 found from the virial theorem is $1.4 \times 10^{13} h$ solar masses. If solid-body rotation would extend to larger distances, the Keplerian mass would become too large to be consistent with the mass of the galaxies in the cluster. It would be of some interest to extend the observations to the outer parts of the cluster.

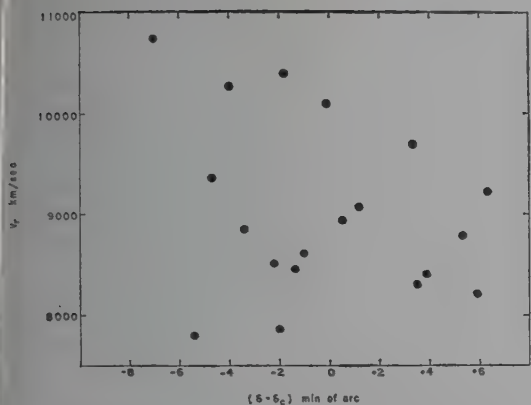


FIG. 4. Radial velocity of galaxies in the cluster Abell 2199 against the distance from NGC 6166 in declination.

The available data are too limited to justify application of the virial theorem to investigate the stability of the cluster. But, since the cluster is somewhat similar to the Coma cluster, a direct comparison with the Coma cluster is informative. The sample variance of the radial velocities in the Coma cluster is 932 km/sec, if four galaxies are omitted which are probably optically superposed (Lovasich, Mayall, Neyman, and Scott 1960). This value is larger than the value 583 km/sec derived by Oort (1958) from the assumption of stability. It is not necessary to discuss here the problem of whether or not the Coma cluster is stable.

If the four galaxies with high velocities are members of the cluster Abell 2199, the sample variance of the radial velocities is 841 km/sec. This is quite comparable to the value for the Coma cluster. Thus, if the Coma cluster is stable, cluster Abell 2199 is probably stable; if the Coma cluster is not stable, cluster Abell 2199 also is unstable.

If, on the other hand, the four galaxies with high velocities are not members of the cluster, the sample variance of the radial velocities is 542 km/sec. This value agrees well with the value expected by Oort for the Coma cluster if it is stable. Thus, if the four galaxies with high velocities are not members, the cluster Abell 2199 may well be stable.

To which extent the difficulties encountered in the discussion of the cluster might arise because the cluster is not isolated cannot be decided. A much larger number of observations, including also the cluster Abell 2197, obviously would be desirable, but it cannot be predicted whether or not the result would justify the large amount of observing time required.

ACKNOWLEDGMENTS

Parts of this work were carried out at the Washburn Observatory of the University of Wisconsin. The author is greatly indebted to C. R. O'Dell for his cooperation in obtaining the photometric data. Thanks are due also to Dr. G. O. Abell and to Dr. E. M. Burbidge for helpful discussions and communication of unpublished results.

REFERENCES

- Abell, G. O. 1958, *Astrophys. J. Suppl.* **3**, 211.
 Edge, D. O., Shakeshaft, J. R., McAdam, W. B., Baldwin, J. E., and Archer, S. 1959, *Mem. Roy. Astron. Soc.* **68**, 37.
 Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* **61**, 97.
 Limber, D. N. and Matthews, W. G. 1960, *Astrophys. J.* **132**, 286.
 Lovasich, J. L., Mayall, N. U., Neyman, J., and Scott, E. L. 1961, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (to be published).
 Minkowski, R. 1958, *Publ. Astron. Soc. Pacific* **70**, 143.
 —. 1961a, Joint Symp. Natl. Acad. Sci. U.S.A. and Acad. Sci. U.S.S.R., Green Bank, West Virginia, May 1961, *Proc. Natl. Acad. Sci.* (to be published).
 —. 1961b, "Problems of extragalactic research," *IAU Symp. No. 15*, 1961 (to be published).
 Oort, J. H. 1958, *La Structure et l'évolution de l'univers*, Solvay Conference (R. Stoops, Brussels, Belgium), p. 163.
 Page, T. 1960, *Astrophys. J.* **132**, 910.

DISCUSSION

ABELL pointed out that cluster A 2199 is in a region of many uncatalogued small clusters and groups (in addition to A 2197 mentioned by Minkowski) that may be at the same distance. The four galaxies with high velocities discussed by Minkowski may not be members of A 2199.

DE VAUCOULEURS commented on the color index of NGC 6166 which, after correction for galactic absorption, is about +0.7, or 0^m3 bluer than normal for a giant elliptical galaxy. It was agreed that this could not be due to $\lambda 3727$ emission, and DE VAUCOULEURS suggested that it might be an indication of continuous synchrotron radiation related to the radio emission.

HARLAN SMITH asked whether the three galaxies associated with NGC 6166 could be nuclear condensations within a common galactic halo, but MINKOWSKI felt that this was ruled out by the velocity dispersion within the group. In answer to AMBARTSUMIAN, MINKOWSKI said that the semistellar nucleus of NGC 6166 is small and too faint to see at the prime focus of the 200-inch telescope, but that it showed clearly on ultraviolet photographs, possibly an indication that it radiates mostly in $\lambda 3727$.

Radio Sources and Clusters of Galaxies

SIDNEY VAN DEN BERGH

David Dunlap Observatory, Richmond Hill, Ontario, Canada

UNFORTUNATELY the positions of most radio sources are not known with high accuracy. It is therefore often difficult to identify a source of radio radiation with a particular extragalactic nebula. For a given positional accuracy the problem of identification becomes progressively more difficult as identifications are pushed to fainter limits. This suggests that it might be profitable to approach the identification problem from a statistical point of view. The process of statistical "identification" of radio sources might proceed in three stages:

(1) Statistical association of radio sources with large regions (superclusters) in which the density of galaxies is higher than average.

(2) Statistical association of radio sources with clusters of galaxies.

(3) Statistical association of radio sources with individual galaxies.

Statistical associations with superclusters might be expected to yield information on the fraction of all radio sources of a given brightness which are associated with galaxies at a certain distance. Statistical associations with clusters of galaxies might yield information on the frequency with which sources of radio radiation occur in clusters of different type and different richness. Such information might contribute to our understanding of the processes which give rise to radio radiation. Finally, the statistics of identifications with individual galaxies may yield information on the nature and duration of strong radio emission.

SURFACE DENSITY OF GALAXIES AT THE POSITIONS OF RADIO SOURCES

The surface density of galaxies brighter than the 18th magnitude (Shane and Wirtanen 1954; Shane, Wirtanen, and Steinlin 1959) has been determined at the positions of radio sources (Mills, Slee, and Hill 1958, 1960) in the zone $-22^\circ < \delta < +10^\circ$. To minimize

the effects of galactic obscuration and contamination of the sample by galactic sources, the investigation was restricted to the regions with galactic latitude $|b^I| > 40^\circ$.

Table I shows the observed and expected frequency with which radio sources, with a flux $S > 20 \times 10^{-26}$ w m⁻² (cps)⁻¹ at 3.5 m, were observed to occur in regions with different surface densities of galaxies. The expected frequency distribution was obtained from the surface densities of galaxies at 596 points which were chosen at random.

Comparison of the observed and expected frequency distributions in Table I shows that there is an excess of bright radio sources in regions with a high surface density of galaxies and a deficiency of bright sources in regions with a low density of galaxies. A χ^2 test shows that the differences between the observed and expected distributions are significant at the 5% level. For faint sources with $10 \times 10^{-26} < S \leq 20 \times 10^{-26}$ w m⁻² (cps)⁻¹ at 3.5 m, no significant difference is found between the observed and the expected frequency distributions. Probably this indicates that most faint radio sources are located in regions which are so distant that they do not contain an appreciable number of galaxies brighter than the 18th mag.

The data for the bright radio sources with $S > 20 \times 10^{-26}$ w m⁻² (cps)⁻¹ may be represented by a model in which half the sources are associated with regions containing galaxies brighter than the 18th mag. and half the sources are so distant that their positions are not correlated with the density of galaxies brighter than the 18th mag. It should, however, be emphasized that this is by no means the only model which would fit the data of Table I. Any model in which a significant fraction of the bright radio sources are associated with regions containing galaxies brighter than the 18th mag. would fit the data. For the adopted model, a satisfactory fit to the data (see Fig. 1) is obtained if

$$P(\sigma) \sim e^{\sigma/30}, \quad (1)$$

in which $P(\sigma)$ is the probability that a bright radio source is associated with a region in which the surface density of galaxies is σ . Equation (1) indicates that the frequency of radio sources is strongly dependent on the space density of galaxies. However it should be emphasized that Eq. (1) cannot be interpreted in any simple way because the observed surface density of galaxies is the result of the superposition of galaxies at different distances. Also the contribution of a region containing a high space density of galaxies, to the observed surface density, will depend on the distance of that region. Finally, the surface density, due to clustering at a given distance, will depend on the size of the grid in which

TABLE I. The observed (O) and expected (E) frequency with which bright radio sources occur in regions with different surface densities of galaxies.

σ	O	E	O-E
0-19	0	0.0	0.0
20-39	12	18.7	-6.7
40-59	51	53.5	-2.5
60-79	27	31.0	-4.0
80-99	13	10.0	+3.0
100-119	7	2.6	+4.4
120-139	5	0.6	+4.4
140-∞	2	0.6	+1.4

he counts are made (one square degree in the present case).

The fact that a significant fraction of all bright radio sources appear to be associated with relatively nearby regions of space indicates that considerable caution should be exercised in interpreting the count-brightness relation for radio sources. The slope of the count-brightness relation will depend, to a certain extent, on the density of galaxies in the vicinity of our Galaxy. The effects of very large scale clustering (van den Bergh 1961a) will therefore be superimposed on effects which are determined by cosmological factors.

RADIO SOURCES AND CLUSTERS OF GALAXIES

Recently Mills (1960) has investigated the coincidences between the positions of radio sources in the Sydney survey of the zone $-20 < \delta < +10^\circ$ and rich clusters of galaxies in Abell's (1958) catalogue. Mills found 55 coincidences between radio sources and clusters of galaxies, whereas only 16 chance coincidences would be expected from his material. A similar analysis can be made of the sources in the Cambridge 3C catalogue (Edge *et al.* 1959). The positions of 282 3C sources with $|b^1| \geq 25^\circ$ were compared with the positions of clusters in Abell's catalogue. Those cases in which the 3C position of a radio source was located within 20 minutes of arc of the center of a cluster were regarded as coincidences. The number of chance coincidences to be expected from the comparison of the two catalogues was estimated from the coincidences between clusters and radio sources with 20^m added to their right ascension. Twenty-seven coincidences between clusters and radio sources were found, whereas the expected number of chance coincidences was found to be only nine. These statistics support Mills' conclusion that the majority of the coincidences between clusters and radio sources are probably real. Additional support for this conclusion is provided by the observation that the mean radio brightness of sources which are associated with nearby clusters is significantly higher than the mean radio brightness of sources which appear to be associated with distant clusters.

Among the 34 bright radio sources with a flux $S > 25 \times 10^{-26} \text{ w m}^{-2} (\text{cps})^{-1}$ at 159 Mc, which are located at galactic latitude $|b^1| \geq 25^\circ$, 8 coincidences with clusters in Abell's catalogue, were found. Of these 34 sources, at least one, 3C48, appears to be galactic rather than extragalactic (Sandage *et al.* 1960). Some are, no doubt, located in very distant clusters which are not visible on the Sky Survey plates. The bright source 3C295 is an example of such a case (Minkowski 1960). Also the bright source 3C274=M87, is located in the nearby Virgo cluster, which is not included in Abell's catalogue of distant clusters. The fraction of all bright radio sources with $|b^1| \geq 25^\circ$, which are located in rich clusters is therefore at least $10/33=0.30$. It has been estimated (van den Bergh 1961b) that the fraction

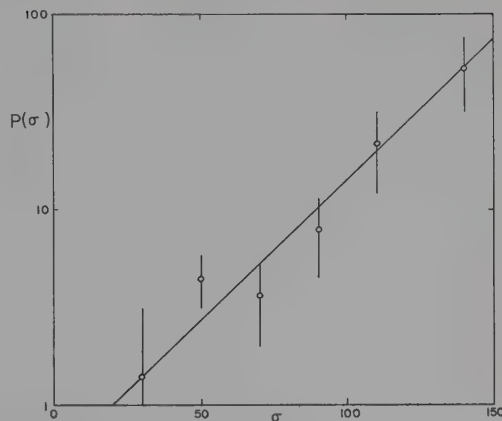


Fig. 1. Probability $P(\sigma)$, on an arbitrary scale, that a bright radio source occurs in a region containing σ galaxies per square degree, for a model in which half of the observed bright radio sources are associated with regions containing galaxies brighter than the 18th mag.

of all galaxies which are located in rich clusters is smaller than 0.10. A similar estimate has been obtained by Minkowski (1961). It therefore appears that galaxies in clusters are more likely to be radio sources than are field galaxies. A similar conclusion appears to be indicated by the strong dependence of the occurrence of radio sources on the surface density of galaxies, which is indicated by Eq. (1).

The fact that cluster galaxies have a higher probability of being radio sources than do field galaxies may be accounted for in at least two ways.

(1) It may be assumed that elliptical galaxies, which occur frequently in clusters, but are relatively rare in the general field, are more frequently associated with strong sources of radio radiation than are spiral galaxies. The evidence of nearby identified radio sources appears to be consistent with this assumption.

(2) It may be assumed that a large fraction of all strong radio sources are due to the collisions of galaxies. Such collisions would be expected to occur most frequently in clusters of galaxies.

It appears possible to distinguish between these two possibilities by considering the frequency with which radio sources occur in clusters of different richness. If a significant fraction of all radio sources are due to collisions, then one would expect the frequency with which radio sources occur in clusters to be strongly dependent on the total cluster population and density. The frequency with which collisions occur in clusters will be proportional to $n\sigma v$, in which n is the number of cluster galaxies, σ the surface density of galaxies in a cluster, and v the velocity dispersion of the cluster galaxies. If the cluster radius is independent of n and if the cluster mass is directly proportional to n , then the frequency of collisions is proportional to n^1 . It appears rather probable that very rich clusters of galaxies may have a

somewhat larger radius than do poorer clusters. The actual collision frequency may therefore depend on a power of n which is slightly less than $\frac{5}{2}$. Lacking detailed knowledge of the dependence of the cluster radius on the cluster density and cluster mass, it might be assumed that the actual collision frequency is proportional to n^2 .

From the 3C catalogue, the catalogue of Mills, Slee, and Hill (1960) and Mills (1960) data are available on 51 coincidences between radio sources with $|b^1| \geq 25^\circ$

TABLE II. Frequency of coincidences with clusters of different richness.

r	N	$C_r(\text{obs})$	Δ_r	C_r
1	50-79	39	11.7	27.3
2	80-129	9	3.6	5.4
3+4+5	130- ∞	3	0.7	2.3

and clusters of galaxies in Abell's "statistically homogeneous sample." From the statistics, approximately 16 of these 51 coincidences are expected to be due to chance. Table II shows how these coincidences are distributed over clusters of different richness. In the table, r is Abell's richness class, N the number of galaxies within two magnitudes of the third brightest cluster member, $C_r(\text{obs})$ the number of coincidences with clusters of richness r , Δ_r the expected number of chance coincidences, and C_r the true number of coincidences between radio sources and clusters of richness r . Δ_r has been obtained from the total expected number of chance coincidences, under the assumption that the number of chance coincidences with clusters of richness r is directly proportional to the number of clusters of richness r in Abell's homogeneous sample.

In Table III the true frequency distribution C_r from Table II is compared with the calculated frequency

TABLE III. Comparison of the true frequency distribution of coincidences C_r with the calculated frequency of coincidences $C_r(\text{calc})$ for the case in which the frequency of collisions is independent of the total cluster population.

r	N	C_r	$C_r(\text{calc})$
1	50-79	27.3	25.4
2	80-129	5.4	8.0
3+4+5	130- ∞	2.3	1.6

distribution of coincidences $C_r(\text{calc})$ which was obtained under the assumption that the probability that a radio source occurs in a cluster is independent of the total cluster population. The agreement is seen to be good. Due to the small number of coincidences which are available for discussion and the resulting uncertainty of the statistics, the case in which the frequency of radio sources in clusters is directly proportional to the number of cluster members cannot be excluded with certainty. A frequency of radio sources proportional to the square

TABLE IV. Probable identifications of NGC galaxies with radio sources.

NGC	$m_r - m_{pg}$	Type	Remarks
474	-4.0	E1t	Fuzzy nucleus surrounded by segments of nebulous ring. Interacting with NGC 470.
533	-3.6	E2:	NGC 545 appears fuzzy; is the brightest member of a small cluster of E galaxies.
545	-4.8	E3:	
547		E1:	
584	-1.9	E4	Brightest member of a small cluster containing mostly E galaxies.
1068	-1.4	Sbp	Seyfert galaxy; in a small cluster.
1218	-4.4	E1	Isolated, not in a cluster.
1275	-6.2	E1p	Brightest member of Perseus cluster. Dark patches and bright knots. Similar to NGC 1316 and NGC 5128?
1316	-4.9	E3p	Dark patches and bright knots. Similar to NGC 1275 and NGC 5128?
1399	-1.6	E0	Apparently normal supergiant Sb in a small loose cluster.
1417	-2.3	Sb I-II:	
2217	-0.5	SBa I	Apparently normal SBa. Not in cluster.
4038	-0.7	Sctt	Colliding supergiant spirals in a small very open cluster.
4039		Stt	
4234	-3.6	SB ⁺ IV	Normal SB dwarf in Virgo cluster. Probably optical.
4261	-4.4	E2	In Virgo cluster.
4486	-5.9	E1	Jet. In Virgo cluster.
4782	-5.0	E0(t?)	Double E
4783		E0(t?)	
5128	-4.0	E0p	Dark patches and bright knots. Similar to NGC 1316 and NGC 1275?
5792	-2.5	Sb(n?)p	Highly peculiar spiral with overextended spiral arms like those in the Seyfert galaxy NGC 4258.
6166	-6.2	E3+E2: +E3:	Multiple E. Brightest member of a cluster.
7177	-2.3	Sb ⁺ II	Isolated Sb. Has peculiar outer arm.
7720	-6.2	E2: (+E0)?	Brightest member of cluster containing E galaxies. Double or star superimposed.

of the cluster population appears to be excluded by the data. It may therefore be concluded that collisions of galaxies probably do not account for a significant fraction of all radio sources.

STATISTICS OF INDIVIDUAL IDENTIFICATIONS

As has recently been re-emphasized by Minkowski (1961), selection effects seriously affect the sample of coincidences between radio sources and faint distant galaxies. It therefore appears prudent to limit discussion to coincidences between radio sources and relatively bright galaxies. Table IV contains a listing of

probable identifications with NGC objects for which $m_r - m_{pg} < 0.0$. The data in the table have been drawn from recent compilations by Bolton (1960), Dewhirst (1959) and Mills, Slee, and Hill (1958, 1960). Whenever possible the galaxies in the table were classified on the blue prints of the Palomar Sky Survey. In a few cases the classification was based on reproductions of plates obtained with large reflectors.

Of the 21 coincidences listed in Table IV, 14 are with elliptical galaxies, i.e., 67%. Of the northern Shapley-Ames galaxies only 22.9% are ellipticals (van den Bergh 1960). One may therefore conclude that elliptical galaxies are more frequently associated with radio sources than are spiral galaxies. The fact that ellipticals are strongly concentrated in clusters probably accounts for the fact that such a large fraction of all bright radio sources are found to be associated with clusters. Inspection of Table IV shows that the values of $m_r - m_{pg}$ are systematically smaller for ellipticals than they are for spirals (see Table V). This suggests that, in the mean, ellipticals are intrinsically stronger sources of radio radiation than are spirals. Of the 14 coincidences with ellipticals at least three and possibly four are associated with double or multiple ellipticals. Among

TABLE V. Frequency of $m_r - m_{pg}$ for NGC galaxies.

$m_r - m_{pg}$	No. ellipticals	No. spirals
-0.1 to -1.5	0	3
-1.6 to -3.0	2	3
-3.1 to -4.5	5	1
-4.6 to -6.0	4	0
-6.1 to -7.5	3	0

the more than 200 elliptical galaxies in the Shapley-Ames catalogue north of declination -27° there are only three or four similarly compact binary systems, with both components ellipticals. Although the numbers are small, and the statistics correspondingly uncertain, the data do appear to favor the suggestion that binary or multiple ellipticals are more likely to be radio sources than are single ellipticals. The data in Table IV may indicate that binary elliptical galaxies are, in the mean, more powerful sources of radio radiation than are single ellipticals. However, the data are not sufficiently numerous to establish this conclusion.

The frequency with which radio sources are associated with clusters of galaxies, which has been discussed in the previous section, indicates that most binary elliptical galaxies should probably not be regarded as colliding galaxies. If binary ellipticals do not represent objects which are colliding, then they are either stable binaries or expanding systems, as has been suggested by Ambartsumian (1956). It would, however, appear to be difficult to reconcile the expansion hypothesis with current ideas on stellar evolution. It is therefore tentatively concluded that close binary and multiple ellipticals are probably stable systems.

REFERENCES

- Abell, G. O. 1958, *Astrophys. J. Suppl.* **3**, 211.
 Ambartsumian, V. A. 1956, *Proceedings of the 5th Conference on Questions of Cosmology* (Moscow), p. 413.
 van den Bergh, S. 1960, *Publs. David Dunlap Obs.* **2**, 159.
 —. 1961a, *Publs. Astron. Soc. Pacific* **73**, 46.
 —. 1961b, *Zs. Astrophys.* (to be published).
 Bolton, J. G. 1960, *Obs. Calif. Inst. Tech., Radio Obs.* No. 5.
 Dewhirst, D. W. 1958, in *Paris Symposium on Radio Astronomy* (Stanford University Press, Stanford, California), p. 507.
 Edge, D. O. Shakeshaft, J. R., McAdam, W. B. Baldwin, J. E., and Archer, S. 1959, *Mem. Roy. Astron. Soc.* **68**, 37.
 Mills, B. Y., Slee, O. B., and Hill, E. R. 1958, *Australian J. Phys.* **11**, 360.
 —. 1960, *ibid.* **13**, 676.
 Mills, B. Y. 1960, *ibid.* **13**, 550.
 Minkowski, R. 1960, *Astrophys. J.* **132**, 908.
 —. Joint Symp. Radio Astronomy Natl. Acad. Sci. U. S. A. and Acad. Sci. U.S.S.R., Green Bank, West Virginia, May 15-19, 1961. *Proc. Natl. Acad. Sci.* (to be published).
 Sandage, A. R., Greenstein, J. L., Matthews, T. A., and Münch, G. 1960, Paper presented to 107th meeting Am. Astron. Society in New York on December 3, 1960.
 Shane, C. D. and Wirtanen, C. A. 1954, *Astron. J.* **59**, 285.
 Shane, C. D., Wirtanen, C. A., and Steinlin, U. 1959, *ibid.* **64**, 197.

DISCUSSION

MILLS remarked that van den Bergh's statistics based on the 3C catalogue are roughly the same as his earlier statistics based on the Sydney catalogue of radio sources. (The 3C catalogue includes sources of smaller angular size than the early Sydney survey made with a pencil beam.) This fact suggests that his identifications of the sources are properly with individual galaxies rather than with intergalactic material in clusters, and this is consistent with later observations of angular size made at Sydney.

DE VAUCOULEURS questioned the classification of NGC 1275, 1316 and 5128 as elliptical galaxies. He and MRS. BURBIDGE emphasized that these and other possible misidentifications may have spuriously increased the proportion of radio sources identified as ellipticals, particularly at large distances where anything without obvious spiral arms is classified E. Under closer examination, many of these may be reclassified as young objects.

VON HOERNER criticized the assumption that collisions of galaxies (resulting in radio sources) should be proportional to a power of the density greater than one. One would expect that power to be about two if each collision resulted in a radio source; but if the interstellar matter in a galaxy plays an important role, only the first collision of a galaxy could produce a radio source, and we would expect a smaller frequency of radio sources in dense clusters because most of the galaxies there have already suffered collisions.

BALDWIN objected to the statement that 50% of radio sources can be identified with optical galaxies brighter than 18 mag.; recent studies at Cambridge show that the proportion is nearer 10%. VAN DEN BERGH agreed that the proportion drops as one goes to

fainter radio sources, partly due, perhaps, to difficulties in interpreting the radio observations. His lower limit of flux density was $25 \times 10^{-26} \text{ W m}^{-2} (\text{cps})^{-1}$.

MINKOWSKI, in a preview of his paper presented at IAU Symposium No. 15 (Proceedings, in press), said that the identified radio sources confirmed the association of such sources with clusters of galaxies. However, the evidence seems to indicate no statistical correlation between identified radio sources and double galaxies when account is taken of the preferential search for double galaxies in making the identifications. He suggested that van den Bergh's contrary conclusion may be due to the fact that his data do not refer to a unit volume, and maintained that sound statistical data can be obtained only from lists of identified radio sources complete to definite limiting magnitudes (radio and photographic) in a specific area of the sky. Moreover, van den Bergh's correlation between radio source and the first power of density of galaxies in clusters would

be expected if radio sources occur at low probability in single galaxies.

VAN DEN BERGH added that Matthews' new results (IAU Symposium No. 15, in press) on the identification of radio sources appear to confirm the conclusion that binary elliptical galaxies have a high probability of being associated with radio sources.

KERR agreed that the question of whether radio sources are single or double galaxies is a complex one. Although the statistics of identified sources do not reveal a clear relation with double galaxies, it is true that a high proportion of radio sources are double *in themselves*. A specially interesting case is Centaurus A (NGC 5128) which appears to consist of a close pair of radio sources in the middle of a much wider pair of radio sources.

AMBARTSUMIAN remarked that recent work at the Burakan Observatory supported van den Bergh's conclusions.

THE ASTRONOMICAL JOURNAL

VOLUME 66, NUMBER 10

DECEMBER, 1961

Stability of Clusters of Galaxies

SIDNEY VAN DEN BERGH

David Dunlap Observatory, Richmond Hill, Ontario, Canada

I. INTRODUCTION

THE following methods may be used to determine the masses of galaxies:

(a) Individual mass determinations of galaxies can be obtained from rotation curves. This method has been applied to a fairly large number of spiral galaxies.

(b) Individual masses may be estimated from the velocity dispersion of stars in the nuclei of elliptical nebulae.

(c) Masses of members of physical pairs of galaxies may be determined from a statistical analysis of the radial velocity differences of the components.

(d) The virial theorem may be used to obtain the mean masses of cluster galaxies, if it is assumed that such clusters are in dynamical equilibrium.

II. MASSES DERIVED FROM ROTATION

A number of investigations of the rotation of individual spiral galaxies have been carried out in recent years, using both optical observations of *HII* regions and 21-cm observations of neutral hydrogen. These investigations all yield mass-to-luminosity ratios in the range $1 < M/L < 30$. (Throughout this paper a Hubble constant $H = 100 \text{ km/sec/Mpc}$ is used and a galactic absorption $A_{\text{pg}} = 0.24 \text{ csc } |b|$ is adopted. All quoted mass-to-luminosity ratios are for photographic light.) In view of the large observational uncertainties involved

in any individual mass determination, the true range of the mass-to-luminosity ratios may be somewhat smaller than is indicated above. Possibly the data indicate a tendency for high-luminosity Sb galaxies to have larger mass-to-luminosity ratios than do low-luminosity Sc galaxies. However the data appear to leave little doubt as to the order of magnitude of the mass-to-luminosity ratios in spiral galaxies.

III. MASS DETERMINATION FROM VELOCITY DISPERSION

Using the velocity dispersion method, Burbidge, Burbidge, and Fish (1961 a, b) have recently studied the mass-to-luminosity ratio of the dwarf elliptical galaxy M32. Even taking into account the inherent observational difficulties associated with this method and the uncertainties in both the distance to and the reddening of M32, the mass-to-luminosity ratio falls in the same range as that which is obtained from the rotation of spirals.

IV. MASSES OF BINARY GALAXIES

A statistical approach to the mass determination of pairs of galaxies has recently been used by Page (1961) to determine the mass-to-luminosity ratios of 42 pairs. The mass-to-luminosity ratio which is obtained in this way is proportional to the square of the velocity difference of the components of the pair. The inclusion in

TABLE I. Data on 23 pure pairs.

NGC	M_{pg}	DDO type and class	$\bar{V}_c$	ΔV	W_Δ	S
2535	-20.1	S(B)ct I:	3983	1	3.44	1.75
2536	-19.2:	Pec				
2693	-20.7	Ep	5082	167	0.83	0.9
2694	-18.4	E0				
2719	-18.9:	Sbt or Sct	3143	137	2.66	0.4
Anon	-18.4:	Pec				
3226	-17.9	E2	1110	217	10.75	2.3
3227	-19.2	Sbnt				
3395	-19.0	Sct	1630	22	6.15	1.3
3396	-18.6	Pec(t)				
3454	-17.0	Sc(p)	1034	54	2.17	3.8
3455	-17.3	Sb(nt)				
Ho 231a	-20.4:	Sp	6086	318	1.27	0.8
231b	-20.1:	Sct				
3769	-17.2	Sbn	750	28	2.24	1.3
Anon	-15.6:	Et or Pec				
4038	-19.8:	Sctt	1452	39	5.72	1.2
4039	-19.6:	Stt				
4105	-19.8	E2	1805	283	1.34	1.3
4106	-19.4	E1t				
4485	-17.2	Irr III-IV:	695	155	1.17	3.5
4490	-19.4	Scn*t III:				
4782	-20.5:	E0(t?)	4194	628	1.00	0.7
4783	-20.7:	E0(t?)				
5194	-20.2	Sc(t) I	586	100	3.76	4.4
5195	-18.6	Pec(t)				
5257	-21.5:	Sbt I: or Sct I:	6645	175	1.03	1.4
5258	-21.1:	Sbt I: or Sct I:				
5426	-19.3	Sct	2211	96	2.19	2.6
5427	-20.0	Sc(t) I				
5480	-19.2	Sc**t I-II:	2010	305	0.54	3.1
5481	-18.6:	E2				
5506	-18.5:	Scp	1987	311	1.19	3.9
5507	-17.9:	E5p				
5574	-17.9	?	1601	185	0.54	2.8
5576	-19.3	E2				
5774	-18.6	S- IV:	1555	40	1.15	4.5
5775	-19.2	Sb or Sc				
5929	-18.4:	Et	2782	175	2.61	0.5
5930	-18.9:	Sbnnt				
5953	-18.8:	Et	2166	30	3.76	0.8
5954	-18.9:	Sct				
6068	-20.2:	Sc I	4186	71	2.45	2.0
Anon	-19.4:	S				
7714	-19.3:	Sctt	2982	38	1.10	2.0
7715	-18.4:	St				

TABLE II. Mass-to-luminosity ratios for pure pairs.

Type of pair	No. of pairs	$\mathcal{M}/L$	$\mathcal{M}/L$ (Page)
E+E	3	50 ± 20	94 ± 38
E+S	6	26 ± 23	48 ± 28
S+S and S+Irr	9	4.8 ± 4.2	0.33 ± 0.35

five pairs in Page's sample, which have velocity differences larger than 500 km/sec, are located in the Virgo cluster suggests that his sample contains some pairs which do not form permanently bound systems. The danger of including such systems is, of course, largest in clusters. All Page's pairs were therefore checked for possible cluster membership on the prints of the Palomar Sky Survey. All probable cluster members and members of multiple systems were eliminated from the sample. During this inspection of the Sky Survey prints it became apparent that a number of systems had been incorrectly classified. All galaxies were therefore reclassified on the DDO system. The data on the remaining 23 pure pairs are given in Table I. The table contains the NGC number, the absolute magnitude, the DDO type and whenever possible the luminosity class, the mean corrected redshift, the velocity difference of the components, the weight of the velocity difference ($W_\Delta = 1.0$ corresponds to an uncertainty of 90 km/sec in the velocity difference) and the apparent separation of the components.

Using the equations given by Page (1961), least-squares solutions were made for the mass-to-luminosity ratios of elliptical, spiral and mixed pairs. The results of these computations are shown in Table II. The principal difference between the present results and those obtained by Page occurs for pairs in which both members of pairs are spirals. The present value $\mathcal{M}/L = 4.8 \pm 4.2$ appears to be quite consistent with the mass-to-luminosity ratios of spirals which are derived from the analysis of their rotation curves.

V. MASSES OBTAINED FROM THE VIRIAL THEOREM

It was first pointed out by Zwicky (1933) and Smith (1936) that the application of the virial theorem to

TABLE III. Mass-to-Luminosity ratios for clusters.

Cluster	Population	$\mathcal{M}/L$	Reference
Coma cluster	E	900 ^a	van den Bergh (1960a)
Virgo cluster	E+S	600	van den Bergh (1960b)
Canes Venatici cluster	S	400	van den Bergh (1960a)
South pole group	S	500-900	de Vaucouleurs (1959)

^a This value of $\mathcal{M}/L$ was obtained from a total cluster mass of $2.5 \times 10^{15} \mathcal{M}_\odot$ (van den Bergh 1960a) and a total cluster luminosity of $2.8 \times 10^{12} L_\odot$. This total cluster luminosity is based on the luminosity given by Schwarzschild (1954) converted to $H=100$ km/sec/Mpc. An improved value of the total cluster luminosity may probably be obtained by fitting Abell's (1958) mean luminosity function of rich clusters to counts by Zwicky (1957) and Shane and Wirtanen (1954). After correction for galactic absorption and K effect this yields a total cluster luminosity of $6.9 \times 10^{12} L_\odot$ and hence a mass-to-light ratio of 360.

the sample of even a small number of optical pairs, or pairs moving in hyperbolic relative orbits, will therefore strongly affect the results. The fact that four out of the

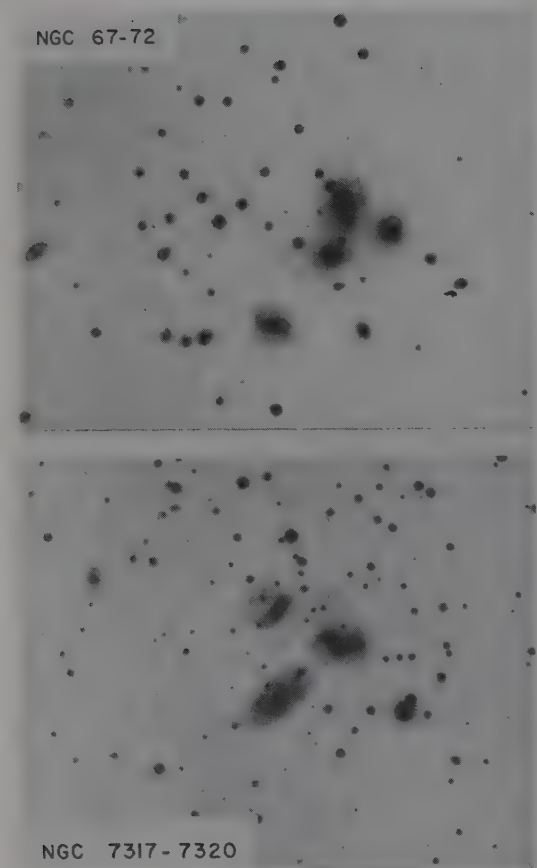


FIG. 1. Reproductions from the red prints of the Palomar Sky Survey of Stephan's Quintet (VV 288) and the cluster VV 166. (Both prints have the same scale.) Copyright National Geographic Society—Palomar Sky Survey.

clusters of galaxies gives extremely high total cluster masses. If it is assumed that this mass is entirely concentrated in galaxies then one obtains remarkably high mass-to-luminosity ratios. Some recent determinations of cluster mass-to-luminosity ratios are listed in Table III. Although the individual mass-to-luminosity ratios given in Table III are subject to considerable uncertainty, they do appear to show that the mass-to-luminosity ratios which are obtained for clusters are

TABLE IV. Listing of supercompact clusters.

Vorontsov-Velyaminov cluster number	α (1950)	δ (1950)	Popu- lation	Remarks
VV 115	15 ^h 57 ^m 0	+20°54'	E+S	Seyfert's cluster
VV 116	09 36.4	-04 37	E+S	
VV 165	14 48.7	+19 17	E+S?	NGC 67-72 Stephan's Quintet
VV 166	00 15.8	+29 48	E+S	
VV 288	22 33.7	+33 42	E+S	
Anon	00 46.8	+23 06	E	

TABLE V. Comparison of clusters VV 166 and VV 288.

Cluster	$\bar{V}_e$	M_{pg} (total)	S^a
VV 166	+6785 km/sec	-21.5:	1.1×10^{22} cm
VV 288	+6695	-22.3:	1.0×10^{22}

^a S is the mean projected separation of the five nuclear constituents.

at least an order of magnitude higher than those which are obtained for individual galaxies which are members of pairs. A number of hypotheses have been advanced to account for the very large masses which are obtained by applying the virial theorem to clusters of galaxies. Zwicky (1933) has proposed that very large amounts of nonluminous intergalactic matter may exist in clusters. On the other hand, Ambartsumian (1958) has suggested that clusters of galaxies may be expanding so that the virial theorem is not applicable.

VI. SUPERCOMPACT CLUSTERS

In view of the recent observations by Burbidge and Burbidge (*Astrophys. J.* **134**, 244, 1961) that NGC 7320 is not a physical member of Stephan's Quintet, the present discussion of this system should be substantially modified.

Recently the Burbidges (1959) have pointed out that supercompact clusters, such as Stephan's Quintet may provide a means of distinguishing between these two hypotheses. Typical supercompact clusters have volumes which are almost 10^4 times smaller than the volumes of normal clusters. It therefore appears unlikely that they could contain very large amounts of intergalactic gas. Also the dimensions of supercompact clusters are so small that such systems could only exist for a very short time if they possessed a positive total energy.

The dynamics of Stephan's Quintet has recently been discussed in detail by Burbidge and Burbidge (1959) and by Limber and Mathews (1960). These authors agree that a rather high mass-to-luminosity ratio is required if Stephan's Quintet is to be regarded as a stationary, i.e., nonexpanding, system. However considerable uncertainty is introduced by the fact that the kinetic energy of the system depends almost entirely on the relative motion of the components of the pair NGC 7318 a/b. A considerable reduction in the mass-to-luminosity ratio of the system may be achieved if it is assumed that the velocity vectors of the components of this pair are parallel or almost parallel to the line of sight. It would therefore be of considerable interest to investigate the dynamics of other supercompact clusters. The supercompact clusters listed in Table IV were found during a systematic search of the Sky Survey prints and would appear to be the most suitable for such an analysis. Only for cluster VV 166 are the data sufficiently complete to allow a comparison with Stephan's Quintet (VV 288). The supercompact clusters VV 166 and VV 288 are surprisingly similar, as can be

TABLE VI. Data on the individual members of clusters VV 166 and VV 288.

NGC	DDO type	M_{pg}	$V_c - \bar{V}_c$	NGC	DDO type	M_{pg}	$V_c - \bar{V}_c$
68	E1	-20.0	-773	7317	E4	-19.5	+308
69	E?	-18.7	+77	7318a	Stt	-20.0	+254 ^a
70	S*t	-19.7	...	7318b	S*tt	-19.9	-790
71	S?	-19.8	+31	7319	S(B)(n)tt	-21.1	+229
72	S(B)	-19.9	+416	7320	Sc*	-21.2	...
Anon	E?	-18.7	+247				

^a Mean of Lick and Mt. Wilson velocities.

from Fig. 1 and the data in Tables V and VI. Both are subclusters within rather poor clusters which contain a mixed population of spirals and ellipticals. On the whole, the galaxies in VV 166 are of slightly earlier type than those in VV 288.

Limber and Mathews (1960) have shown that the kinetic and potential energies of a cluster of galaxies may be expressed as

$$T \simeq \frac{1}{2} \sum_i m_i V_{ri}^2, \quad (1)$$

$$\Omega \simeq - \frac{1}{2} \sum_{i,j \text{ pairs}} \frac{G m_i m_j}{r_{ij}'}, \quad (2)$$

which V_{ri} is the radial velocity of the i th component and r_{ij}' is the projected separation of components i and j . They also showed that, for a quintet of mass points which are in equilibrium, the relation

$$2T \simeq -\Omega \quad (3)$$

is satisfied at any instant, to within an accuracy of about 10%. Equations (1), (2), and (3) were used to compute the mass-to-luminosity ratios for cluster VV 166 and Stephan's Quintet. For the sake of simplicity it was assumed that, within each cluster, all galaxies have the same mass-to-luminosity ratio. The separations of the components were measured on the prints of the Palomar Sky Survey. Equipartition of energy was assumed to obtain the kinetic energies of NGC 70 and NGC 7320, for which no radial velocity measurements are available. The computed energies and mass-to-luminosity ratios are given in Table VII.

Due to the uncertainties in the projection factors and the directions of the velocity vectors, no great accuracy can be claimed for these results. However the data for cluster VV 166 do appear to confirm the high mass-to-luminosity ratio which has been found for Stephan's Quintet.

Inspection of the photographs of both clusters reveals the presence of a considerable amount of luminous intergalactic material (tidal arms). The inclusion of

this diffuse light would tend to increase the total cluster luminosity and hence decrease the mass-to-luminosity ratio. Clearly additional photometric observations are highly desirable. Radial velocity measurements for NGC 70 and NGC 7320 would also add considerably to the reliability of the results.

Two peculiarities of supercompact clusters are of some interest:

(1) Most supercompact clusters in Table IV contain a mixed population of spirals and ellipticals, whereas ordinary dense clusters, such as the Coma cluster, contain a pure elliptical population in their dense nuclear regions.

(2) Both clusters VV 166 and VV 288 contain more bright galaxies and fewer faint galaxies, than does the general luminosity function of galaxies. In this respect these superdense clusters of galaxies are similar to galactic star clusters which also exhibit a deficiency of faint members.

It is not clear if supercompact clusters are in any way related to such objects as NGC 6166. In the radio source NGC 6166 the mean separation of the components is smaller than that in a typical supercompact cluster by one order of magnitude. Possibly this object should therefore be regarded as a supergiant elliptical galaxy with multiple nuclei, rather than as a supercompact cluster. None of the supercompact clusters in Table IV are associated with known radio sources. A systematic search of the prints of the Palomar Sky Survey turned up only six supercompact clusters. Such clusters must be extremely rare. It is estimated that less than a dozen such objects, with redshifts smaller than 10 000 km/sec, exist in the entire sky.

VII. MASS-TO-LUMINOSITY RATIO

The data on mass-to-luminosity ratios which have been discussed in previous sections are summarized in Table VIII.

TABLE VII. Energy and mass-to-light ratios of two supercompact clusters.

Cluster	$2T$	$-\Omega$	M/L
VV 166	$7.8 \times 10^{49} M/L$	$2.2 \times 10^{57} (M/L)^2$	350
VV 288	$8.8 \times 10^{49} M/L$	$9.3 \times 10^{57} (M/L)^2$	100

TABLE VIII. Summary of data on mass-to-luminosity ratios.

Clusters of galaxies	$400 \leq M/L \leq 900$	see Table III
Supercompact clusters	$100 \leq M/L \leq 350$	see Table VII
Binaries in clusters	$M/L < 230$	van den Bergh 1960b
Noncluster binaries	$M/L < 70$	see Table II
Single galaxies	$1 < M/L < 30$	see Sec. II

All methods, except that based on the application of the virial theorem to clusters, agree in yielding $\mathfrak{M}/L < 100$ for ellipticals and $\mathfrak{M}/L \ll 100$ for spirals. No large systematic differences appear to be indicated between isolated individual galaxies and individual cluster galaxies. The conclusion that the large masses, which are found by the application of the virial theorem to clusters, are not associated with the individual cluster galaxies therefore appears inescapable.

Three possible explanations for this discrepancy will now be discussed briefly:

Hypothesis 1. *Intergalactic forces are not predominantly gravitational over distances in excess of 10 kpc.* The separations of the members of supercompact clusters are of the same order as those encountered in typical binary galaxies. Even if nongravitational forces are important, one would therefore expect to obtain similar mass-to-luminosity ratios for members of supercompact clusters and for members of binary systems. The data in Table VII appear to show that this is not the case.

Hypothesis 2. *Most of the mass of clusters of galaxies is due to the presence of nonluminous intergalactic matter.* Since the volumes of typical supercompact clusters are almost 10^4 times smaller than those of normal clusters it is difficult to see how they could contain a sufficient amount of intergalactic material to account for the high mass-to-luminosity ratios which appear to be indicated by the observations.

Hypothesis 3. *Clusters of galaxies are expanding systems with a positive total energy to which the virial theorem is not applicable.* Although attractive at first sight this hypothesis meets with a number of serious difficulties. These difficulties are briefly discussed below.

A. Time Scale

If the members of supercompact clusters have normal mass-to-luminosity ratios then their mutual gravitational attraction is quite insufficient to hold them together. In first approximation such systems may then be regarded as being in free expansion. The time scale for this expansion is of the order of

$$\tau = D/V. \quad (4)$$

For a typical supercompact cluster $D \simeq 50$ kpc and $V \simeq 500$ km/sec, so that $\tau \simeq 1 \times 10^8$ years. On the basis of current theories of stellar evolution such an age appears incompatible with the existence of an apparently normal giant elliptical galaxy in both Stephan's Quintet and in cluster VV 166, unless one assumes that the expansion age of these clusters is very different from the age of their constituents. Application of Eq. (4) to rich clusters of galaxies yields expansion ages of a few times 10^9 years, i.e., ages which are smaller than the currently accepted age of the universe by about an order of magnitude.

B. Population of Intercluster Regions

It is well known that most giant ellipticals are located in clusters, whereas the population of the general field consists predominantly of spirals. If the ages of clusters are much shorter than the age of the universe, then the number of clusters which now exists must be much smaller than the number of all clusters that ever existed. Most giant elliptical galaxies which were once located in clusters, which have since dispersed, should now be found in the general field. The assumption that rich clusters of galaxies are dispersed with a time scale short compared with the age of the universe would therefore appear to contradict the observation that most giant ellipticals are, at the present time, located in clusters.

C. Velocity Dispersion of Field Galaxies

If clusters of galaxies are systems of positive energy, then some cluster galaxies will leave clusters with very high space velocities. The presence of an appreciable number of such cluster escapees among the population of field galaxies would lead to a rather high velocity dispersion. This appears to be in contradiction with a recent investigation by Neyman and Scott (1961), who find $\sigma^2 = -66 \text{ (km/sec)}^2 \pm 121 \text{ (km/sec)}^2$ for field galaxies.

The evidence presented above makes it appear unlikely that either dense rich clusters or small supercompact clusters are expanding rapidly. The rather awkward situation with which we are apparently faced may perhaps be summarized as follows:

- (1) The masses of cluster galaxies are too low to prevent such clusters from expanding rapidly.
- (2) The assumption that clusters of galaxies are indeed expanding rapidly leads to predictions which appear to be in conflict with observation.

REFERENCES

- Abell, G. O. 1958, *Astrophys. J. Suppl.* **3**, 211.
 Ambartsumian, V. A. 1958, *La Structure et l'évolution de l'univers*, Solvay Conference (R. Stoops, Brussels, Belgium), p. 241.
 van den Bergh, 1960a, *Astrophys. J.* **131**, 558.
 —. 1960b, *Monthly Notices Roy. Astron. Soc.* **121**, 387.
 Burbidge, G. R. and Burbidge, E. M. 1959, *Astrophys. J.* **130**, 15.
 Burbidge, E. M., Burbidge, G. R., and Fish, R. A. 1961a, *ibid.* **133**, 393.
 —. 1961b, *ibid.* **133**, 1092.
 Limber, D. N. and Mathews, W. G. 1960, *ibid.* **132**, 286.
 Neyman, J. and Scott, E. L. 1961, *Astron. J.* **66**, 148.
 Page, T. L. 1961, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (to be published).
 Schwarzschild, M. 1954, *Astron. J.* **59**, 273.
 Shane, C. D. and Wirtanen, C. A. 1954, *ibid.* **59**, 285.
 Smith, S. 1936, *Astrophys. J.* **83**, 23.
 de Vaucouleurs, G. 1959, *ibid.* **131**, 718.
 Zwicky, F. 1933, *Helv. Phys. Acta* **6**, 110.
 —. 1957, *Morphological Astronomy* (Springer-Verlag, Berlin), p. 41.

DISCUSSION

G. BURBIDGE asked why the values of M/L for double galaxies differ so markedly from those recently published by Page (*Astrophys. J.* **132**, 910, 1960). PAGE answered that statistical fluctuations could easily account for this, as indicated by the rms errors given above, but disputed the large number of "optical pairs" omitted by van den Bergh from his list of double galaxies.

In answer to OORT, VAN DEN BERGH explained that he obtained the high value of $M/L=900$ for the Coma cluster on the assumption of an isotropic velocity distribution and a density distribution similar to that of a polytrope of index 5. Assuming a uniform density distribution, would give M/L one-half as great. It was agreed that there is a large uncertainty in L due to errors in measured total magnitudes of members of the Coma cluster. VAN DEN BERGH also mentioned that one velocity from the Humason-Mayall list was rejected as a foreground object projected on the Coma cluster. HOLMBERG commented that these high values of M/L for clusters should be reduced, as shown in his paper below.

BOK protested the discussion of time scales as short as 10^8 years, noting that even the youngest galaxies, such as the Large Magellanic Cloud, contain old stars as well as the better-known, young associations of stars and gas clouds. He questioned whether any galaxy could be formed in its present state in as little as 10^8 years.

HARLAN SMITH referred to the views of I. Shklovskii,

in which the radio galaxies are extremely young objects in their first burst of formation of stars and supernovae. On this view there must be *two* classes of elliptical galaxies: first, the embryonic type, just condensing, with the vaguely elliptical form observed in most of the identified radio sources. Galaxies of this type are later to develop spiral or irregular forms depending on initial conditions as yet unidentified. Evidence of this ancestral form is found in the spheroidal population II distribution in our own and other galaxies. Second, the conventional elliptical galaxies may be much older systems, and nearby examples of this type, such as M32 and NGC 205, have perhaps influenced us unduly to believe that the elliptical form of a galaxy always implies great age. From Shklovskii's point of view, then, it may well be that the youngest galaxies of all are amorphous, possibly unstable systems of roughly elliptical form.

DE VAUCOULEURS questioned whether there could be two such different classes of elliptical galaxies, since the *colors* of elliptical galaxies are so homogeneous. G. BURBIDGE answered that a sample of very young stars condensing on to the main sequence might well have the same integrated color as a population II sample consisting of old main-sequence and red-giant stars. This would require a detailed investigation of T Tauri stars and other specific types, as well as the expected luminosity functions. MRS. BURBIDGE noted that some irregular galaxies which are in rapid rotation must be very young, since they would achieve some measure of axial symmetry in a very few rotation periods.

Virial Theorem and the Stability of Clusters of Galaxies

D. NELSON LIMBER

Verkes Observatory, Williams Bay, Wisconsin

FOR more than a quarter of a century considerable use has been made of the virial theorem in attempts to determine the masses of clusters of galaxies. More recently, the values derived for these masses for a number of clusters have been questioned by some workers who have argued that the masses so obtained are unreasonably large and that it seems preferable to conclude that the clusters in question have positive energy and are unstable and that the application of the virial theorem in the form appropriate for stability for the determination of the masses of the clusters is therefore invalid. In opposition to this latter view, still others who have considered the problem have argued that the indirect evidence for the stability of these clusters is so strong that the masses so derived must have physical significance and must be accounted for in terms of the member galaxies and, if need be, unseen intracluster matter. The existence of this conference bears testimony to the fact that this question is still an open one.

In the present paper no attempt is made to resolve this question once and for all. Instead, I shall confine my discussion to a consideration of several aspects of the virial theorem itself and to a consideration of the application of the virial theorem to two small clusters of galaxies that seem to be of particular interest and importance: the M81 group and the local system.

VIRIAL THEOREM

A. Forms of the Virial Theorem

General Tensor Form

For the present let us consider the virial theorem as it applies to a self-gravitating system of n point masses. Let $m^{(\alpha)}$ denote the mass of that one of these point masses that is labeled by the index α , where $\alpha = 1, 2, \dots, n$, and let $\mathbf{x}^{(\alpha)}(t) \equiv (x_1^{(\alpha)}, x_2^{(\alpha)}, x_3^{(\alpha)})$ denote the position vector of this point mass with respect to the center of mass of the system at time t .

Chandrasekhar (1960) has shown that there exists a general tensor form for the virial theorem for such a system. In terms of the tensors

$$T_{ij} \equiv \frac{1}{2} \sum_{\alpha=1}^n m^{(\alpha)} \dot{x}_i^{(\alpha)} \dot{x}_j^{(\alpha)}, \quad (1)$$

$$\Omega_{ij} \equiv -\frac{1}{2} G \sum_{\alpha=1}^n \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^n m^{(\alpha)} m^{(\beta)} \times \frac{(x_i^{(\alpha)} - x_i^{(\beta)})(x_j^{(\alpha)} - x_j^{(\beta)})}{|\mathbf{x}^{(\alpha)} - \mathbf{x}^{(\beta)}|}, \quad (2)$$

and the inertia tensor

$$I_{ij} \equiv \sum_{\alpha=1}^n m^{(\alpha)} x_i^{(\alpha)} x_j^{(\alpha)}, \quad (3)$$

the appropriate tensor form for the virial theorem is

$$\frac{1}{2} \frac{d^2 I_{ij}}{dt^2} = 2T_{ij} + \Omega_{ij}. \quad (4)$$

If the system is stable against systematic expansion or contraction during an appropriate time interval, then it follows from Eqs. (3) and (4) that

$$2\langle T_{ij} \rangle + \langle \Omega_{ij} \rangle = 0, \quad (5)$$

where the symbol $\langle \dots \rangle$ denotes a time average of the bracketed quantity over such a time interval.

The tensor form for the virial theorem thus provides nine component equations. In particular, the stability of the system requires that Eq. (5) hold for each of these nine components.

Contracted Scalar Form

In the treatment of the virial theorem from the standpoint of the general tensor form, the usual scalar forms follow from the contraction of the corresponding tensor forms. To see this, we note first from Eqs. (1), (2), and (3) that

$$T_{ii} \equiv \frac{1}{2} \sum_{\alpha=1}^n m^{(\alpha)} |\dot{\mathbf{x}}|^2 \quad (6)$$

$$\equiv T,$$

$$\Omega_{ii} \equiv -\frac{1}{2} G \sum_{\alpha=1}^n \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^n \frac{m^{(\alpha)} m^{(\beta)}}{|\mathbf{x}^{(\alpha)} - \mathbf{x}^{(\beta)}|} \quad (7)$$

$$\equiv \Omega,$$

and

$$I_{ii} \equiv \sum_{\alpha=1}^n m^{(\alpha)} |\mathbf{x}^{(\alpha)}|^2 \quad (8)$$

$$\equiv I,$$

where the summation convention has been used, where T and Ω are the total kinetic and potential energies of the system, and where I is the polar moment of inertia. Using the results contained in Eqs. (6), (7), and (8) in the two tensor forms for the virial theorem as given in Eqs. (4) and (5), we obtain the two corresponding scalar forms

$$\frac{1}{2} (d^2 I / dt^2) = 2T + \Omega, \quad (9)$$

and, for stability,

$$2\langle T \rangle + \langle \Omega \rangle = 0. \quad (10)$$

It is sometimes convenient in practice to express this last scalar form for stability in a somewhat different form. For this purpose, let $\langle V_G^2 \rangle$ denote the mean-square velocity of the point masses—the mean being obtained by weighting the velocities of the different point masses proportionally to their masses and by averaging over the time interval in question, and let $\mathcal{M}_G$ and R denote the total mass of all the galaxies in the system and an appropriately defined outer radius, respectively. Equation (10) can be alternatively expressed in terms of these quantities as

$$\langle V_G^2 \rangle = C_{GG} G (\mathcal{M}_G / R), \quad (11)$$

where C_{GG} is a dimensionless constant that is defined with respect to the value taken for R and that characterizes the mean spatial distribution of the mass in the form of galaxies within the system during the given time scale.

In this latter form that applies to the case of stability, the virial theorem makes it possible to determine the total mass of the system in terms of the mean-square velocity, the radius, and the relative spatial distribution of the mass within the system. It is in this sense that the virial theorem is most often applied to clusters of galaxies.

Modified Scalar Form to Include Effects of Intracluster Gas

The foregoing developments represent idealizations to the problem of clusters of galaxies. First, of course, the galaxies within a cluster are not point masses to nearly the same degree of approximation as is the case for clusters of stars. The dimensions of the individual galaxies in clusters are very often not too much smaller than the separations of neighboring galaxies in these clusters. Related to this is the point that it is not yet completely clear whether the total energies of clusters of galaxies whose members are considered as point masses are essentially constant subsequent to the formation of the cluster or whether these total energies can change significantly in the course of time as the result of different kinds of coupling between the motions and spatial distributions of the galaxies considered as point masses, on the one hand, and internal processes within the individual galaxies, on the other.

To the present, neither of these possible complications has been investigated in detail. However, on the basis of order-of-magnitude estimates for some of the more predictable of these effects, there is no evidence that the foregoing forms of the virial theorem require important modifications for these reasons for the purposes for which they are now used.

The situation may sometimes be different insofar as the neglect of another factor in these forms for the virial theorem is concerned. There is definite evidence for the

presence of intracluster gas and dust in at least several galactic star clusters, and it does not appear unreasonable to assume that there may be significant intracluster substrata of gas or of intergalactic stars associated with clusters of galaxies. If such substrata should include significant fractions of the total masses of clusters, then this matter—in addition to that in the form of galaxies—would play a major role in determining the spatial and velocity distributions of the member galaxies.

If it can be assumed—as seems reasonable—that the only important interaction of a substratum of this kind upon the dynamics of the galaxies is gravitational, then it can be shown (Limber 1959) that the stability of the system of the galaxies in such a cluster leads to an extended form for the virial theorem that relates the mean-square velocity of the galaxies, the radius of the cluster, the separate relative spatial distributions of the components of mass in the form of galaxies and in the form of the intracluster substratum, and the separate masses of these two components. This result can be conveniently expressed as a simple modification of the virial theorem in the form of Eq. (11) as follows:

$$\langle V_G^2 \rangle = C_{GG} G \frac{\mathcal{M}_G}{R} \left[1 + \left(\frac{C_{Gg} + D_{Gg}}{C_{GG}} \right) \frac{\mathcal{M}_g}{\mathcal{M}_G} \right], \quad (12)$$

where the symbols that have been used before have their previous meanings, where $\mathcal{M}_g$ is the total mass of the substratum, and where C_{Gg} and D_{Gg} are dimensionless constants that are appropriately defined functions of the relative spatial distributions of both the mass in the form of galaxies and the mass in the form of the substratum—the two distributions being different in general.

In the paper to which reference has been made concerning this modified form for the virial theorem, it is shown how the dimensionless coefficients that appear in Eqs. (11) and (12) can be readily derived from the relative spatial distributions for the two components of mass for cases in which the two distributions are spherically symmetric and have the same center in at least a first approximation.

A comparison of the corresponding expressions for the modified and for the usual forms for the virial theorem [Eqs. (12) and (11)] shows that the effectiveness—gram for gram—of the component of mass in an intracluster substratum relative to that in the form of galaxies in influencing the velocity dispersion for the galaxies will depend upon the relative spatial distributions of the mass in the form of galaxies and that in the form of the substratum. In the limit in which the spatial distribution of the intracluster substratum is much less centrally condensed than is the distribution of the galaxies, the intracluster substratum becomes of no consequence—gram for gram—relative to the mass in the form of galaxies. On the other hand, in the limit in which the substratum is far more centrally condensed than is the distribution of the galaxies, it can

be shown (Limber 1959) that the mass in the substratum can never be more than about three times as effective—gram for gram—as the mass in the form of the galaxies insofar as its effects upon the velocity dispersion of the galaxies is concerned. Finally, it follows from the modified form for the virial theorem—as it must—that if the two components have the same relative spatial distribution, then the masses in each of the two components are equally effective in this sense.

The modified form for the virial theorem and results such as these that follow from it will be important if it is found that intracluster substrata containing significant fractions of the total masses of clusters are associated with at least some clusters. For the present, the actual evidence for the presence of such intracluster substrata is only of an indirect kind, at best.

B. Application of the Virial Theorem to Clusters of Galaxies

Nature of the Available Observational Data

In practice the virial theorem as applied to clusters of galaxies has usually been used to derive estimates for the mass of the cluster through Eq. (10) [or through Eq. (11)]. That the masses so derived could either be accepted as meaningful determinations or could be compared with independent estimates for the purpose of testing the applicability of the virial theorem in this form is not important for the purpose at hand.

In the use of the virial theorem in this way, it is necessary to obtain from the observations estimates for the spatial velocities of the member galaxies and for their mutual spatial separations. In almost every case the observations can give only the radial components of these velocities and the components of these separations perpendicular to the line of sight from the observer. The two remaining components of the velocities and the line-of-sight component of the separations must thus be estimated from the observable components through some kind of symmetry arguments. In the case of clusters containing many member galaxies, the averaging is usually sufficient to ensure that the mean projection factors so derived are accurate.

The situation can be quite different for clusters containing only a few member galaxies. Here the uncertainties in any set of mean projection factors can become large and important. In particular, the assumption that the cluster is approximately spherically symmetric—even in the sense of a time average—need not be a valid approximation for such clusters. In these cases it is tempting to seek in the tensor form of the virial theorem for the case of stability, Eq. (5), one or more component equations that will depend less on uncertain projection factors than does the usual scalar form.

The only component equation from Eq. (5) that seems to hold promise in this respect is the form

$$2\langle\tau_{11}\rangle + \langle\Omega_{11}\rangle = 0, \quad (13)$$

where the Cartesian axis characterized by the subscript 1 is taken to be parallel to the line of sight. In this form τ_{11} can be expressed directly in terms of the observed radial velocities without the necessity of introducing any uncertain projection factor. The situation with regard to the evaluation of Ω_{11} is decidedly less favorable. The evaluation of the factor $|\mathbf{x}^{(\alpha)} - \mathbf{x}^{(\beta)}|$ that appears in the denominator of Ω_{11} [cf. Eq. (2)] requires that an appropriate projection factor be used. Still less satisfactory is the fact that the x factor in the numerator of this expression is a function only of unobservables—separations along the line of sight. The best *a priori* estimate for this factor in the numerator of Ω_{11} is probably the mean of the corresponding factors in the expressions for Ω_{22} and Ω_{33} , which latter factors are observables.

In view of the approximations that are required in order to make practical use of the virial theorem in the form of Eq. (13) for the purpose of analyzing clusters of galaxies, there is some question as to whether, in practice, the virial theorem in this form is superior to, or even as good as, the usual form as given in Eq. (10). The practical use of either form requires assumptions concerning projection factors; however, the required assumptions enter in different ways in the two different analyses. Nevertheless, if in each of the analyses the assumption of spherical symmetry is made whenever an argument pertaining to symmetry is required, it can be shown that the two different kinds of analysis lead to identical numerical conclusions concerning the mass or stability of the cluster. The question as to which of the two forms of the virial theorem is superior in practice is thus an academic one, and there is no reason for preferring the tensor form of the virial theorem to the usual form in the problem being considered here.

Required Time-Averages

This latter scalar form for the virial theorem for stability involves time averages for the total kinetic energy T and for the gravitational potential energy Ω . For the case of any particular group or cluster of galaxies, the observations can only provide information concerning these energies for what is equivalent to a given instant: the present epoch. It is thus of interest to be able to estimate for any given group or cluster—tentatively assumed to be stable—by just how much the ratio $2T/|\Omega|$ of the instantaneous kinetic and potential energies can be expected to deviate from the value $+1$ —the value appropriate to the ratio $2\langle T \rangle / \langle |\Omega| \rangle$ of the corresponding time averages according to this form for the virial theorem for the case of stability.

For this purpose, let

$$\Lambda \equiv 2T/|\Omega| - 1 \quad (14)$$

measure the deviation of an instantaneous value of $2T/|\Omega|$ from unity. In terms of the quantity Λ so intro-

duced, the problem just posed is one of estimating the most likely range of values of Λ about zero for a given cluster in equilibrium. Clearly, the detailed evaluation of the most likely range of Λ for such a cluster would require, in general, a much more detailed knowledge of the dynamics of the cluster than is available. However, it appears that a useful first approximation to this range in Λ for a given cluster can be obtained from the following considerations.

As for any other stable cluster, the limiting case of a cluster consisting of only two galaxies that can be treated as point masses must satisfy the virial theorem as given in Eq. (10) if the system has negative total available energy, i.e., if

$$E \equiv T + \Omega < 0. \quad (15)$$

It can be shown without great difficulty that for such a two-body system with negative total available energy,

$$-e \leq \Lambda \leq +e, \quad (16)$$

where e is the eccentricity of the closed elliptical orbit. The quantity Λ takes on the value $-e$ when the point masses are at their greatest separation ($\sim$ apastron), while Λ has the value $+e$ when the point masses are closest together ($\sim$ periastron). Thus, for the two-body problem, the permitted range for Λ —and its probability distribution as well—can be simply expressed in terms of the orbital eccentricity.

No such simple result exists for groups and clusters having more than two members. Nevertheless, it appears that a reasonable first approximation to the expected range of Λ for such systems can be obtained by considering a group or cluster of n members as equivalent for this purpose to $(n-1)$ two-body systems. Here, if we let $\langle e \rangle$ denote an appropriate mean or characteristic value for the *eccentricities* of the orbits of the n members about the center of mass of the cluster, then the instantaneous values of Λ that characterize the cluster as a whole can be expected to satisfy a relation of the form

$$[\Lambda^2 \langle e \rangle]^{\frac{1}{2}} \lesssim \langle e \rangle / (n-1)^{\frac{1}{2}}. \quad (17)$$

Thus, for example, for a quintet we should expect the root-mean-square value of Λ to be less than about 0.25 for a value of $\langle e \rangle$ of 0.5, or less than about 0.45 for $\langle e \rangle$ equal to 0.9.

Estimates of this kind for the expected range of Λ for groups and clusters of galaxies may prove useful in the analysis of groups containing only a small number of members. However, it can be shown that for such groups the uncertainty in the estimate for $2\langle T \rangle / \langle |\Omega| \rangle$ due to the possible range of Λ is equal to or less than the corresponding uncertainty introduced into the analysis by reason of the uncertainties in the projection factors that are required in the evaluation of T and Ω from the observational data.

Clearly, from these considerations or from quite general considerations, the expected range in Λ for clusters containing many members becomes negligibly small, in general. This is also true, of course, for the uncertainties arising from the uncertainties in the projection factors.

ANALYSIS OF TWO SMALL CLUSTERS OF PARTICULAR INTEREST

A. General

In any consideration of the application of the virial theorem to groups and clusters of galaxies, the M81 group and the local system stand out as being of particular interest for several reasons. First, the galaxies in these two groups are near enough that reasonable estimates for the masses of the larger and brighter member galaxies can be derived from studies of their internal motions. Again, because of their nearness to us, these two groups are particularly well suited for studies of the structures of, and stellar populations within, their key member galaxies. Finally, in the case of the local system, and because of our position within one of its two most massive members, the uncertainty in the estimate for the total gravitational potential energy that enters because of the uncertainty in the projection factors for the separations of the member galaxies is largely absent.

B. M81 Group

The importance of the M81 group insofar as analysis by means of the virial theorem is concerned was recognized several years ago by Ambartsumian (1959). From his analysis of the cluster, he concluded that the masses of the member galaxies alone were not sufficient to hold the cluster together and that the cluster was disrupting. Let us here analyze this group in somewhat greater detail.

Observational Data

Let it be assumed for the present that the members of this group that are relevant insofar as its stability is concerned are NGC 3031 (M81), NGC 3034 (M82), NGC 3077, and NGC 2976. The nebular types of these four galaxies in the Hubble system have been given as (Humason *et al.* 1956) Sb, Irr, Irr, and Sc, respectively, while Morgan's corresponding classifications (1958) are kS4, aI, Ep, and aI. I shall take for the radial velocities of these galaxies with respect to the sun the following values that have been derived from the work of Humason, Mayall, and Sandage (1956): -55 km/sec, $+263$ km/sec, -158 km/sec, and $+42$ km/sec, respectively; for the calculation of the relative luminosities of these galaxies, I shall use Holmberg's (1950) values for the photographic apparent magnitudes: 7.85, 9.20, 10.57, and 10.73, respectively. Finally, for the distance to the group, I shall take 2.2×10^6 parsecs (Limber 1960).

Kinetic and Potential Energies and the Virial Theorem

Münch (1959) has derived an estimate for the mass of M81 from an analysis of its internal motions. For the distance assumed here, his value is $1.4 \times 10^{11} M_{\odot}$. Trial estimates for the masses of the other three member galaxies will follow from estimates for their mass-to-light ratios.

Tentatively, let it be assumed that the mass-to-light ratios for NGC 3034, 3077, and 2976 are 20, 50, and 20, respectively. These values refer to photographic light and are in units of solar mass and luminosity. On the basis of what is known today concerning mass-to-light ratios, these estimates appear to be quite conservative in that they appear to provide overestimates for the corresponding masses. The mass-to-light ratio for M81 follows directly as 20.

These observational data and assumptions lead to a value for the total kinetic energy of the M81 group with respect to its center of mass of about 1.0×10^{59} ergs. The corresponding value for the gravitational potential energy follows as -2.3×10^{58} ergs. Both estimates have been derived on the basis of what appear to be, on statistical grounds, the best choices for the relevant projection factors. It follows that the corresponding best estimate for $2T/|\Omega|$ is about 9.0. Thus, if the analysis given is correct except for a common multiplicative correction factor for the masses of the four galaxies, then for the stability of the cluster, this common factor would be about 9; i.e., stability would then be possible only if each of the four masses were about 9 times larger than the values actually used. This would imply a mass for M81 itself of about $1.2 \times 10^{12} M_{\odot}$, which is to be compared with the value $1.4 \times 10^{11} M_{\odot}$ that Münch obtained from the study of its internal motions. It should be noted here that the uncertainty in the distance to the group affects both of these determinations by the same factor, so that the ratio of these two values is invariant to changes in the distance. The corresponding value of the mass-to-light ratio for the group as a whole for stability follows as 200.

Possible Interpretations of these Results

At least at first glance, the most natural and least *ad hoc* interpretation of the analysis for the M81 group is that this group does not satisfy the virial theorem for stability, that it has positive energy and is disrupting. However, this interpretation is not without difficulties.

Let us introduce the concept of a characteristic time scale τ_c for a group or cluster such that during a time τ_c a galaxy moving with a constant component of velocity in the plane of the sky equal to the presently estimated root-mean-square value for this component for the cluster members will move a distance in the plane of the sky equal to the present projected mean distance of the cluster members from their center of mass. Clearly, if a group or cluster is so unstable that

$T \gg |\Omega|$, then the present instability cannot have persisted for an interval appreciably greater than the characteristic time scale for that group or cluster.

In the case of the M81 group, the characteristic time scale is about 2×10^8 years if the foregoing analysis is taken at its face value. This time scale is uncomfortably short—being about that for the Pleiades cluster—and raises serious difficulties concerning the interpretation of the foregoing results in terms of instability. Thus, it is exceedingly difficult to believe that M81, itself, can be nearly this young; and although one cannot yet rule out the possibility that the other members of the system are this young, even this does not appear very probable.

It should be noted that the apparent instability for the group results almost entirely from the relatively large velocity of M82 with respect to those of the other members. This suggests that a possible solution of the dilemma lies in assuming that the large relative velocity of M81 is largely confined to the radial direction. However, while such an assumption can be used to overcome the difficulties associated with a time scale of 2×10^8 years, it also acts to reduce the value of the calculated ratio for $2T/|\Omega|$ and to thereby raise the possibility of explaining the group in terms of stability.

A possibility that deserves consideration is that M82 received its present kinetic energy long after M81 was formed. Whether, in this line of thinking, it is more reasonable to assume that M82 was formed at the time that it received its present kinetic energy some 2×10^8 years ago or whether it is more reasonable to conclude that this galaxy, too, was formed long before the system received its present energy is not clear. Both alternatives encounter difficulties of their own. On the first of these assumptions, M82 came into being as a galaxy long after at least one other member of the group was formed. This explanation seems to require the existence of an appreciable quantity of gaseous intracluster—or intercluster—matter as recently as 2×10^8 years ago. If the case here is to be argued for, then it becomes more difficult to oppose the view that such a group is stable because of the presence of large masses of unseen intracluster matter. On the other hand, if it is assumed that M82 received its present kinetic energy long after its formation as a galaxy, then the question of the mechanism responsible for this acceleration must be dealt with.

The arguments in support of an interpretation of the M81 group in terms of stability are no less beset with difficulties. In one view, one can argue that the group is stable by reason of the mutual attractions of its member galaxies alone for a mass of M81 equal to that derived by Münch. Support for such an interpretation would have to be found in terms of one or more of the following possibilities. The accuracy of the observed radial velocities might be questioned, or it might be argued that the *actual* value for the relevant projection

factors and for the correction factor for obtaining the required time averages for T and Ω have not been shown to be inconsistent with the stability criterion in the form of Eq. (10). Finally, it might be argued that the actual mass-to-light ratio for one, or more, of the three companions of M81 is very much greater than even the value assumed in the present analysis.

To my knowledge, there is no reason at present for believing that errors in the observed radial velocities can account by themselves for the result obtained. On the other hand, it appears that one cannot rule out the possibility that, by themselves, the uncertainties in the projection factors and in the factor that enters in estimating the necessary time averages can conceivably account for the results obtained here in terms of stability. However, this alternative is not a comfortable one; it may be called upon to save an occasional small group from instability, but it is not able to save the great majority of such clusters whose stabilities are now in question. Finally, although the possibility cannot be ruled out that some member of the group other than M81 may have a much larger mass-to-light ratio than has been assumed here, there is no direct evidence for the existence of such large values for any galaxies. It may conceivably be of significance in this connection that M82 appears to be anomalous insofar as its reddening is concerned.

If one rules out the possibility that M82 is not a physical member of the group, then there appear to be only two further interpretations of the analysis of this group that hold any promise. Either our direct determinations of the masses of galaxies from studies of internal motions do not include large fractions of the total masses of such galaxies that should properly be associated with them, or there is a relatively large mass of unseen intracluster matter. To my knowledge, there is at present no direct evidence in support of the first of these alternatives, and without such support it appears quite artificial. The second of these possibilities has been put forward to account for the stability of a number of clusters of galaxies. However, although this latter possibility seems less artificial than the former, it, too, has very little direct evidence in its support. But it is not impossible that intergalactic and intracluster matter plays a much more significant role in the universe at the present time than is generally believed.

This concludes my discussion of the application of the virial theorem to the M81 group. I have not tried to draw any kind of definite conclusion because I have not felt that the evidence pointed sufficiently clearly to any particular one of the possible interpretations. Once the evidence has been examined and the alternatives have been considered, one can hardly keep from leaning toward one or another of these alternatives. But in the case of a particular cluster such as the M81 group, I feel that additional evidence will be required before any one of the several alternative interpretations becomes compelling.

C. Local System

Analyses of the foregoing kind have been applied to the local system of galaxies by Kahn and Woltjer (1959) and by Godfredsen (1961). Kahn and Woltjer concluded that the system must be stable and that this was possible only if there were a large mass of unseen matter associated with the system. Godfredsen argued, on the other hand, that in view of the uncertainties involved, it was quite consistent to conclude that the local system was stable under the mutual attractions of its member galaxies alone. Let us here turn to an analysis of the local system and examine the evidence.

Observational Data

Values for the masses of several members of the local group have been derived from studies of internal motions (Brandt 1960; Schmidt 1956; de Vaucouleurs, 1959, 1960; Burbidge *et al.* 1961). If these values are tentatively accepted and if the masses of the remaining members are derived from what appear to be reasonable estimates for their mass-to-light ratios, it can be shown that the stabilities of M31 and of the Galaxy within the local system depend—to the required accuracy—only upon the masses of these two galaxies and upon their relative velocity and separation. Tentatively, it will be assumed that this is the case, and the stability of these two galaxies within the local system will therefore be treated in terms of the stability of these two galaxies considered as constituting a simple double galaxy.

The values that will be tentatively used here for the masses of M31 and of the Galaxy are $4.0 \times 10^{11} M_{\odot}$ and $1.0 \times 10^{11} M_{\odot}$, respectively—the former being that derived by Schmidt (1956) while the latter has been taken from Brandt's study (1960). The separation of the two galaxies will be taken to be 6×10^5 parsecs, the value appropriate to the above determination for the mass of M31. These values suffice to determine the gravitational potential energy of M31 and the Galaxy—considered as making up a double galaxy. In this one case, no uncertain projection factor enters into the determination of Ω .

The evaluation of the relative radial velocity of M31 with respect to the Galaxy requires both the radial velocity of M31 with respect to the local standard of rest near the sun and also the circular velocity about the center of the Galaxy for the solar neighborhood. It appears that the best available value for the first of these quantities—and a very accurate one—is that derived by van de Hulst, Raimond, and van Woerden (1957) from a study of the 21-cm radiation from M31. This study gives for the radial velocity of M31 with respect to the local standard of rest near the sun the value -296 ± 3 km/sec. This value should be compared with the corresponding optically determined values of -262 ± 20 km/sec and -286 ± 30 km/sec derived at Mt. Wilson and at Lick, respectively.

The direction of the velocity in the solar neighborhood that corresponds to circular motion about the center of the galaxy is 39° from the line of sight to M31, so that this circular velocity plays an important role in the derivation of the relative velocity of the center of mass of M31 with respect to that for the Galaxy. Because the radial velocity of M31 with respect to the local standard of rest near the sun is one of approach, it follows that the larger this circular velocity is taken to be, the smaller will be the absolute value of the relative velocity of the centers of mass of the two systems and, hence, the smaller will be the kinetic energy of this two-body system.

The most used of recent estimates for the circular velocity is the value 216 km/sec given by Schmidt (1956). However, since questions have been raised concerning the accuracy of this value, the present results will be carried through for the three following values for the circular velocity: 200, 250, and 300 km/sec.

Kinetic and Potential Energies and the Virial Theorem

For the above data, the gravitational potential energy Ω for M31 and the Galaxy considered as a double galaxy follows directly as -5.8×10^{57} ergs. The calculation of the kinetic energy of the system with respect to its center of mass is not so direct since some estimate must be made for the ratio of the square of the relative spatial velocity of the two galaxies to the square of the corresponding radial component. On statistical grounds the best estimate for this ratio is *three*, and it is this value that will be used in the present calculations. For this choice, we obtain for the kinetic energy of the system the values 4.6×10^{58} , 2.4×10^{58} , and 9.1×10^{57} ergs for the three choices for the circular velocity: 200, 250, and 300 km/sec, respectively. The corresponding values for the ratio $2T/|\Omega|$ follow as 16., 8.3, and 3.2, respectively; and if the virial theorem is to be satisfied in the form given in Eq. (10), then these last values also represent the factors by which the assumed masses would have to be increased in order to satisfy this form for the virial theorem if common factors are taken for the two masses and if the rest of the foregoing analysis is left unchanged.

Possible Interpretations of these Results

As in the case of the M81 group, the interpretation of these results that seems least artificial at first sight is that the local system has positive energy and is disrupting. But here two factors act together to raise serious difficulties for any simple interpretation along these lines. First, it is only for this system that the sense of the velocities with respect to the center of mass can be determined, and it is one of approach. According to the interpretation in terms of positive energy, this relative, line-of-sight velocity must have been one of approach since the time at which the positive energy was made available to the system and will continue as

a velocity of approach for the next three or four billion years. Because of this, the interpretation in terms of positive energy leads to the conclusion that the separation of M31 and the Galaxy was more than twice its present value at the time that the solar system was formed and was about equal to the present distance to the M81 group some ten billion years ago. This latter time interval can hardly be significantly greater than the present age of the Galaxy and of M31.

These conclusions that seem to follow from an interpretation in terms of positive energy also appear to raise difficulties concerning the process by which the present positive energy was made available to the system. For example, what are the implications of the requirement that momentum be conserved? It hardly appears that M31 and the Galaxy can have acted in concert to conserve momentum at the time at which the positive energy was made available, and alternative mechanisms seem to require appreciable quantities of mass in the form of unseen matter.

The interpretation of the local system in terms of stability for the assumed masses for M31 and for the Galaxy and without recourse to large quantities of unseen matter within the local system seems to be possibly only in terms of one or more of the four following explanations. First, stability under these conditions would be possible if the radial velocity of M31 with respect to the local standard of rest near the sun were much larger algebraically than the value of -296 km/sec that has been used here. However, even for a conservatively high estimate of 250 km/sec for the circular velocity, the required radial velocity would have to be significantly larger than -250 km/sec in order to account for our results by itself. It hardly seems possible that this can be the case.

Second, stability can be achieved if the circular velocity can be shown to be sufficiently large; but this effect by itself could explain the results only if the circular velocity were significantly in excess of 300 km/sec. There seems to be little independent support for such a value.

The ratio of $2\langle T \rangle / \langle |\Omega| \rangle$ can be reduced by at most a factor of 6 through optimum choices for the projection factor for the velocities and for the factor that enters in converting the observed estimates for T and Ω into the required time averages. Even such a factor, by itself, cannot produce stability for values of the circular velocity of 250 km/sec and less. Nevertheless, factors of 2 or even 3 can be obtained in this way without too great an improbability, and such factors—when operating together with one or more of the other possibilities and uncertainties being discussed here—may make it possible to interpret this particular system in terms of stability without undue violence to the data.

Fourth and last, in this connection, it should be noted that the radial velocities of the two companions of M31—M32 and NGC 205—and of the two companions of the Galaxy—the Magellanic Clouds—are all such as

to reduce the relative velocity of approach of M31 and the Galaxy for circular velocities less than about 270 km/sec if the double galaxy under consideration is extended to include M31, M32, and NGC 205 as one component and the Galaxy and the Magellanic Clouds as the other. However, as was noted earlier, for what appear to be the best available estimates for the masses of these companion systems, the results of the foregoing analysis for $2T/|\Omega|$ are altered by less than 20%. In order for these companions to play a significant role in accounting for the stability of the system, it would be necessary for them to contain many times the mass that has been assumed for them here.

Let us turn now to a last general type of interpretation of the present analysis: an interpretation in terms of the stability of the local system by reason of the assumed existence in this system of a large mass of unseen matter. We shall include here the possibility that the mass assumed for the Galaxy is in error, the possibility that the masses derived for M31 and for the Galaxy from studies of their internal motions do not include large masses that are properly associated with these two individual galaxies, and the possibility that there is a large mass of unseen matter in the local system that is not in the form of the brighter galaxies.

Brandt has suggested that because of the observational uncertainties, the mass of the Galaxy may be uncertain by almost an order of magnitude, but to explain the present results in terms of stability in this way alone, the mass of the Galaxy would have to be about $4 \times 10^{12} M_{\odot}$ —a value that is exceedingly difficult to accept. There is no direct evidence for the second of the above possibilities, and it has been mentioned only because of the difficulties associated with the other possible explanations for this system and for other groups and clusters of galaxies as well.

Finally, we come to the possible interpretation of the foregoing analysis in terms of the stability of the system by reason of the existence of a large mass of unseen matter within the system but not associated with the brighter galaxies. Probably the strongest argument in support of this interpretation is the indirect argument that all of the alternative interpretations of the analysis seem to encounter serious difficulties. Such an argument cannot be very satisfying by itself until more direct evidence can be found. In the case of this particular system Kahn and Woltjer (1959) have argued that there is additional evidence in support of this interpretation—evidence in the form of an observed distortion of the galactic disk which they interpret in terms of a dynamical interaction between the Galaxy and an unseen intergalactic substratum of gas. However, this evidence, too, is indirect; and the observational data upon which it rests is not unambiguous insofar as its explanation is concerned.

With these remarks, I shall conclude my discussion of the interpretation of the results that follow from the application of the virial theorem to the local system.

As in the case of the similar discussion of the M81 group, I feel that the evidence is not yet sufficient to support strongly any one of the alternative interpretations of the results that have been obtained. What does seem clear from this analysis is that these results will find their explanation either in significant revisions of parameters that we had believed were reasonably well tied down, in the operation of some unknown process that makes available to newly formed galaxies large kinetic energies, or in the existence of a great mass of unseen matter whose role in the evolution of the system as a whole and of the particular galaxies within it would be great.

A possibility that has been tacitly ignored in the foregoing discussion is that the local system is not a single physical system and that M31 and the Galaxy are relatively close together only through chance. In view of the previous results, this suggestion may deserve more serious consideration.

This research was supported in part by the United States Air Force under contract.

REFERENCES

- Ambartsumian, V. A. 1958, *La Structure et l'évolution de l'univers*, Solvay Conference (R. Stoops, Brussels, Belgium), p. 163.
 Brandt, J. C. 1960, *Astrophys. J.* **131**, 293.
 Burbidge, E. M., Burbidge, G. R., and Fish, R. A. 1961, *ibid.* **133**, 393.
 Chandrasekhar, S. 1960, *J. Math. Anal. Appl.* **1**, 240.
 de Vaucouleurs, G. 1959, *Astrophys. J.* **130**, 728.
 ——. 1960, *ibid.* **131**, 265.
 Godfredsen, E. A. 1961, *ibid.* **134**, 257.
 Holmberg, E. 1950, *Lund Medd. Ser. II*, No. 128.
 Humason, H. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* **61**, 97.
 Kahn, F. D. and Woltjer, L. 1959, *Astrophys. J.* **130**, 705.
 Limber, D. N. 1959, *ibid.* **130**, 414.
 ——. 1960, *ibid.* **131**, 168.
 Morgan, W. W. 1958, *Publs. Astron. Soc. Pacific* **70**, 364.
 Münch, G. 1959, *ibid.* **71**, 101.
 Schmidt, M. 1956, *Bull. Astron. Inst. Neth.* **13**, 15.
 van de Hulst, H. C., Raimond, E., and van de Woerden, H. 1957, *ibid.* **14**, 1.

DISCUSSION

ABELL asked whether an upper limit could be set to the density of "invisible matter" between galaxies where there is no star formation, and it was pointed out that this would depend on physical conditions in the invisible matter. Ionized hydrogen at very high temperature provides no useful upper limit. LIMBER added that concentrating intergalactic matter toward the center of a cluster increases its effectiveness in holding the cluster together up to a limit of about three times the effectiveness of more dispersed matter.

OORT pointed out that the radial velocity of the center of gravity of M82 recently measured in 21-cm radiation is +190 km/sec, which is 90 km/sec lower than the optical value, possibly because of optical obscuration of the southwest (approaching) side of M82. The radio value of M82's velocity is much closer to the

velocity of M81. LIMBER agreed that this new result is important though it reduces the derived best estimates of $2T/\Omega$ only from 9 to about 6.

G. BURBIDGE added that a provisional rotation curve has been obtained for M82 by N. U. Mayall. This leads to a mass estimate of about 1.5×10^{10} suns and small M/L , leaving only NGC 3077 and 2976 that may have high M/L .

ROBERTS asked whether the circular velocity of our galaxy at the sun could not possibly be as high as 300 km/sec and LIMBER replied that, if the Dutch value of the relative radial velocity is used, the circular velocity must be well in excess of 300, in order to achieve consistency.

SCHMIDT noted that this would of course increase the mass of the galaxy to be used in Limber's analysis.

FISH suggested that some of the present dwarf

galaxies may have previously been much more massive, having since lost the bulk of their matter by ejection of supernova envelopes. However, this cannot produce a large effect, since if their masses were much larger than 10^{10} suns (a value small compared to the mass of M31 and the galaxy) no escape of material would have taken place.

VON HOERNER described the results of his numerical calculations simulating motions in systems of 16 mass points. Starting with negative total energy, he found that Limber's parameter $\Lambda = 2T/\Omega$ deviated from the expected average value of 1.0, but never by more than a factor of 2. He concluded that deviations from the time averages in the virial theorem could be no larger than this, but PAGE noted that larger fluctuations might take place in systems with fewer than 16 masses, or when the masses varied widely.

Magnitude-Redshift Relation of Galaxies in Clusters in the Presence of Instability and Absorption

JERZY NEYMAN AND ELIZABETH L. SCOTT
University of California, Berkeley 4, California

INTRODUCTION

THE usual method of studying stability of systems of galaxies is indirect: Tentatively assuming stability, the virial theorem is applied in order to estimate the average masses of presumed members of the system. If the estimate so derived exceeds considerably those obtained by other methods, the conclusion is drawn that, unless the system of galaxies contains a very considerable amount of invisible matter, it must have positive energy and be expanding.

In 1954 a more direct method for studying stability (Neyman and Scott 1954) was introduced. Originally, it was meant for a very particular purpose, namely to obtain evidence for or against the hypothesis that the systems of galaxies, that is, clusters and groups, participate in the general expansion of the universe. Under certain specific assumptions about the structure of clusters, about the distribution of radial velocities and about the luminosity function, a function of the observations was determined representing a criterion for testing the hypothesis of stability more suitable than any other criterion. The use of this particular criterion ensures the greatest possible probability of detecting instability if such is present and of moderate degree. If the linear diameter of the system of galaxies studied is a small fraction of this system's distance from the observer, the criterion deduced is approximately equal to the regression coefficient of the radial velocity of the system's members on their apparent magnitude corrected for redshift.

Because no dynamical considerations are involved in the deduction of the test, it may be labeled kinematical test.

At the time the original paper was in preparation, the richest material for any one system of galaxies was that for the Coma cluster and it contained the data for only 12 objects. The kinematic test applied to these observations gave no indication of instability. Subsequently, when the number of observations increased and, at the same time, the question of stability of systems of galaxies acquired particular interest in connection with the Ambartsumian hypothesis, the kinematic test was applied to the vastly richer empirical material. The relevant paper (Lovasich, Mayall, Neyman, and Scott 1961) is now in press.

As is true with every statistical method, the validity of the kinematical test is subject to certain conditions. The principal conditions are (i) that the objects treated as members of a given system really belong to this system, and (ii) that the selection of galaxies for which the measurements of the apparent magnitude and of

redshift are made did not distort the distribution of redshift relative to the apparent magnitude. Naturally, the validity of the test based on the virial theorem is subject to similar restrictions.

The first purpose of the present paper is to present a heuristic explanation of the working of the kinematical test. The second purpose is to report on the empirical results of the test applied to data on eight clusters and on 10 smaller groups of galaxies. Most of these results are given in the last-mentioned paper. However, since the preparation of that paper our views have undergone a certain evolution and the present interpretation of some of the results is slightly different.

It will be seen that the evidence regarding large clusters is indistinct. If the Virgo cluster is taken as a single system and if it is treated on its own, separately from other clusters, there appears to be an indication of expansion, but it is very mild, with the probability of error in asserting instability having an upper bound of the order of one in four. If one recalculates the data taking into account the suggestion of de Vaucouleurs that in reality there are two distinct systems, one composed of spirals and the other of elliptical galaxies, the mild indications of instability persist only for the ellipticals. For the Coma cluster, after the elimination of four outlier galaxies, the test gives a mild indication of contraction. Also, the joint treatment of all eight clusters fails to give any clear indication of instability.

The above results relating to large clusters contrast with those obtained for small groups. Originally the analysis was performed for the totality of 15 groups of galaxies for which there are at least three objects with measured luminosity and redshift. No indication of instability was obtained. However, the data for five of these groups show convincing evidence that the objects attributed to some groups were subjected to a strong selection for similarity in redshift, thus violating one of the basic conditions of validity of the kinematic test. When these five groups were eliminated, the test gave a strong indication that groups are rapidly expanding, the probability of error in asserting instability being less than 0.02.

The concept of stability contemplated in the application of the kinematic test differs essentially from that of dynamic stability and refers to the motions within the cluster that prevail at the moment of observation. If at the given moment the motions of some particles towards the center of the system are counterbalanced by the motions of some other particles away from the center, then at that moment the system formed by these particles is kinematically stable. If the motions away

from the center predominate then the cluster is expanding and vice versa. Thus, with the exception of two instants of time, a dynamically stable system of two bodies, with elliptical orbits about their center of gravity, will be "kinematically unstable": Over a period of time it will be contracting and over the next period of time expanding.

HEURISTIC EXPLANATION OF THE WORKING OF THE KINEMATIC TEST

In the present section we explain the concept of kinematic stability of a system of particles and, heuristically, show how the regression coefficient of radial velocity on the apparent magnitude indicates expansion or contraction. The problem of optimality of this criterion is not treated here and the reader is referred to the earlier papers mentioned above.

Our present explanation begins in terms of the theoretical model of a system of galaxies. Subsequently we discuss the relation between the probabilistic concepts and the phenomena actually occurring in systems of galaxies. It will be seen that the kinematic test may be confidently applied to large clusters of galaxies treated individually. Contrary to this, individual treatment of small groups of galaxies, for example of pairs, is likely to be misleading. With reference to small groups the kinematic test is expected to be reliable only if applied to a number of such groups treated simultaneously.

The theoretical structure of systems of particles considered in this paper is visualized as being composed of circular rings. In each ring, the density of particles per unit volume (really the probability density) is constant along the ring. However, this density may well vary from ring to ring. For any galaxy, within a ring of radius η the velocity at any given moment is postulated to be the resultant of the following four components:

(1) The velocity u with which the system as a whole recedes from the observer.

(2) The rotational velocity ω of the ring, possibly depending upon η and on the ring, but the same for all galaxies within the ring.

(3) Velocity which we shall call the velocity of instability. This component is located on the line connecting the center of the ring with the position of the galaxy, is supposed constant for all the galaxies in the ring, and is denoted by $\theta\eta$, suggesting the possibility that it is proportional to the radius of the ring. However, this proportionality is not postulated and the coefficient θ , called the instability factor, may well vary from ring to ring. If $\theta=0$ then the given ring is kinematically stable. If $\theta>0$, the ring expands. If $\theta<0$, it contracts. If θ has the same value for all rings, then in the evolution of the system this coefficient plays a role somewhat analogous to that of the Hubble constant H . In particular, if $\theta=H$, then the system considered simply participates in the general expansion of the universe.

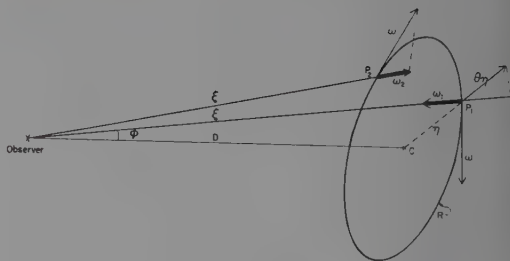


FIG. 1.

(4) Random velocity component v . It is postulated that the projection of v on any line has expected value zero and a fixed variance σ_v^2 . Also it is postulated that the random velocities for any two galaxies are mutually independent.

Using the above hypotheses, we shall now evaluate the regression coefficient of the radial velocity of a system member on this particle's distance from the observer. The position of the observer with regard to the system of particles is arbitrary.

With reference to Fig. 1, consider a possible position P_1 of a particle within the system, located within a given ring R of radius η . For a fixed position of the observer, let ξ , D , and ϕ denote, respectively, the distance of P_1 , the distance of the ring's center C , and the angular separation between P_1 and C .

The radial velocity ρ_1 of a particle at P_1 may be represented as follows:

$$\rho_1 = u \cos \phi + \omega_1 + \theta(\xi - D \cos \phi) + v_1, \quad (1)$$

where ω_1 is the contribution of the rotational velocity of the ring, where the third term is the contribution of the velocity of instability, and where v_1 is the random component with expectation zero.

Simple geometrical considerations indicate that the ring R must have another point P_2 (possibly coinciding with P_1) characterized by the same distance ξ from the observer and the same angular separation ϕ .

The radial velocity of a particle at P_2 is

$$\rho_2 = u \cos \phi + \omega_2 + \theta(\xi - D \cos \phi) + v_2, \quad (2)$$

where the symbols have meanings similar to those in (1). Simple calculations show that $\omega_1 = -\omega_2$ and, in particular, if P_2 coincides with P_1 , then $\omega_1 = \omega_2 = 0$. Because of the assumed equal density of galaxies along any ring, and because $E(v) = 0$, it follows that for fixed ϕ and ξ the expected value of ρ is

$$E[\rho | \xi, \phi] = \frac{1}{2}[E(\rho_1) + E(\rho_2)] \\ = u \cos \phi + \theta(\xi - D \cos \phi). \quad (3)$$

That much is exact. However if the dimensions of the systems considered are small compared to their distances, then $\cos \phi$ is very close to unity. In these circumstances we have the approximate formula

$$E(\rho | \xi) = u + \theta(\xi - D), \quad (4)$$

representing the regression of the radial velocity ρ on the distance ξ of a galaxy from the observer. It is seen that this regression is linear and that the coefficient of regression is equal to the factor of instability θ .

Formula (4) being approximately true for any ring taken separately, with its own expansion coefficient θ , it can be averaged either over the whole system or over some selected part of it. If the coefficient θ has the same value for all rings then the resulting average regression equation will be linear and will coincide with (4). On the other hand, if θ varies from ring to ring in a systematic manner then this linearity may be distorted.

In practical applications it is only the average regression that may be studied. In this connection it is important to notice that if the average regression coefficient of ρ on ξ is not zero then this means that at least some of the rings are unstable and that the sign of the average regression coefficient indicates the kind of instability that prevails: expansion if this sign is positive and contraction if it is negative. Also, non-linearity of the average regression indicates that the particular rings composing the system have different factors of instability. On the other hand, the reverse conclusions are not necessarily true and, in particular, the equality to zero of the average regression coefficient of ρ on ξ does not necessarily mean that all rings of the system are stable.

Since the distances ξ of particular galaxies are unobservable, the above results can only be used in connection with some distance indicator, which, however, must be independent of the radial velocity. The distance indicator that is currently available is the apparent magnitude, provided it is reliably corrected for the effect of redshift. The calculations reported below are all based on K corrections given in Humason, Mayall, and Sandage (1956).

Let m stand for the apparent magnitude of a cluster galaxy, corrected for redshift. We assume that it is connected with the absolute magnitude M by the following approximate relation:

$$m = M - 5 + 5 \log_{10} \xi + C + \tau(\xi - D), \quad (5)$$

where C and τ are approximately constant and characterize the absorption of light by the invisible matter within the cluster. If $\xi = D$ then this absorption is C mag. The symbol τ stands for the average dimming per unit distance. We assume that the absolute magnitude M is a random variable independent of the position of the galaxy.

Assuming that the linear dimensions of the cluster are small compared to its distance D , we write

$$\xi = D + (\xi - D) \text{ and}$$

$$\begin{aligned} 5 \log_{10} \xi &= 5 \log_{10} D + a \log_{10} [1 + (\xi - D)/D] \\ &\approx 5 \log_{10} D + (a/D)(\xi - D), \end{aligned} \quad (6)$$

where $a = 5 \log_{10} e = 2.1715$. Substituting in (5) we

obtain, approximately,

$$m = M - 5 + 5 \log_{10} D + C + (a/D + \tau)(\xi - D). \quad (7)$$

Restricting ourselves to galaxies located in a single ring and comparing (7) with (4) we see that the covariance of m and ρ is equal to

$$\text{Cov}(m, \rho) = \theta(a/D + \tau)\sigma_{\xi}^2, \quad (8)$$

where σ_{ξ} is the dispersion of the distances from the observer of those galaxies that are located within the given ring. The variance of m is

$$\text{Var}(m) = \sigma_M^2 + (a/D + \tau)^2 \sigma_{\xi}^2, \quad (9)$$

and the regression coefficient of ρ on m , say B , is

$$\begin{aligned} B &= \text{Cov}(m, \rho) / \text{Var}(m) \\ &= \theta \sigma_{\xi} \frac{aq + \tau \sigma_{\xi}}{\sigma_M^2 + (aq + \tau \sigma_{\xi})^2}, \end{aligned} \quad (10)$$

where, for brevity, $q = \sigma_{\xi}/D$. If θ is the same for all rings then the over-all regression coefficient of ρ on m will have the same form (10) with the exception that now σ_{ξ} will denote the dispersion of the distance ξ from the observer not limited to galaxies in a single ring but referring to the whole system. If the value of θ varies from ring to ring, then the over-all covariance of m and ρ will be a weighted mean of covariances corresponding to particular rings with the variance (9) retaining its form provided that σ_{ξ} is treated as the over-all dispersion of ξ .

It follows that the remarks made above regarding the meaning of the average regression of ρ on ξ apply, with obvious modifications, to the average regression coefficient of ρ on m . If this average regression coefficient is not zero, then at least some rings must have $\theta \neq 0$. If this average coefficient is positive then among the rings of the system there prevails expansion, etc.

With these remarks in mind, it is interesting to consider formula (10) on the tentative assumption that for all rings the instability factor θ is the same, in which case (10) represents the over-all regression coefficient of ρ on m .

It is seen that, if one treats σ_{ξ} as the "typical" distance of a galaxy from the center of the system then the first factor $\theta \sigma_{\xi}$ represents the instability velocity of a galaxy at this typical distance from the center. For all systems likely to be studied at any time $q = \sigma_{\xi}/D$ is a small fraction diminishing with the distance D of the system. $\tau \sigma_{\xi}$ is the dimming of light due to internal absorption over the typical distance within the system. We shall call this term "typical absorption" within the system. If one assumes that the diameter of a system is of the order of $7\sigma_{\xi}$ and if one accepts Zwicky's estimate (1957) that the galaxies located behind a large cluster have their magnitudes dimmed by about 0.7 mag., then the typical absorption $\tau \sigma_{\xi}$ must be of the order of 0.1 mag. Naturally, $\tau \sigma_{\xi}$ may well vary from one system to

TABLE I. Values of the regression B of the radial velocity on apparent magnitude of cluster galaxies.

Mean radial velocity u km/sec	$ \theta = H$				$ \theta = 10H$			
	$\tau = 0$	$\tau = 10^{-7}$	$\tau = 2 \times 10^{-7}$	$\tau = 0$	$\tau = 10^{-7}$	$\tau = 2 \times 10^{-7}$	$\tau = 0$	$\tau = 10^{-7}$
1000	10.40	14.81	18.83	104.00	148.10	188.34		
5000	2.13	6.96	11.59	21.26	69.58	115.91		
10 000	1.06	5.92	10.61	10.64	59.21	106.08		
15 000	0.71	5.57	10.28	7.09	55.74	102.78		
20 000	0.53	5.40	10.11	5.32	53.99	101.12		

the next. The last quantity in (10) to be considered is σ_M , the dispersion of the absolute magnitude of cluster galaxies. Our tentative estimate is $\sigma_M = 1.25$. It is based on the estimates of σ_M for particular types of field galaxies (Neyman and Scott 1959), all of the order of 1.0 mag., and on the consideration that particular systems may well contain galaxies of several different types. It follows that the denominator in (10) is somewhat larger than unity. With these estimates it is seen that the regression coefficient B must be of the order of the product of instability velocity at the typical distance from the center by a fraction depending upon the distance of the system and upon the typical absorption, perhaps of the order of 0.1. In the absence of absorption this fraction is ordinarily very small indeed.

The natural standard of comparison for the value of the instability factor θ is the Hubble constant H . Tables I and II give, respectively, the values of B computed for several combinations of values of the intervening constants, and the standard errors of the estimate of B that one can compute from the observations. Consider the case that θ and, therefore, B are not zero, so that the cluster is unstable. In order that this fact can be easily detected empirically, the value of B should be larger than the standard error of its estimate. While the paper of Lovasich, Mayall, Neyman, and Scott (1961) gives more details on this point, Tables I and II are indicative.

Table I is computed using formula (10) with the substitution of $\sigma_\xi = 7 \times 10^5$ parsecs, a value that fits Shane's counts for the Coma cluster and for several other clusters. For small groups this value is likely to be too large, leading to an overestimate of B . The other adopted values of the intervening constants are $\sigma_M = 1.25$, and $H = 10^{-4}$ km/sec/parsec. The values of τ contemplated were suggested by the estimate of Zwicky.

TABLE II. Typical values of the standard error of the estimate of B .

Number of observations n	4	9	16	25	36	49	64	81	100
Large clusters, $\sigma_1 = 600$ km/sec									
$S(m) = 1.0$ mag.	300	200	150	120	100	85	75	67	60
$S(m) = 0.5$ mag.	600	400	300	240	200	170	150	133	120
Small groups, $\sigma_1 = 300$ km/sec									
$S(m) = 1.0$ mag.	150	100	75	60	50	43	37	33	30
$S(m) = 0.5$ mag.	300	200	150	120	100	85	75	67	60

Using observational data for a given cluster an estimate $\hat{B}$ of the regression coefficient B may be computed. The standard error of $\hat{B}$ computed using n pairs of measurements (m_i, ρ_i) , $i = 1, 2, \dots, n$, is given by the formula, say

$$\sigma_B = \frac{\sigma_1}{[\sum (m_i - \bar{m})^2]^{\frac{1}{2}}} = \frac{\sigma_1}{S(m)n^{\frac{1}{2}}}, \tag{11}$$

where $S(m)$ represents the sample dispersion of the apparent magnitude and σ_1 the dispersion of the random component of the radial velocity of the cluster members. In Table II we adopted typical but somewhat optimistic values for σ_1 , namely 600 km/sec for large clusters and 300 km/sec for groups. For nearby clusters the values of $S(m)$ we encountered are of the order of unity. However, when the distance of a system of galaxies is large, the radial velocities are easily measured only for the brightest members and this selection reduces the value of $S(m)$ to about 0.5 mag. or less.

Comparing Tables I and II, it is seen that if the systems studied are unstable but the rate θ of expansion or contraction is of the order of magnitude of the Hubble constant, then, whether the system is a large cluster or a small group and whether it contains absorbing matter in quantities comparable to the estimate of Zwicky or not, it is more or less hopeless to expect that the kinematic test will detect instability. Possibility of such detection appears only if the instability factor θ is substantially larger than the Hubble constant. Thus, for $|\theta| = 10H$ with $\tau = 2 \times 10^{-7}$ mag./parsec a nearby system may have $B = 188$ km/sec/mag. If this system is a large cluster, then with $n = 25$ observations, the standard error of the estimate of B is 120 km/sec/mag., less than the value of B itself. If this system is a small group, then the standard error is only 60 km/sec/mag. and the chance of detecting instability is excellent.

The interesting detail of Tables I and II is that, with radial velocities of systems of galaxies exceeding 5000 km/sec, the detection of instability when $|\theta| = 10H$ using moderate numbers of observations remains hopeless unless the systems contain invisible matter with absorption coefficient τ of the order of 2×10^{-7} mag./parsec. Thus, if the observations yield an estimate of B that is significantly different from zero, this is evidence both of instability and of the presence within the system of a substantial amount of absorbing matter.

All the above discussion applies strictly to the theoretical model of a system of galaxies, and it is essential to be clear about the possible relation between the conclusions so reached and the frequency relations obtaining in real clusters and real groups of galaxies. Although the delimitation between large clusters and small groups cannot be made sharp, it appears appropriate to consider these two categories separately.

For a big cluster, exemplified by Coma, it seems to us very intuitive that the idealization in terms of circular rings, each with an approximately uniform

density (strictly speaking, constant probability density) of galaxies is entirely realistic. Naturally, it is impossible to expect that to each galaxy at point P_1 in Fig. 1 there will correspond another galaxy exactly at point P_2 and that projections of the rotational velocities of the two on the appropriate lines of sight will balance exactly. However, any such exactitude is not needed for the statistical test. The test will be valid if the galaxies that are actually observable have positions and velocities comparable to those in a random sample from the distribution postulated in the model and, with a large membership of a system, this is very likely to be true.

The situation is entirely different if the system considered is very small. The extreme situation occurs when the system is composed of just three members G_1 , G_2 , G_3 as illustrated in Fig. 2. Here the three galaxies are supposed to be of the same mass, with G_1 being located at the center of gravity of G_2 and G_3 , each having a circular orbit around G_1 . The system is kinematically stable at all times. However, the assumptions of a random structure are not satisfied and, while the whole system may be regarded as composed of a single "ring" considered above, the density of galaxies along this ring is far from uniform: There are only two such galaxies located at the two ends of a diameter. As a result, if the kinematic test is applied to any such system, its outcome will be misleading. If the three galaxies happen to be equally bright then the correlation between the radial velocity and the apparent magnitude will be perfect at all times. According to the time that the observations are made, the regression coefficient of ρ on m will be positive or will be negative. The result of the application of the test will be: with a negligible probability of error one can assert that the true regression of ρ on m is positive (or negative). While this assertion will be correct, it would not be correct to replace it with an assertion of kinematic instability.

The situation illustrated in Fig. 2 is extreme. However, with obvious modifications, the reasoning applies to all small systems in which internal motion may be expected to follow a regular pattern. If any such system is treated individually, the regularity of internal motions may easily imply a nonzero correlation, occasionally positive and occasionally negative, between the radial velocity and magnitude, without any connection with kinematic instability. In order to validate this connection, it is necessary to create the opportunity for the correlations due to the regularity of internal motions to average out. Although no definite evidence exists, it is usual to act on the assumption that the orientation of systems of galaxies is random so that, for example, triplets illustrated in Fig. 2 contemplated from a given point are in a clockwise motion as frequently as they are in the opposite motion. On this assumption, then, if one applies the kinematic test simultaneously to a reasonably large number of small systems of galaxies, the magnitude-redshift correlations due to the regularity of internal motions will have a

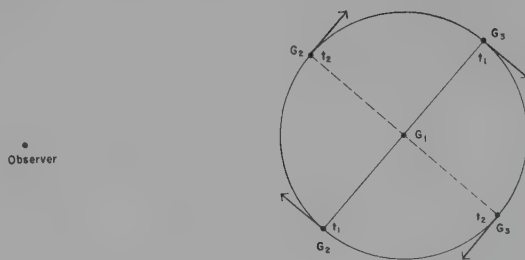


FIG. 2. Dependence on time of the magnitude-redshift relation in a small stable system.

chance of averaging out and, if they do not, this would be evidence of, so to speak, collective instability of the systems considered. The number of systems necessary for an effective averaging out of correlations due to regularity of motions is difficult to determine. However, it is clear that the smaller the systems considered, the larger must be the number of such systems treated together.

EMPIRICAL EVIDENCE

The observational data used in the present paper is mostly that analyzed by Lovasich, Mayall, Neyman, and Scott (1961). It refers to all systems of galaxies for which it was possible to find in the literature the apparent magnitude and the redshift for at least three presumed members of the system. The principal source of data is the catalogues of Humason and of Mayall published in 1956. Additional radial velocities for 28 presumed members of the Coma cluster are due to Mayall (1961), with the magnitudes measured by Kron, Mayall, and Walker. Some magnitudes were taken from the papers by Stebbins and Whitford (1952) and by Pettit (1954). The radial velocities for the cluster "Around NGC 6166" are due to Minkowski and the apparent magnitudes to Abell, both supplied in advance of publication. It is a pleasure to record here our indebtedness. Also Dr. Abell was kind enough to supply us with unpublished apparent magnitudes for several objects in the Hercules and the Coma Borealis clusters and we are indebted to Mrs. E. M. Burbidge for her kind help in matching the apparent magnitudes with the radial velocities for several objects in the Hercules cluster.

With one exception, the calculations for particular systems were based on all the objects that observational astronomers assigned to this system. The exception is the object NGC 4258 attributed to the Ursa Major Cloud by Humason, Mayall, and Sandage (1956). The reason for the exclusion of this object is that on the scatter diagram in Fig. 3 it corresponds to an outlier point far to the left of all the others which form a generally cohesive group.

One other case of exclusion may be mentioned. This refers to the four galaxies NGC 4670, Anon 1, NGC 4725, and NGC 4793 in the region of the Coma cluster.

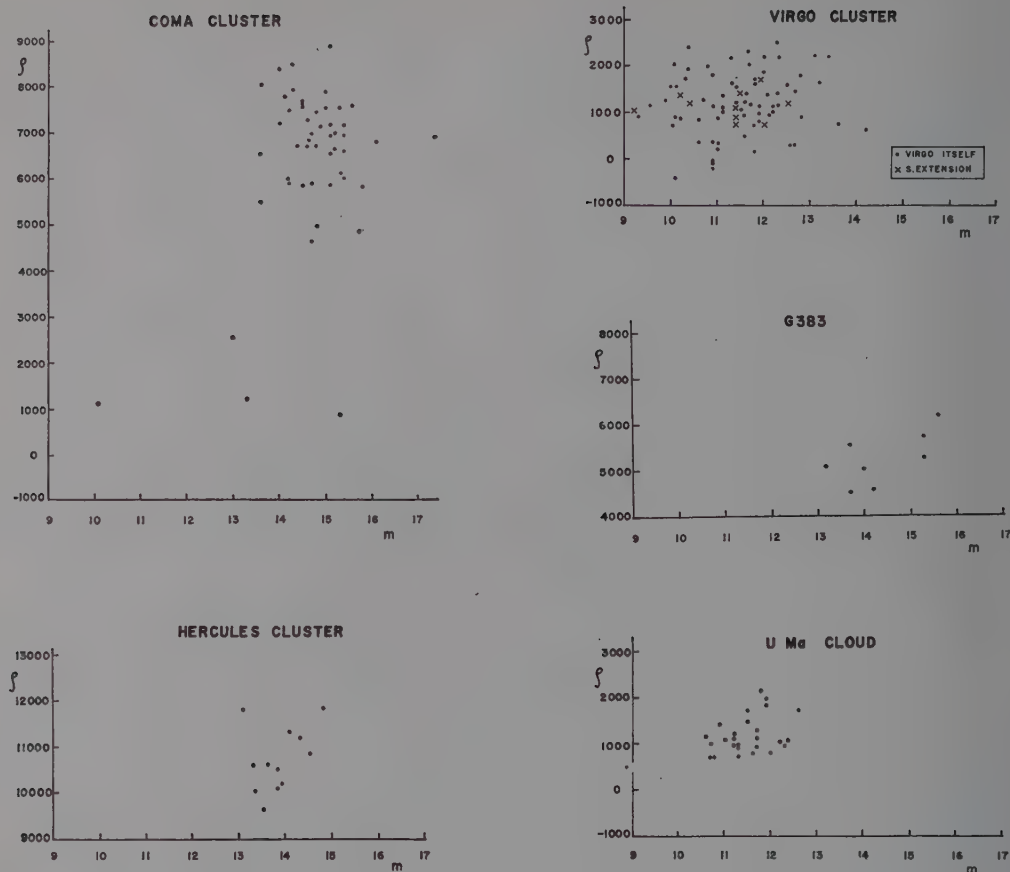


FIG. 3. Distribution of radial velocity and apparent magnitude for several clusters and groups.

They were observed by Mayall as presumed members of Coma. However, the subsequent analysis of the spectra suggested to Mayall that the four galaxies are subluminal foreground systems, not in the Coma cluster. In the scatter diagram in Fig. 3 these four objects correspond to the four outlier points far removed from the other 46.

Tables III and IV list separately the eight clusters and the 15 groups included in the calculations. In each table the successive columns give the total number n of objects for which measurements are available, the mean radial velocity ρ , in km/sec, the mean apparent magnitude, its sample dispersion and its sample covariance

TABLE III. Basic statistics for the eight clusters studied.

Cluster	n	ρ km/sec	m	$S(m)$	$S(m, \rho)$	$S(\rho)$
Virgo	80	1197	11.4	1.00	92.4	641
Perseus	4	5516	13.2	0.79	109.8	520
Coma	46	6866	14.8	0.69	-102.1	932
Around NGC 6166	19	9028	16.3	0.42	93.7	864
Hercules	15	10776	15.2	0.52	40.2	652
Pegasus II	3	12821	15.0	0.40	-103.0	662
Ursa Major I	3	15334	16.1	0.52	-248.2	481
Cor. Bor.	8	21651	17.4	0.32	-84.2	1294

with ρ . In all cases the apparent magnitude was not corrected for redshift and the appropriate corrections were introduced in the calculations. The corrections are based on the data published in Humason, Mayall, and Sandage (1956); they do not affect the figures in Tables III and IV materially. The last columns in the two tables give the sample dispersion of the radial velocity.

The data in Tables III and IV exhibit two unsatisfactory features. One is that, as the distance of the systems considered grows, the value of $S(m)$ the sample dispersion of the apparent magnitude is markedly decreased. With reference to formula (11) giving the standard error of the estimate $\hat{B}$ of the regression coefficient of ρ on m , it is seen that, other things being equal, this parameter must increase with the distance. In fact (see Table III), a single observation for the Virgo cluster with $S(m)$ about unity is worth three observations for Coma Borealis, where $S(m)$ is of the order of about $\frac{1}{3}$. It seems very probable that the notable decrease of $S(m)$ with the distance is the effect of the difficulty in measuring redshift for faint objects. The result is that for very distant systems only the very

brightest objects are observed so that their apparent magnitude has about the same value. While this circumstance diminishes the power of the test for stability, it does not invalidate the test.

The second unsatisfactory feature, notable in Table IV, indicates a circumstance that may invalidate the kinematic test. This feature is a steady increase in $S(\rho)$ observable for increasing values of ρ , that is, for increasing distance of the systems.

Various remarks in the writings of observational astronomers indicate that, frequently, when a presumed member of a system has radial velocity very different from that of the remaining presumed members of the system, a decision is made that the particular object does not belong to the system. In other words, the usual method of assigning membership to a cluster or group involves selection for similarity in redshift. This selection may or may not be satisfactory. In particular, if pushed too far, it will distort the conditional distribution of redshift, given the apparent magnitude of the

TABLE V. Results for the eight clusters studied.

Cluster	<i>n</i>	ρ , km/sec	$\hat{B}$	Standard error of $\hat{B}$	$P\{T\}$	$\hat{\theta}$
Virgo	80	1197	92	76	0.24	16 <i>H</i>
Virgo: Spirals	19	1306	-2	210	0.94	-3 <i>H</i>
Virgo: Ellipticals	34	1016	81	108	0.45	12 <i>H</i>
Perseus	4	5516	178	580	0.77	145 <i>H</i>
Coma	46	6866	-212	199	0.24	-250 <i>H</i>
Around NGC 6166	19	9028	539	446	0.29	661 <i>H</i>
Hercules	15	10776	147	404	0.76	204 <i>H</i>
Pegasus II	3	12821	-644	1394	0.78	-448 <i>H</i>
Ursa Major I	3	15334	-919	1073	0.39	-2122 <i>H</i>
Corona Borealis	8	21651	-809	924	0.26	-3398 <i>H</i>
All clusters	178		21	139	0.39	14 <i>H</i>

$S(\rho) = 70$ km/sec. Finally, then, the alternative calculations were performed for 10 groups other than the five mentioned. In Table IV the excluded groups are marked with asterisks.

RESULTS

The numerical results obtained are summarized in Table V, referring to clusters and in Table VI referring to groups of galaxies. The latter table gives the details only for the 10 groups not apparently affected by selection for similarity of redshift. As to the remaining five groups marked with asterisks in Table IV, it should suffice to say that the calculations based on the totality of data, including the five starred groups, gave no indication of instability: the probability of error in asserting instability has the upper bound 0.77.

The particular entries in the two tables are: the number n of observations, the mean radial velocity in km/sec, the estimate $\hat{B}$ of the regression coefficient of the radial velocity of the apparent magnitude, the estimate of the standard error of $\hat{B}$, the "significance probability" P , and a very tentative estimate $\hat{\theta}$ of the instability factor θ . It must be explained that for the individual clusters for which a substantial number of observations is available, that is, for Virgo and for Coma, the estimate of the standard error of $\hat{B}$ was computed individually, using the data for the particular cluster in order to estimate the dispersion σ_1 . On the other hand, for all other systems in each table the standard error of $\hat{B}$ was estimated using all the data in the table on the (not very reliable) assumption that in all the systems involved the random velocity component

TABLE VI. Results for 10 of the groups studied.

Group	<i>n</i>	ρ , km/sec	$\hat{B}$	Standard error of $\hat{B}$	$P\{T\}$	$\hat{\theta}$
Ursa Major Cloud	26	1208	259	144	0.09	47 <i>H</i>
G 1068	3	1604	432	273	0.12	106 <i>H</i>
Fornax	3	1631	-2020	2853	0.47	-505 <i>H</i>
G 5846	8	1808	91	175	0.62	24 <i>H</i>
G 7619	6	3836	107	334	0.57	62 <i>H</i>
G 6928	3	4643	-147	279	0.60	-105 <i>H</i>
G 507	3	4665	-489	387	0.21	-346 <i>H</i>
G 383	8	5264	413	169	0.02	331 <i>H</i>
Stephan's Quintet	4	6735	-107	340	0.74	-117 <i>H</i>
G 68	5	6785	269	305	0.39	273 <i>H</i>
All 10 groups	69		162	38	0.02	51 <i>H</i>

TABLE IV. Basic statistics for 15 groups of galaxies.

Group	<i>n</i>	ρ , km/sec	<i>m</i>	$S(m)$	$S(m, \rho)$	$S(\rho)$
*M81	7	172	10.0	1.39	-14.0	132
*M101	6	478	10.5	1.59	-29.5	102
*G 1023	5	513	10.5	0.87	-51.1	257
*Leo Group	18	788	11.1	0.98	36.9	264
*Ursa Maj. Cl.	26	1208	11.5	0.56	81.1	403
*G 3190	3	1222	12.2	0.58	-40.8	70
G 1068	3	1604	10.8	1.05	472.1	453
Fornax	3	1631	11.0	0.10	-20.2	299
G 5846	8	1808	12.2	0.87	69.2	384
G 7619	6	3836	13.3	0.98	102.8	285
G 6928	3	4643	13.8	1.02	-153.7	308
G 507	3	4665	13.1	0.74	-265.7	416
G 383	8	5264	14.4	0.90	334.9	570
Stephan's Q.	4	6735	14.2	0.68	-50.3	470
G 68	5	6785	14.9	0.66	117.5	458

cluster members, and this would invalidate the conclusions drawn from the application of the kinematic test.

The figures given in the last column of Table IV show a systematic increase over the top few entries and suggest to us that, probably, the assignment of membership to the nearby groups was accomplished under selection for similarity in redshift that was much stronger than for the more distant groups. Whether the assignment of members to the other groups left the distribution of ρ unaffected is, of course, uncertain. However, in view of the evidence in Table IV, it seemed advisable to perform the test for the groups of galaxies twice, once using all the material available and then for only those groups in which the values of $S(\rho)$ do not suggest excessive selection for similarity of redshift. Generally, the groups to be eliminated from this study are the four nearer groups, with redshift between 170 and 790 km/sec. However, G 3190, with mean redshift 1222 km/sec, is also suspect because of the very tiny dispersion of

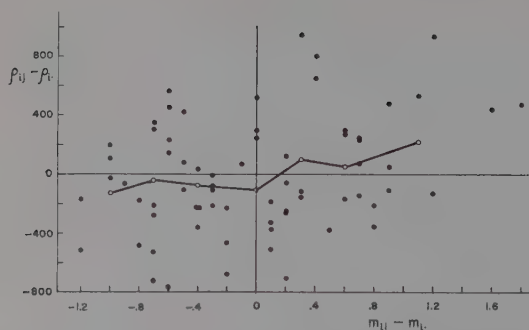


FIG. 4. Scatter diagram of radial velocity and apparent magnitude for the ten groups studied. Dots: Individual deviations from mean for group; circles: normal points.

of the radial velocity has the same dispersion σ_1 . The significance probability means the upper bound of the probability of error in asserting kinematic instability.

The estimates $\hat{\theta}$ of the instability factor are those given in Lovasich, Mayall, Neyman, and Scott (1961). While giving them here, we view them with a certain amount of skepticism for the reason that, contrary to the regression estimates B , they depend not only on the immediately available data, that is, on apparent magnitudes and radial velocities of particular galaxies, but also on *presumed values* of certain other parameters not estimated from the same data. These parameters are $\sigma_E = 6.8 \times 10^5$, $\sigma_M = 1.25$, and $\tau = 0$.

Inspection of Table V shows that, with a risk of error with probability possibly equal to one in four, we may assert instability of the Virgo cluster as a whole and of the Coma cluster as represented by 46 objects. In the first case the indicated instability is expansion. In the second case the regression coefficient B is negative and indicates contraction. This latter finding is interesting because it is consistent with the theoretical deductions of van Albada as reported in another paper in this volume.

Following the suggestion of de Vaucouleurs (1961) that what is commonly called the Virgo cluster is, in reality, a combination of two clusters, one composed of spiral and the other of elliptical galaxies, the analysis was repeated for these two groups of galaxies separately. It was found that the group of 19 spirals (as identified by de Vaucouleurs) does not show any indication of instability. On the other hand, for the ellipticals the indications of expansion persists.

The joint treatment of all the clusters shows no indication of instability.

As to the results relating to groups of galaxies, only the figures relating to the totality of 10 groups are relevant for judging stability. It is interesting that in this case the probability of error in asserting instability is really small, not more than 0.02. The kind of instability indicated is expansion. As mentioned, because of the considerable uncertainty regarding the constants needed to estimate θ , we do not consider the estimate

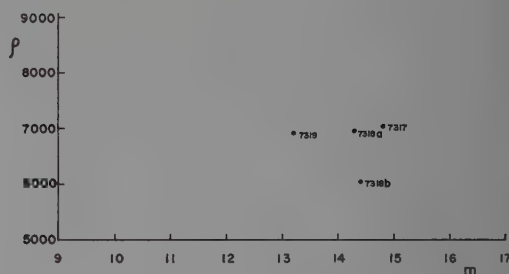


FIG. 5. Scatter diagram and identification of the four available galaxies in Stephan's Quintet.

given as trustworthy. However, its value of 51.4 H km/sec/megaparsec (which is 5140 if $H = 100$) is a rate very considerably faster than that of the general expansion of the universe.

While the over-all regression coefficient for the 10 groups and the significance probability of 0.02 summarize the information provided by the data, it is interesting to examine them optically. Figure 4 gives the combined scatter diagram of points representing the 69 members of the 10 groups studied. In order to eliminate the effect of increasing distance of the groups on both the apparent magnitude and the radial velocity, Fig. 4 reflects the deviations of these quantities from their respective means computed separately for each of the systems studied. The seven circles connected with a polygon represent "normal points."

While the positive correlation between the deviations of ρ and m from their means appears obvious, the question arises as to whether it really means kinematic instability of the systems of galaxies studied. In other words, the question is whether there is any alternative phenomenon that could have caused the exhibited correlation, other than the suspected expansion. Undoubtedly, such factors exist. The worst suspect is the possible inclusion in each, or in most, of the systems studied of some foreground and/or some background objects. However, in order that this factor work, it is necessary that the number of mistakenly attributed objects be substantial. Another possibility is that the measurements of the radial velocity and of apparent magnitude are affected by some unsuspected correlated errors. Still another possibility is that the K corrections evaluated by Sandage and introduced in the calculations are insufficient, whereby the galaxies with relatively large radial velocities appear fainter than they should. Again, a rather large change would be required.

If the possibility of these and of similar other factors is not admitted, then there remains expansion of the systems. As evidenced by the data in Tables I and II, in order that the expansion be noticeable with the available volume of observational material, this expansion must be rapid, at least of the order of 10 times the general expansion of the universe and, most probably, accompanied by a substantial amount of internal absorption.

Before concluding we wish to present a separate scatter diagram of magnitude and redshift for Stephan's Quintet, which has been the subject of recent studies by Ambartsumian (1957), the Burbidges (1961), and by Limber and Mathews (1960). There are only four objects available to us and they are identified in Fig. 5. It will be seen that, although strongly suspected of expansion on dynamical grounds, the kinematical aspect of the situation is far from indicating instability. With rare exceptions, the scatter diagrams drawn for other small systems have an appearance similar to that for Stephan's Quintet. The distinct impression of positive correlation emerges only if a number of such diagrams are superimposed so that their centers of gravity coincide. This is just what is represented in Fig. 4.

REFERENCES

- Ambartsumian, V. A. 1957, *Ann. Acad. Sci. Armenian S.S.R. Ser. Phys.-Math. Sci.* 11, 1.
 Burbidge, E. M. and Burbidge, G. R. 1961, *Astrophys. J.* **130**, 15.
 de Vaucouleurs, G. 1961, *Astrophys. J. Suppl.* **6**, 213.
 Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* **61**, 97.
 Limber, D. N. and Mathews, W. G. 1960, *Astrophys. J.* **132**, 286.
 Lovasich, J. L., Mayall, N. U., Neyman, J., and Scott, E. L. 1961, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (University of California Press, Berkeley, California), Vol. 3, p. 187.
 Neyman, J. and Scott, E. L. 1954, *Studies in Mathematical and Mechanics Presented to Richard von Mises* (Academic Press Inc., New York).
 —. 1959, *Handbuch der Physik*, edited by S. Flügge (Springer Verlag, Berlin), Vol. 53, p. 416.
 Pettit, E. 1954, *Astrophys. J.* **120**, 413.
 Stebbins, J. and Whitford, A. E. 1952, *ibid.* **115**, 284.
 Zwicky, F. 1957, *Morphological Astronomy* (Springer-Verlag, Berlin).

DISCUSSION

PAGE asked whether the elimination of outliers—supposed projected foreground or background objects recognized by discordant velocities—would not in itself introduce unwanted selection effects. NEYMAN advised

that the investigator try calculations with and without outliers, then make up his mind "which he likes best," while retaining both.

HOLMBERG, referring to his later paper below, interpreted the correlation between radial velocities of galaxies in clusters and their "nuclear magnitude" as evidence of systematic errors in the measurement of radial velocity rather than as a result of expansion of the clusters.

DE VAUCOULEURS estimated that $0^m.7$ intergalactic absorption in the Coma cluster would indicate intergalactic material of total mass approximately 1% of the cluster's mass (derived from the virial theorem), if the absorbing material is of the same composition as interstellar material in our galaxy (see *Astrophys. J.* **131**, 1960, 589).

In answer to G. BURBIDGE, NEYMAN defended the application of a statistical test for expansion to small groups of only a few galaxies each. He argued that a systematic expansion would be detected from observations of a large number of such small groups. However, PAGE pointed out that the power of the test is reduced by the variability in absolute magnitude. SCOTT inserted that all such variability is taken into account in the power computations, and that the power can be >0.9 with 10 groups and $\theta \geq 10H$. DE VAUCOULEURS questioned the effect of rotation of clusters on the results of the test. In answer, NEYMAN stated that rotation would not affect the test—and could not be detected by it. Likewise, the exchange of galaxies between cluster and field, as proposed by Lemaitre in a later paper below, would not be detected.

McCREA asked what the test would show if clusters were fortuitous and all galaxies took part in the general expansion of the universe. NEYMAN answered that if there were no intergalactic absorption then $\hat{\theta}$ would be equal to the Hubble constant, 10^{-4} in his units, which is much smaller than values observed.

Evolution of Clusters of Galaxies under Gravitational Forces

G. B. VAN ALBADA

Astronomical Institute, Amsterdam, The Netherlands

I. INTRODUCTION

A THEORETICAL study relating to the formation and evolution of clusters of galaxies has been published last year (Van Albada 1960). As a continuation of this investigation, a series of numerical integrations has been made at the Mathematical Center at Amsterdam. Before entering into a discussion of these integrations and of the results obtained by them, we will dwell for a moment on the theoretical foundations.

In paper I we considered the formation and evolution of clusters of galaxies under the influence of their own gravitational field. The starting point was an approximately homogeneous field of galaxies, embedded in an expanding universe. If the velocity dispersion of the galaxies is equal to σ and if the mean density of the universe is ρ , then instabilities may develop in regions whose characteristic dimensions are of the order

$$r \approx \sigma / (G\rho)^{1/2}. \quad (1)$$

If R is the radius of the expanding universe, ρ is proportional to R^{-3} while σ varies as R^{-1} . It follows that r/R is proportional to $R^{-1/2}$; i.e., the instability progresses in the direction of regions of smaller relative dimensions.

In the early evolutionary stages of the universe, only regions of very large relative dimensions could have been unstable. The actual development of the instabilities may have been prevented by radiation pressure. Only after the universe had expanded to a density of the order 10^{-24} or 10^{-25} g/cm³ could radiation escape from the potentially unstable regions and could the instability develop. It must be assumed that at this critical epoch the galaxies existed already, possibly only as eddies in a gaseous medium.

In the computation of the further development of the instabilities, the system of galaxies has been treated as a kind of "gas" with the individual galaxies as the constituent "particles." We may express this idea by saying that, instead of following the individual galaxies along their entire orbits, we fix our attention on a given macroscopic mass element and consider only those galaxies which, at any given instant, are within this element. This may be achieved if we start out with an arbitrary galaxy and "jump over" every time when we come close to another galaxy. Thus we are thrown back and forth by a series of "quasi collisions." The most important difference with an actual gas is that, in these quasi collisions, the velocities of the two particles are simply interchanged and not otherwise altered.

It should be noted in this connection that the effects of actual collisions and close encounters have been

neglected. There are reasons to expect that these effects are of relatively little importance. The main reason for this is that the time of relaxation of the system is much longer than the time of free pulsation.

The mechanism of the evolution of the "quasi gas" may now be pictured as follows. Local expansions or contractions lead to local "adiabatic" changes in the velocity dispersion. Thus a compression by a factor 2 in the x direction, accompanied by an expansion by a factor 3 in the direction of y leads to a doubling of σ_x , while σ_y is reduced by a factor 3. Anisotropic expansions or contractions thus lead to an anisotropic pressure tensor which, in its turn, influences the mass motion.

Pressure gradients will be built up by nonuniform expansions and contractions, but the mixing of galaxies from different regions will tend to reduce these gradients and the anisotropy of the pressure tensor. This mixing process may be compared to the thermal flow in an actual gas. It is possible to take this phenomenon into account, but the resulting equations are very involved. A general treatment would lead to a system of partial differential equations of order 10. If spherical symmetry is assumed, the order of the system reduces to six. In the numerical computations which have been made up to now, we have assumed spherical symmetry and neglected the "thermal flow." These are two serious limitations which should be kept in mind.

With these assumptions, the problem becomes relatively simple. We have to consider radial and tangential motions. The mass motions are only in the radial direction. If a mass element whose initial distance from the center of the configuration is equal to r_0 is displaced to r , the compression is $\partial r / \partial r_0$ in the radial and r/r_0 in the tangential direction. Thus for the velocity dispersions in these two directions we find

$$\sigma_{\text{rad}}/\sigma_0 = (\partial r / \partial r_0)^{-1} \quad \text{and} \quad \sigma_{\text{tan}}/\sigma_0 = (r/r_0)^{-1}. \quad (2)$$

Here, as elsewhere, subscript 0 refers to the initial stage.

If the radial and tangential components of the pressure are denoted by P and Q , we find from Eqs. (2) and the equation of continuity

$$P/P_0 = (\rho/\rho_0)^3 (\bar{\rho}/\rho_0)^{-1}, \quad Q/P_0 = (\rho/\rho_0) (\bar{\rho}/\rho_0)^{1/2}. \quad (3)$$

Here $\bar{\rho}$ is the average density inside the sphere of radius r .

From Eqs. (2) and (3) we see that in the outer regions of a cluster, where $\rho < \bar{\rho}$, the radial component of the pressure is smaller than the tangential component; i.e., the peculiar motions are mainly in the tangential direction.

The mass motion is described by the equation

$$\frac{dV}{dt} = -\frac{Gm}{r^2} - \frac{1}{\rho} \frac{\partial P}{\partial r} + 2 \frac{Q-P}{\rho r}, \quad (4)$$

where m is the total mass inside sphere r .

Expressing the pressure and the density by the quantities r/r_0 and $\partial r/\partial r_0$, we finally obtain

$$\begin{aligned} \frac{d^2 r}{dt^2} + \frac{4\pi}{3} G \rho_0 \frac{r_0^3}{r^2} \\ = \sigma_0^2 \left[3 \frac{\partial^2 r}{\partial r_0^2} \left(\frac{\partial r}{\partial r_0} \right)^{-4} - \frac{2}{r_0} \left(\frac{\partial r}{\partial r_0} \right)^{-3} + \frac{2}{r_0} \left(\frac{r}{r_0} \right)^{-3} \right]. \end{aligned} \quad (5)$$

This equation has essentially the form of a wave equation. The velocity of propagation of a "sound wave" is equal to $\sigma_{\text{rad}} \sqrt{3}$.

By studying this equation we hope to be able to describe the development of an initial small perturbation of the homogeneous field into a real cluster of galaxies.

II. TRANSFORMATION AND INTEGRATION OF THE WAVE EQUATION

As in paper I, we introduce dimensionless variables x , u , and ξ defined by

$$r_0 = \lambda x, \quad r = (R/R_0) \lambda y, \quad (6)$$

$$\xi = (1/c)(dR/dt). \quad (7)$$

Here λ is an adjustable parameter with the dimension of length. By changing its value, we change the linear scale of the disturbance. R_0 is the radius of the universe at some arbitrarily chosen epoch, and c is the velocity of light. The quantity ξ replaces the time coordinate, but it should be noted that ξ decreases with time. The quantity y measures distance relative to the radius of the expanding universe.

With these new variables, Eq. (5) takes the form

$$\ddot{y} = -\frac{2}{\xi^2 - \epsilon} \left(\frac{x^3}{y^2} - y \right) + k^2 \left(\frac{y''}{y^4} - \frac{2}{3xy^3} + \frac{2x^2}{3y^3} \right), \quad (8)$$

where dots and primes denote derivatives with respect to ξ and to x , respectively.

The parameter ϵ refers to the model of the universe. Its values for a spherical, a Euclidean and a hyperbolic universe are -1 , 0 , and $+1$, respectively.

The quantity k is a dimensionless constant, which is inversely proportional to λ and thus depends on the size of the perturbed region. The product $k^2(\xi^2 - \epsilon)$ is inversely proportional to R . When this quantity decreases below 6, the region characterized by the parameter k becomes gravitationally unstable.

If ϵ differs from zero, Eq. (8) cannot be integrated unless the quantity k is specified first. For a Euclidean

universe, however, where $\epsilon=0$, the quantities k and ξ occur only in the form of the product $k\xi$. As a matter of fact, the separate quantities k and ξ have no meaning for a Euclidean universe. The product, however, or better $k\xi/6^{1/2}$, indicates the inverse size of the region under consideration as compared to the radius of instability at the same epoch.

For $\epsilon \neq 0$, a particular integration of Eq. (8) can be used only to describe the subsequent stages of development in one particular region. If $\epsilon=0$, however, the same integration may also be used to describe the state of regions of various sizes at the same epoch. Hence these integrations have a very wide field of application. For this reason, we have devoted considerable attention to the special case of a Euclidean universe.

To start the integration of Eq. (8), we must specify the initial conditions. For this purpose, we have made use of the solution of the linearized equation studied in paper I. We thus write

$$\begin{aligned} y &= x - ax^{-1/2} J_{3/2}(x), \\ &\text{(initial conditions)} \quad (8) \\ \frac{1}{k} \frac{\partial y}{\partial \xi} &= bx^{-1/2} J_{3/2}(x). \end{aligned}$$

We started the integrations somewhat beyond the limit of stability, at $k\xi=1$. It was assumed that at the corresponding epoch the amplitude of the disturbance was still small.

The most extensive integrations were made with $a=1.00$, $b=1.62$. These values correspond to an initial disturbance of the average density in the region of about 7% (cf. I, p. 193/194). We have also made some computations with a four times smaller perturbation.

The displacement $y-x$ as given by Eq. (8) passes through zero near $x=4.49$ and reaches maximum values near $x=2.08$ and $x=5.94$. These points remain remarkably stationary when the integration is carried on beyond the linear approximation. Physically, this means that a cluster collects its material exclusively from the region with $x < 4.5$ and mainly from the region with $x < 2.1$. It is clear that the regions beyond $x=4.5$ have practically no influence on the development of the cluster.

The integrations have been performed with a rectangular net. In view of what has been said above, it seemed possible to use a fixed array for x . The end point of the array was chosen at $x=6.0$, because small uncertainties in the estimates of y' and y'' , which have to be made at this point, introduce very little error into the solution. It has been verified that the error which is introduced in this way is insignificant.

The main series of integrations has been made with a step in x equal to 0.1. The step in ξ is limited by two considerations. One restriction follows from the condition of numerical stability of the solution. Physically, this condition may be expressed by the requirement

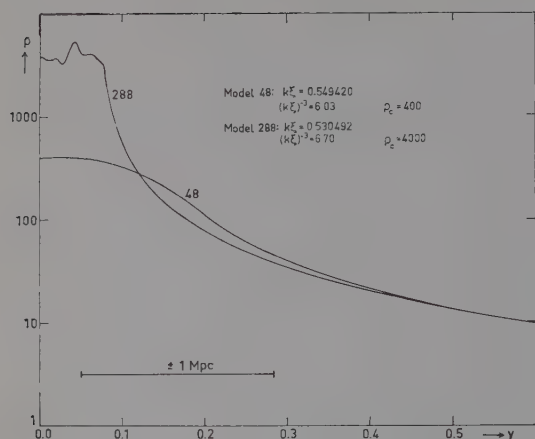


Fig. 1. Density distributions, models 48 and 288.

that a "sound wave" is not permitted to travel from one net point to the next in one time step.

Since the "velocity of sound" is inversely proportional to $\partial r / \partial r_0$ and the distance between adjacent net points is directly proportional to the same quantity, the maximum possible step in $k\xi$ is directly proportional to $(\partial r / \partial r_0)^2$. Compression factors of the order of 100 occur in our models, as should be the case if these models are to represent clusters like Coma, with a central density of the order 10^5 times the mean density of the universe. This means, however, that the maximum possible step in $k\xi$ becomes of the order of 10^{-4} times the step in x . Considering that the range in $k\xi$ to be covered is of the order unity, it is understandable that only a fraction of the evolutionary history of dense clusters could be covered by the integrations.

One might try to overcome this difficulty by interchanging the roles of x and y in Eq. (8), i.e., by using the transformed equation where x is the dependent and y the independent variable. This means that, instead of using comoving coordinates, we use a coordinate system fixed in space (i.e., only expanding with the universe). Because the distance between adjacent net points is then fixed, the permissible step in $k\xi$ will be proportional to $\partial r / \partial r_0$ only.

The difficulty with this procedure is that the transformed differential equation is elliptic in those regions of space where the mass motions are "supersonic." We will see later that this happens in the extensive outer regions of the cluster, which may be described as the "corona." In these regions, no numerically stable solution of the transformed equation can be obtained. With the original equation (7) no such difficulties occur.

Even if the condition for numerical stability is satisfied, we still have to put additional restrictions on the step in $k\xi$ when adjacent net points come very close and when their velocities and accelerations are large. If this additional restriction is neglected, we may even get crossovers of the net points. We have to choose the

time step sufficiently small so that the change in the relative positions of the net points remains under control. In certain parts of the integration this condition may be more severe than the condition of numerical stability.

The integrations were made on the X1 of the Mathematical Center at Amsterdam. The program was written in ALGOL, which was translated by the machine in its own code.

III. RESULTS OF THE INTEGRATIONS: GENERAL TRENDS OF DEVELOPMENT

The main series of integrations, for a Euclidean universe and with initial conditions specified by $a=1.00$, $b=1.62$ could be carried on for the range $k\xi=1.00 \dots 0.5278$. These figures may be transformed to a time scale by observing that the "absolute" time t is proportional to ξ^{-3} . The absolute length of the time interval and the dimensions of the configuration will be discussed later (Sec. VI).

The first part of the computation, when the compression is still moderate, progresses rapidly. The models with $k\xi > 0.55$ are characterized by a very regular structure, similar to the structure already described in paper I. In these models, the density over the greater part of the configuration decreases as a moderate negative power of r , so that a plot of $\log \rho$ vs $\log r$ shows only a slight curvature.

For smaller values of $k\xi$, i.e., for relatively later epochs, the cluster develops a different structure with a clear division between a "nucleus" and a "corona." The corona maintains the regular structure described above, but the nucleus exhibits large internal fluctuations in density, whereas its over-all structure is nearly homogeneous. The nucleus stands out sharply against the corona: An almost sudden drop in density by a factor 10 or so is not unusual. Figure 1 shows the density distributions in models 48 and 288, respectively, as typical examples of the two different structures.

The reasons for the development of this composite structure can be understood as follows. While, under the influence of the gravitational attraction of the cluster, matter is streaming inwards, the pressure in the central regions of the cluster gradually increases until it balances or even overshoots the gravitational force. The cluster may thus be expected to oscillate more or less wildly around an equilibrium structure.

A first understanding of the evolution of the cluster may thus be reached by studying possible equilibrium structures. For this purpose, we return to Eqs. (3) and (4).

We readily see that an equilibrium structure of finite mass and dimensions and with zero density at the surface should be possible. At the boundary of such a structure we would have $P/\rho=0$ and $\partial P/\rho \partial r=0$, as follows from the first equation (3). On the other hand, Q/ρ would be finite. If then we assume that the left-hand

side of Eq. (4) vanishes, we obtain

$$\frac{Gm}{r^2} = -\frac{2P_0}{r\rho_0} \left(\frac{\bar{\rho}}{\rho_0} \right)^{\frac{5}{3}}. \quad (9)$$

It should be noted that the value of the ratio $P_0/\rho_0^{5/3}$ which occurs in this expression does not depend on the epoch to which it is referred, because in isotropic adiabatic expansion P varies as $\rho^{5/3}$.

Combining this equation with the obvious relation $m = (4\pi/3)\bar{\rho}r^3$, we find that, given $P_0/\rho_0^{5/3}$, the radius r and the mean density $\bar{\rho}$ are completely determined by m . In particular, we find that $m r^3 = \text{const}$; hence the mean density of such an equilibrium structure is proportional to the square of its mass.

We may look at this result from a different angle. When a cluster collects its material from distant regions, we have to take into account that the average angular momentum of the incoming galaxies with respect to the center of the cluster is proportional to their initial distance from the center, say, r_0 . At the same time, the total mass of the cluster will be proportional to the volume from which it has collected its material, i.e., to r_0^3 . We may now ask under what conditions the angular momentum of galaxies coming from a distance r_0 will be so large that they will move in free orbits along the boundary of a cluster which collected its material from the sphere r_0 . We then find proportionality between m/r^2 and r_0^2/r^3 , or between r_0 and $1/r$, where r is the radius to which the cluster contracts. This is the same relation which we found above.

Introducing our dimensionless variables, Eq. (9) may be reduced to the form

$$xy = \frac{1}{3}k^2(\xi^2 - \epsilon). \quad (10)$$

Considering that the model for which the integration has been made develops its nucleus for $k\xi \approx 0.5$, we find $xy \approx 0.08$. At the time when the nucleus developed, its boundary was at $x = 1.2$ approximately. From this we find $y = 0.07$, corresponding to a compression by a factor 17 or an average density of 5000 for the nucleus. This is a good approximation of the value which was found in the actual integration. It should be noted that all densities are expressed in the average density of the universe at the same epoch.

It should now be mentioned that a true equilibrium structure never develops. At the very best, the acceleration dV/dt will be zero, but then the contraction of the nucleus continues. Then, in a second approximation, we may describe the evolution of the nucleus as a series of subsequent equilibrium structures. Because of the contraction of the nucleus, the boundary value of y decreases, and then Eq. (10) shows that the boundary value of x increases. Expressed otherwise, accretion occurs at the boundary of the nucleus. This conclusion is confirmed (better, was first obtained) by the numerical integrations.

The above considerations suggest an interesting conclusion. Let us consider two series of integrations, differing only on account of the initial conditions. We assume that the amplitude of the original perturbation was much smaller in series B than in A . Then the development of the nucleus will occur at a smaller value of $k\xi$ in series B than in A .

We now compare phases for which the accretion of the nucleus in the two series of models has proceeded to the same fraction of the available mass, i.e., to the same value of x . Then Eq. (10) shows that the boundary of the nucleus occurs at a smaller value of y in series B than in series A . This evidently means that the nucleus in series B is more compressed.

Hence we arrive at the seemingly paradoxical conclusion that a small amplitude of the original perturbation favors a strong development of the cluster.

If we look at this result from a different angle, it loses its paradoxical tinge. The development of cluster B is slower than that of A and hence the expansion of the universe will proceed farther during the development of B than during the growth of A . Since the compression y/x determines the relative density with respect to the universe as a whole, the stronger development of cluster B has to be understood in a relative sense only. Indeed, Eq. (9) shows that equilibrium nuclei with the same mass will also have the same absolute density.

The consideration of relative densities is, of course, meaningful. We would not consider Coma as a cluster if the same high space density of galaxies would prevail throughout the universe. Moreover, the above considerations are also of importance in a comparison of two clusters of different masses at the same epoch.

The perturbations A and B which we considered need not have the same scale. It would be natural to suppose that a perturbation in a region of large relative dimensions would have a small amplitude. This would be in conformity with the idea that the original perturbations are to be interpreted as statistical fluctuations in the number of galaxies. If it is assumed that perturbation B has a large-scale parameter, i.e., a small value of k , the simultaneous state of A and B will be described by a larger value of $k\xi$ for A than for B . Then our previous statements apply to this case.

It is interesting to note that there is no well-defined lower limit for the amplitude of the original perturbation which may give rise to the growth of a cluster. In practice there are two limiting factors: first of all the competition with clusters growing in neighboring regions and second the limitation imposed by the available time.

In order to test the above reasoning, a numerical integration has been made for an initial perturbation which was about four times smaller than was assumed in the main series. It was found that the development of the nucleus began at a value of $k\xi$ which was twice as small as in the main series. But the value of x at the boundary of the nucleus was also twice as small. As

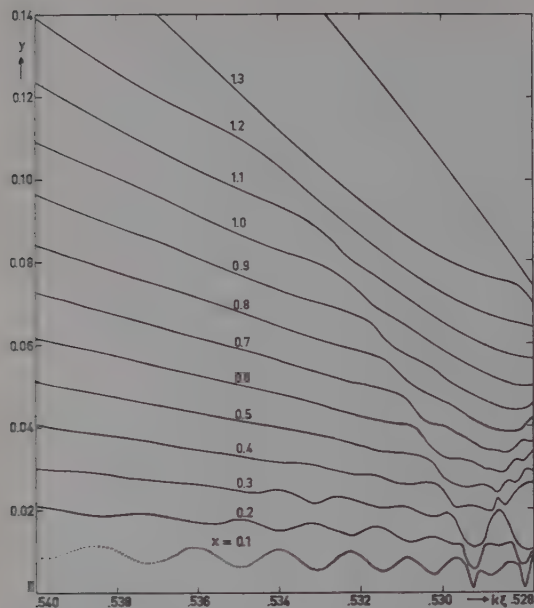


FIG. 2. y as a function of $k\xi$ for various values of x . x =initial distance from center; y =distance at epoch.

a result, y was also twice as small, and the compression y/x had approximately the same value.

The reason for this result may be understood as follows. The nucleus of the cluster makes its first appearance when the density of the equilibrium configuration is about equal to the central density of the preceding model. We have seen that these preceding models are characterized by a very regular structure in which ρ varies as a moderate negative power of r . These models do not strongly depend on the initial conditions, except for the time which they take for their development. Their densities at their most advanced stages, where these types of models cease to be applicable, differ only by moderate factors. Hence the first appearance of the nucleus occurs at an approximately fixed value of the relative density, i.e., of y/x .

For this reason, our previous statement must be amended slightly. We consider clusters with different total masses. The first development of the nucleus will occur at approximately the same relative density in all clusters. The mass ratio nucleus/corona at this stage will depend on the total mass of the cluster. If now we consider different clusters at one particular epoch only, we may say that if we find two clusters of different total masses in which the nuclei are just beginning to develop, then the two nuclei will practically have the same mass and density and the difference in mass between the two clusters will be due entirely to the corona. We may also see, however, other clusters in more advanced stages of evolution, and then we see that the mass and density of the nucleus is larger, its density being proportional to the square of

its mass. Our integrations have not proceeded far enough to see what happens when the entire corona has been consumed, but we may expect that in this case also the cluster of larger mass will have the higher density.

We now comment on the nearly homogeneous structure of the nucleus. In the nucleus we expect an approximate balance between gravitation and pressure—at least on the average. The pressure gives two contributions in Eq. (4): the gradient of the radial pressure and the term due to the anisotropy. Since the tangential pressure is larger than the radial pressure, the anisotropic term already compensates part of the gravitational force. Therefore the gradient in the radial pressure need not be very large. Furthermore, Eq. (3) shows that the radial pressure depends very strongly on the density. Hence the density gradient can be very small.

Equation (4) may also be written in the form

$$\frac{dV}{dt} = -\frac{Gm}{r^2} - 3\frac{P}{\rho^2}\frac{\partial\rho}{\partial r} + 2\frac{P}{\bar{\rho}r}\left(\frac{\bar{\rho}}{\rho} - 1\right)^2\left(\frac{\bar{\rho}}{\rho} + 2\right). \quad (11)$$

This equation shows that, apart from the term contributed by the density gradient, there is always an outward pressure, unless ρ and $\bar{\rho}$ happen to be exactly equal. The term with the pressure gradient, moreover, is provided with a factor 3, as if this were the ratio of specific heats. Disregarding the last term in (11), the equilibrium structure would be a polytrope $n=\frac{1}{2}$; the actual configurations should be still closer to homogeneity.

We conclude this section with a remark about the influence which our various approximations may have had on these results. The most dangerous approximation is probably the neglect of the "thermal flow." The "thermal flow" would tend to reduce the pressure gradient and thus have two effects: The cluster would tend to contract to higher densities and the nucleus would not quite so closely approach homogeneity. Only detailed calculations could show just how large these differences would be.

IV. PULSATIONS AND WAVE PHENOMENA

In the preceding section we described the development of the nucleus of the cluster as a series of subsequent equilibrium structures. The actual computations show, however, that such an equilibrium configuration can only be considered as a kind of average condition.

In Fig. 2 we show a series of curves giving y as a function of $k\xi$ for various values of x . Only the part of the integration for $k\xi < 0.54$ is shown, because this is the region where the interesting phenomena occur.

At the left-hand side of the figure most of the lines $x=\text{const}$ have a regular downward slope. The spacings between subsequent lines $x=0.0$ to $x=1.2$ increase

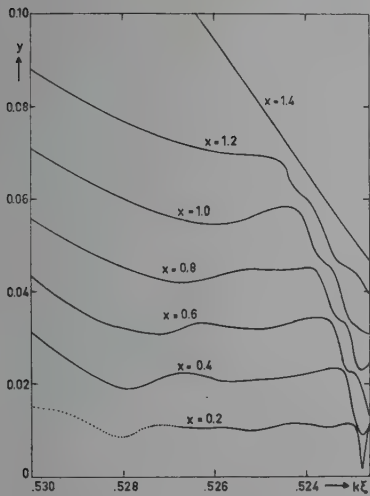


FIG. 3. y as a function of $k\xi$. x is steps of 0.2.

only very gradually, but for larger values of x the spacing rapidly increases. This phenomenon indicates that the nucleus of the cluster is already well developed. Its boundary is near $x=1.2$.

Near the center of the configuration, the line $x=0.1$ and, to a lesser degree, also $x=0.2$, shows waves. Here we observe the expected oscillation about the equilibrium configuration.

In order to see how the finite step used in the integration influence the results, we also have made integrations with steps in x equal to 0.2 and to 0.05. The integration with the coarse step showed very little of the pulsation. With step 0.05, the pulsation started earlier and extended farther into the nucleus. It thus seems that the use of a finite step width has reduced the importance of the pulsation.

Nevertheless, it is not quite clear what significance should be attached to this phenomenon. First of all, the number of galaxies inside $x=0.1$ is very small, so small indeed, that it is very doubtful whether we can attach any real significance to the behavior of this point. In the second place, the assumption of spherical symmetry may have had considerable influence on this result. If spherical symmetry is assumed, it is evident that waves moving away from the center disperse rapidly, and hence possible pulsations must be limited to the very center of the configuration. If there is no perfect spherical symmetry, there is no reason why density fluctuations should not occur at other points as well. Perhaps all that can be concluded from this phenomenon of pulsation is that the structure of the nucleus may not be entirely smooth, but that local congestions may occur. This may have a bearing on the formation of subgroups inside the cluster. For the moment, however, we cannot consider this more than a suggestion.

At the right-hand side of the figure we see how the

galaxies which come in from the corona produce strong waves when they enter the nucleus. These waves proceed towards the center of the configuration and become stronger in the same proportion as the wave front contracts. Arriving at the center, they induce an exceedingly strong compression. After this the wave moves out again and then becomes gradually weaker. The figure shows, however, the production of secondary waves, which collide with the primary waves: The pattern of motions becomes exceedingly complicated. We may add that the details are meaningless: The finite step simulates galaxies from the corona coming in bunches instead of in a gradual stream. A comparison with Fig. 3, where a step 0.2 in x has been used, shows an entirely different pattern in detail. The integration with $\Delta x=0.05$ and with correspondingly smaller steps in time is in progress but it proceeds slowly and has not been continued far enough to reach the stage where the waves from the corona appear. Level and slope of the curves for $\Delta x=0.2$ do not agree with those for $\Delta x=0.1$. The agreement between the series $\Delta x=0.1$ and $\Delta x=0.05$ is much better in this respect.

As it stands, we can only say that we have here an additional and much stronger reason to believe that the structure of the nucleus cannot be smooth. We do not know whether the pressure waves would still occur if we could solve the differential equation instead of the difference equation. This is of little interest, however, since in reality the stream of galaxies from the corona will never be smooth. Our computations indicate that density fluctuations in the incoming stream will be bunched and enhanced as the stream penetrates deeper into the nucleus. In our computations, which treat this problem as a problem of gas motion, we find how the pressure gradients which assume very large values tend to disperse the mass concentrations, but are not large enough to be really effective. If we would have taken into account the "thermal flow," the pressure gradients would be reduced and the bunching might become even more pronounced. Dynamically, the bunching can be understood as the result of the convergence of orbits towards the center; the increase in the orbital velocity is not sufficient to compensate this effect.

Thus the nucleus of the cluster might be a place where groups of galaxies are formed. After leaving the nuclear region, these groups will be partially dispersed, but some groups which have acquired a stable structure may perhaps remain. We must stress, however, the tentative character of this explanation of the formation of groups.

V. INTERNAL VELOCITIES AND THE VIRIAL THEOREM

Our integrations do not only give results for the density inside the cluster, but also for the velocities of expansion and contraction and for the velocity dispersion.

A glance at Fig. 2 shows that the mass motions in

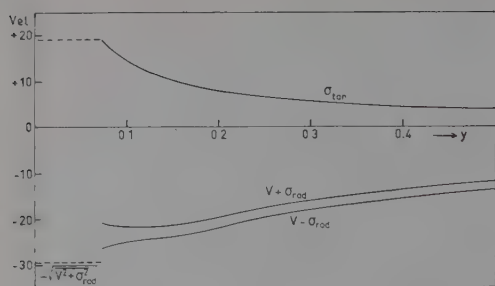


FIG. 4. Theoretical velocity of contraction and velocity dispersions.

the nucleus are very complicated. There is no point in studying these expansions and contractions in detail. It is important to note, however, that their velocities are smaller than the local velocity dispersions. The mass motions in the nucleus are subsonic.

Because of the over-all homogeneity of the nucleus, the over-all pressure is almost constant and the difference between P and Q is small.

When we proceed from the nucleus into the corona, the density drops with a large factor. As we see from Eqs. (2) or (3), this drop is accompanied by a large drop in σ_{rad} . σ_{tan} also decreases, but more slowly.

The mass motion in the corona is a contraction. The velocity of contraction is "supersonic" in the corona, but becomes "subsonic" inside the nuclear region. This is due to the rapid change in σ_{rad} . As we have seen, the "velocity of sound" is equal to $\sigma_{\text{rad}}\sqrt{3}$.

We may compare these results with the data obtained by Mayall (1960) in the Coma cluster. We have not made a projection of the models on the sky, but give results for the radial and tangential motions separately (Fig. 4).

For the nucleus, where the velocities change chaotically from point to point, the average values of $(\langle\sigma_{\text{tan}}^2\rangle)^{\frac{1}{2}}$ and $(\langle\sigma_{\text{rad}}^2\rangle + \langle V^2\rangle)^{\frac{1}{2}}$ are indicated. Outside the nucleus,

we show σ_{tan} and $V \pm \sigma_{\text{rad}}$ as a function of y . All velocities are given in units σ , the velocity dispersion of "field" galaxies, which may be of the order of 50 km/sec.

In the corona, V is negative (contraction). Its absolute value is much larger than $\sigma_{\text{rad}}\sqrt{3}$, hence the mass motions are "supersonic." This is no longer true in the nucleus where in this model $(\langle V^2\rangle)^{\frac{1}{2}}$ and $(\langle\sigma_{\text{rad}}^2\rangle)^{\frac{1}{2}}$ are about equal.

In comparing with the observations, it should be noted that also V contributes to the observed velocity dispersion. In addition it should be taken into account that we observe a mixture of radial and tangential motions, in a ratio which depends on the distance from the center.

It is clear that the theoretical curves show a great similarity to Mayall's diagram of observed velocities in the Coma cluster (Fig. 5).

Attempts to determine the mass of clusters of galaxies have always been based on an application of the virial theorem. Our integrations can now be used to estimate the possible errors in these determinations.

In order to calculate the correction factors, we have assumed that in the application of the virial theorem the space density of the cluster has been evaluated exactly, as can be done in principle. We also have assumed that the cluster was cut off at a given distance from the center, either near the boundary of the nucleus, or well within the corona. We further assumed that the discrimination between cluster members and non-members was perfect. Under these ideal conditions the total observed velocity dispersion would be

$$\sigma_{\text{obs}}^2 = \frac{1}{3}\langle V^2 \rangle + \frac{1}{3}\langle \sigma_{\text{rad}}^2 \rangle + \frac{2}{3}\langle \sigma_{\text{tan}}^2 \rangle. \quad (12)$$

If then the virial theorem is applied in the usual way—the gravitational energy for the mass inside the given boundary is assumed to be evaluated exactly—the mass of the cluster will be overestimated by a factor α , where

$$\alpha = \frac{\frac{1}{k^2(\xi^2 - \epsilon)} \int \frac{x^2}{y'^2} dx + \frac{2}{3} \int \frac{x^4}{y^2} dx + \int \left[2 \frac{k\xi}{k^2(\xi^2 - \epsilon)} y - \frac{\partial y}{k \partial \xi} \right] x^2 dx}{\int \frac{x^5}{y} dx}. \quad (13)$$

There is no point in listing the exact results which we found for α , because these values depend on many circumstances. If the cluster is cut off at the boundary of the nucleus, α is usually larger than unity. It increases when the nucleus contracts. We found values ranging from 1.5 for a very "young" nucleus to about 3 for the most condensed states which were considered. How much larger the value of α might be for still more advanced models is hard to say, but it is difficult to imagine that α could become very large.

If the cluster is cut off somewhere in the corona, the value of α obtained is considerably smaller. For clusters with a very extended corona, α may become smaller than unity. The values which we found ranged from 0.5 to 2.

It is impossible, therefore, to give any general rules about the correction factor which has to be applied to a mass which is determined from the virial theorem. Only detailed comparisons of individual clusters with dynamical models could lead to a determination of this factor.

VI. DIMENSIONS AND TIME SCALE

Up to now we have given a semiquantitative description of the structure of clusters of galaxies as it follows from our models. What is still lacking is a determination of the scale of these formations.

From our transformation formulas it may be derived that the unit of length of our y scale is equal to

$$\Delta = \frac{2\sigma\sqrt{3}}{H} \frac{k\xi}{k^2(\xi^2 - \epsilon)} \quad (14)$$

Here σ is the velocity dispersion for "field" galaxies at the present epoch, and H is the Hubble constant. If, as in paper I we write $\sigma = 50 j$ km/sec, $H = 75 h$ km/sec Mpc, we find

$$\Delta = \frac{2.31 j}{h} \frac{k\xi}{k^2(\xi^2 - \epsilon)} \text{ Mpc.} \quad (15)$$

As an example, we cite our final model of the main series of integrations. For this model, the boundary of the nucleus occurs near $y=0.07$. Since $k\xi=0.53$, $\epsilon=0$, we find for the radius of the nucleus 0.3 Mpc (assuming $j/h=1$). If the distance of the Coma cluster is 89 Mpc and if the radius of the nucleus is $20'$, as determined from Mayall's plot, we find a diameter of 0.5 Mpc. For the corona no definite boundary can be indicated. If we choose the point $x=2.0$, where the density is still 10 times higher than the mean density of the universe, we have $y=0.7$, corresponding to a distance of 3 Mpc. In the Coma cluster, this distance would correspond to about two degrees from the center. Thus the dimensions found for the clusters are at least of the right order of magnitude. The example which we cited was, of course, completely arbitrary.

As to the time scale, we can choose any model from our series for an existing cluster. In doing so, we choose the time scale arbitrarily, but fix the distance scale through the formulas given above. It then becomes a point, however, how long the various stages in the evolution of the cluster have lasted. If, e.g., we choose our final model for an existing cluster, $k\xi=0.53$ corresponds to the present epoch. Since this model applies to a Euclidean universe and because of our choice of H , this corresponds to an "absolute time" of 9×10^9 years. Then the first occurrence of gravitational instability in this region, at $k\xi=6^{\frac{1}{2}}$, occurred at an "absolute time" 9×10^7 years (t is proportional to ξ^{-3}). The first development of the nucleus occurred around $k\xi=0.55$, or about 10^9 years ago.

From these calculations we see that our final model represents a cluster with a still rather "young" nucleus. Nevertheless, the small difference between $k\xi=0.55$ and 0.53 represents a much greater amount of time than one would be inclined to think: The time interval corresponds to a fraction $3 \times 0.02/0.53$ of the present age of the universe. In our final model the galaxies are still streaming in from the corona, and if the numerical

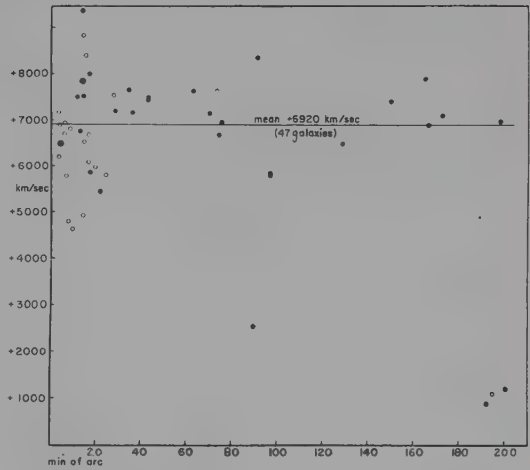


FIG. 5. Mayall's radial velocity measures in Coma.

difficulties can be overcome, more advanced models may certainly be determined.

An interesting question is, whether the cluster will ultimately disintegrate. The increase of the "virial factor" from 1 to 3 in our models indicates that this is very likely to occur; in our final model the nucleus has already acquired positive energy.

VII. SOME FINAL CONSIDERATIONS

In the preceding sections we have referred only to models calculated for a Euclidean universe. We have also calculated some cluster models for a spherical and for a hyperbolic universe, but it would be premature to make a comparison with the other models. There are so many variables in the problem that a discussion of the influence of the universe model will be possible only after many more integrations have been made.

Several serious problems face us when we want to continue this series of integrations and improve on the results. Of these we mention:

(1) The complications arising from "thermal flow." The system of differential equations to be solved will be of the order six instead of two when "thermal flow" is taken into account. Nevertheless, the inclusion of this phenomenon is probably essential.

(2) The great variety in the initial conditions. As we have seen, a mere change in the amplitude of the initial perturbation already changes the solution drastically. We also have to consider the possibility of "nonmonochromatic" initial perturbations and the influence of the model of the universe.

(3) Deviations from spherical symmetry may have serious consequences already for a single cluster, especially in connection with the problem of the formation of subgroups. Another important problem is the mutual influence of disturbances in neighboring regions which

may be connected with the problem of multiple clustering.

(4) The very steep density gradients which occur in part of the integrations do not have a physical meaning, except perhaps in as far as they indicate the possible formation of subgroups. These steep gradients complicate the numerical solution greatly. They may have been enhanced by shortcomings in the technique of the integration process. It will be necessary to find better methods to deal with this situation.

For these reasons, the computations whose results have been presented here can only be regarded as provisional. Nevertheless, the results seem to indicate that clusters of galaxies can be formed by "gravitational amplification" of statistical density fluctuations in an initial homogeneous field of galaxies. The general similarity of the structures which were derived theoretically with actual clusters, the general agreement of the calculated dimensions, the conformity of the pattern of internal motions with the observational results obtained by Mayall for the Coma cluster all favor this explanation. The attempt to continue and to improve these provisional computations therefore seems to be justified.

ACKNOWLEDGMENTS

In conclusion, we wish to express our sincere thanks to the staff of the Mathematical Center at Amsterdam and in particular to its Acting Director, Dr. A. van Wijngaarden, for the very kind and efficient help in carrying out the computations.

REFERENCES

- Mayall, N. U. 1960, *Colloq. intern. centre natl. recherche sci. (Paris)* 95, 40.
 Van Albada, G. B. 1960, *Bull. Astron. Inst. Neth.* 15, 165 (to be cited as paper I).

DISCUSSION

In answer to questions by G. BURBIDGE and PAGE, VAN ALBADA explained that his analysis starts with galaxies condensed sufficiently so that they may be treated as mass points. Tidal forces and any effects of intergalactic gas are ignored. NEYMAN complimented this paper on its interesting theory and pointed out that some of its predictions appear to be consistent with empirical findings presented in the paper by NEYMAN and SCOTT (the Coma cluster taken with outliers appears to be expanding; the Coma cluster taken without outliers appears to be contracting). He asked what the theory predicted for the distribution of numbers of galaxies in a cluster and was told that the theory could not properly account for small groups as independent units because they imply much smaller relative velocities than those assumed (which corre-

spond to a mean of 50 km/sec for a "homogenized" universe).

LEMAITRE asked whether the cosmological constant could be nonzero in this theory and VAN ALBADA said that it had been tried but favored the occurrence of large-scale instabilities.

ABELL proposed that galaxies farther than 100' from the center of the Coma cluster may be field galaxies whose redshifts happen to coincide with the general motion of the cluster. If so, the comparison between the theory and Mayall's plot of observed velocities in Fig. 5 is invalid, but VAN ALBADA felt that the provisional fit within 100' of the center is sufficient corroboration. The elimination of some galaxies at larger distances would not spoil the comparison.

VON HOERNER asked which assumed parameters set the predicted dimensions of a cluster in this theory. VAN ALBADA replied: (1) the mean density—here assumed to be such that space is flat (Euclidean universe); (2) the excess density above the mean—here assumed to be from 2 to 7%; and (3) the "homogenized" velocity distribution (that which is left if matter were distributed uniformly)—here assumed to be 50 km/sec. Changing these assumed values has only a moderate effect on the predicted dimensions of a cluster; a factor of 10 is certainly out of the question. In answer to G. BURBIDGE, he said that the derived time scale for a cluster like the Coma cluster is of the order of the present age of the universe. The Virgo cluster may have started from different initial conditions, evolved faster, and now be disintegrating.

HECKMANN felt that the assumption of sphericity is an oversimplification. He expects that initial fluctuations of a different shape could have a completely different evolution as well as different time scale.

MCCRAE found it remarkable that the initial density should be so uniform after galaxies (which are deviations from uniformity) have condensed. VAN ALBADA related this to the earlier state of the primordial material, which was kept uniform by high radiation density.

MICHIE asked whether the transport of energy had been studied, since the nucleus of a cluster gains energy in time on this theory. VAN ALBADA said that this has not yet been done for technical reasons, but that the effect is likely to be of importance (see paper).

DE VAUCOULEURS raised the question of whether the nucleus of the Coma cluster could be recognized. The disagreement between counts of various observers was ascribed by PAGE and ABELL to different limiting magnitudes and different methods of drawing in the background field beyond 100' from the center. It was agreed that projection would tend to smooth out the nuclear peak of density predicted by van Albada's theory.

On the Possible Influence of a General Rotation on the Expansion of the Universe

O. HECKMANN

Berkeley Astronomical Department and Hamburger Sternwarte

I

WHEN in the late twenties and early thirties of our century the idea of an expanding universe gained more and more support theoretically as well as observationally, there was no unanimity concerning the cosmogonical importance of the singularity shown in the basic equations. Eddington and some of his pupils tried to understand the origin of cosmical condensations as the catalytic factor which caused the unstable static Einstein universe to expand. This hypothetical picture of the early stages of the universe is almost forgotten today. Lemaître very early developed the picture of the primeval atom out of which the expansion started, whereas the large scale condensations—according to him—were formed during that intermediary stage, when the cosmical repulsion of the Λ force and the normal gravitational attractions almost neutralized one another.

Both Eddington and Lemaître needed the Λ term of Einstein's field equations. But this term became unfashionable in the course of time by reasons almost as irrational as reasons of fashion used to be. Excluding the term from the basic equations then seemed to leave the only possibility of considering the singularity, the "big bang," as an inevitable and inescapable event which theorists had to take seriously. One result was the " α - β - γ theory" about the origin of chemical elements. For a while a tendency prevailed to make the ages of rocks and meteorites coincide with "the" age of the universe (as derived from the inverse of Hubble's constant).

The steady-state theory, as a by-product of its fight against relativistic cosmology, brought forward new ideas about the stellar origin of elements which nevertheless could have been combined with relativistic cosmology as well.

At present two main difficulties are felt:

(1) The age of the oldest star clusters, computed along current evolutionary ideas, is considerably larger than "the" age of the universe derived from Hubble's constant. How could they have survived in that narrow "bottle neck" of the universe?

(2) We do not yet have a generally accepted theory connecting the dynamics of galaxies or any stellar systems with the theory of an expanding universe. Yet we have indications (Lifshitz 1946; Bonner 1957; Adams *et al.* 1958) that condensations in an approximately homogeneous universe will find it difficult to increase in the course of expansion on purely gravitational reasons, whereas gravitational "instabilities" could have played an effective role during a stage of a universal contraction. [For a different approach, com-

pare Lemaître (see quotation Adams *et al.*, 1958, p. 1), and van Albada (1960).]

II

In the following paragraphs I am going to look at the "big bang" somewhat more closely. Is it an inescapable consequence of our theoretical ideas? Or is it a special feature only occurring in oversimplified models?

We are going to use classical mechanics throughout the analysis, indicating at the same time corrections which could possibly be caused by relativity theory. Our main result will be that possibly the "big bang" might be avoided and that the narrowness of the "bottle neck" might be much less dangerous than has been thought before, so that condensations, which easily originated gravitationally during a contraction prior to the state of maximum density, might go unaffected through that phase of the universe.

It is usual to describe the matter in the universe as a continuous compressible medium. This description involves far-reaching assumptions which ought to be enumerated carefully. May it suffice here, however, to state that the treatment of cosmological problems as hydrodynamical ones is so general in the literature of the past 30–40 years that we content ourselves with the same basic concepts and laws. If and when we aim at treating cosmology in its proper sense we have to use the basic concepts and laws in such a way as not to exclude the treatment of infinite systems. [See Layzer (1954) and McCrea (1955). These authors doubt the legitimacy of infinite homogeneous Newtonian systems. But it was pointed out by Heckmann and Schücking (1955, 1956) that the difficulties disappear with adequate boundary conditions.]

Let x_i be Cartesian coordinates in Euclidean 3-space, t the Newtonian time. The density of matter $\rho(x_j, t)$, the flow vector $v_i(x_j, t)$, the pressure tensor p_{ik} (as respective moments of the order zero, one, and two in the distribution function of particles which constitute the cosmic medium), obey the hydrodynamic equations (in Euler's form)

$$\rho_{|0} + (\rho v_i)_{|i} = 0, \quad (1)$$

and

$$Dv_i/Dt = v_{i|0} + v_{i|k}v_k = -\Phi_{|i} - (1/\rho)P_{i|k|k} \quad (2)$$

with

$$P_{ik} = p_{ik} + \eta q_{ik}. \quad (2a)$$

Φ will later be identified with the gravitational potential. p_{ik} will degenerate in special cases into $\delta_{ik}p$, where p is an isotropic pressure. q_{ik} is the shear tensor, defined by

$$q_{ik} = \frac{1}{2}(v_{i|k} + v_{k|i}) - \frac{1}{3}\delta_{ik}v_{j|j}; \quad q_{ii} = 0. \quad (2b)$$

η is the coefficient of internal friction. The theory of η is not without complications but need not be considered here. Obviously we write $\rho_{|0}$ instead of $\partial\rho/\partial t$ and $f_{|i}$ instead of $\partial f/\partial x_i$. Einstein's summing convention is used and δ_{ik} is Kronecker's δ .

The equations (1) and (2) do not only show the well-known invariance against Galilei transformations, but a more general one with respect to the group

$$x_i = x'_i + a_i, \quad (3)$$

where the vector components a_i are arbitrary functions of t . This invariance demands that we let Φ be transformed into

$$\Phi' = \Phi + \ddot{a}_k x_k, \quad (4)$$

the dots indicating ordinary derivatives with respect to time. In order to make Φ the gravitational potential of those masses which constitute the material system considered, we have to combine Φ and ρ by Poisson's equation. If we include the so called Λ_0 term, the classical analogue to Einstein's corresponding term, we have

$$\Phi_{|i|k} + \Lambda_0 = 4\pi G\rho; \quad G = \text{gravitational constant.} \quad (5)$$

Λ_0 may be positive, zero, or negative. Its value is unknown, yet must be small in order to leave uninfluenced the representation of gravitational phenomena in our neighborhood. Equation (5) remains invariant under the transformation (3) because of (4). If we want to extend the validity of (1), (2), and (5) to infinite systems we have to abandon the usual boundary conditions $\Phi \rightarrow 0$ if $r \rightarrow \infty$. Instead we have to introduce boundary conditions which—in addition to the validity of (5)—concern only the second derivatives $\Phi_{|i|k}$ of Φ . Consequently, we allow Φ to contain an arbitrary linear term in x_j . This corresponds exactly to the transformation of the potential given by (4), where such a linear term is produced by (3). The linear term represents a homogeneous gravitational field and is unobservable by any experimental means because in it all bodies are falling with the same acceleration. Now (1), (2), and (5) together with boundary conditions at infinity for $\Phi_{|i|k}$ provide the apparatus which allows the description of finite and infinite systems.

Another form of (1), (2), and (5) will be useful in the following: Let us decompose the derivatives of the flow vector into the three parts, shear, rotation, and expansion:

$$v_{|i|k} = q_{ik} + w_{ik} + \frac{1}{3}v_{j|j}\delta_{ik}, \quad (6)$$

q_{ik} is defined by (2b), and

$$w_{ik} = \frac{1}{2}(v_{i|k} - v_{k|i}) = -w_{ki} \quad (6a)$$

are the components of the angular velocity. Without loss of generality we can define an always positive scalar function $R(x_j, t)$ by

$$v_{j|j} = 3 \frac{1}{R} \frac{DR}{Dt}, \quad (7)$$

where D/Dt represents here and in all following cases the substantial derivative

$$DR/Dt = R_{|0} + R_{i|j}v_i. \quad (8)$$

This brings the equation (1) of continuity into the form

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \frac{3}{R} \frac{DR}{Dt} = 0, \quad (9)$$

the integration of which gives

$$(4\pi/3)\rho R^3 = \mathfrak{M} = \text{const} > 0. \quad (10)$$

The interpretation is (independent of pressure, shear, rotation, and potential) that the mass content remains constant in each volume the surface of which is comoving with the flow v_i .

If next we make use of the fact that the gradient $\Phi_{|i|}$ has a vanishing rotation, $\Phi_{|i|k} = \Phi_{|k|i}$, we get after some lengthy algebra

$$Ds_i/Dt = q_{ik}s_k + \frac{1}{2}R^2r_{[j|k]}\epsilon_{ijk}. \quad (11)$$

Here we have introduced the completely skew symmetric pseudotensor ϵ_{ijk} whose nonvanishing components are

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = -\epsilon_{213} = -\epsilon_{132} = -\epsilon_{321} = 1. \quad (11a)$$

Further we have put

$$s_i = R^2 \epsilon_{ijk} w_{jk}, \quad (11b)$$

$$r_{[j|k]} = r_{j,k} - r_{k,j}; \quad r_i = (1/\rho)P_{ij|j}. \quad (11c)$$

r_i contains all terms describing pressure and density gradients as well as the influence of friction. If the pressure is isotropic, and if there exists an equation of state $p = f(\rho)$, the friction term in r_i is the only remaining one.

The last fundamental equation is gained simply by inserting (2) into (5). After a few reductions we get

$$\frac{3}{R} \frac{D^2 R}{Dt^2} = \Lambda_0 - 3G \frac{\mathfrak{M}}{R^3} - q_{ik}q_{ik} + \frac{2s_i s_i}{R^4} + r_{i|i}. \quad (12)$$

The set (10), (11), and (12) is completely equivalent to (1), (2), and (5). Equations (11) express the vortex theorems of Helmholtz. Equation (12) shows how the temporal expansion of a comoving element is influenced by the cosmic Λ_0 force, ordinary gravitation, shear, rotation, gradients of density and pressure, and friction.

III

Theoretical cosmology is a very rudimentary form of hydrodynamics. Its basic idea is that cosmic phenomena are more and more simplified if we take averages over larger and larger volume elements.

Empirically there is some (necessarily inexact) evidence that:

(1) The density of the observable part of the universe can be considered as roughly homogeneous. Though the stochastic fluctuations are large, it seems to be sufficient to consider them as limited in volume and amount [see Neyman and Scott (1959) and Zwicky (1958)].

(2) There is no dependence of the redshift on spatial directions. Until now it seems to be sufficient to consider the redshift as isotropic. [Apparent deviations from isotropy are discussed by de Vaucouleurs (1958).] If we want to express these statements with the help of our hydrodynamic apparatus we have to average Eqs. (10), (11), and (12) over sufficiently large volume elements in order to make the density homogeneous and the flow isotropic. About the random motions, expressed by $\dot{p}_{ik}$, we know very little; but we have no empirical evidence that they depend considerably on the space coordinates.

Consequently the interpretation of (10), (11), and (12) as describing the behavior of cosmic matter averaged over sufficiently large volumes allows us to state: (a) Eq. (10) will remain unchanged, but R is a function of t only; (b) all terms depending on q_{ik} have to be neglected, so that (11) is simplified to $s_i = \alpha_i = \text{const}$, and (12) becomes with $\alpha_i \alpha_i = K^2$,

$$\frac{\ddot{R}}{R} = \Lambda_0 - 3G \frac{\mathfrak{M}}{R^3} + \frac{2K^2}{R^4}.$$

A rotation therefore expressed by $K \neq 0$ is the remaining feature besides an isotropic expansion or contraction. From a kinematic standpoint, without the use of (10), (11), (12), it would have been sufficient to assume that the q_{ik} are zero in order to prove that R is a function of time only.

The last equation can immediately be integrated and gives

$$\frac{1}{2} \dot{R}^2 = \frac{\Lambda_0}{6} R^2 + \frac{G\mathfrak{M}}{R} - \frac{1}{3} \frac{K^2}{R^2} + h; \quad h = \text{const} \geq 0. \quad (13)$$

It contains the law of conservation of energy, stating how kinetic and potential energy of matter in a co-moving (=coexpanding) volume are counterbalanced by the energy of the Λ_0 force and of rotation. The most remarkable feature in our connection is the rotation term: Eq. (13) has no singularity for $R \rightarrow 0$; the left side is always positive; the rotation term is always negative. Its value is limited by that value of R which makes the right-hand side of (13) equal to zero. This minimum value of R describes the maximum contraction of the universe. [The importance of a universal rotation was first indicated by Heckmann and Schücking (1956).]

Equation (10) retains its form exactly in Einstein's theory. If one neglects the term with r_i , the same is true for (11) and (12). If in Einstein's theory we specialize for homogeneity and let the rotation term vanish, the corresponding anisotropic models all have more or less complicated singularities [compare Heckmann and

Schücking (see quotation Adams *et al.*, 1958, p. 149)]. But it is not possible in Einstein's theory to make the shear terms vanish independently of the rotation terms [see Schücking (1957)]. This is the reason why in relativistic mechanics the influence of the rotational term on the singularities of (12) is not yet understood. The following considerations suffer, therefore, from a lack of consistency. We shall have to use a few relativistic concepts (e.g., the notion of the gravitational energy of isotropic radiation). But we have no exact knowledge about the mutual relativistic influence of shear and rotation. Instead we shall use (12) in its classical context.

IV

We are going to estimate the possible amount of a universal rotation in two different ways: The first one starts by comparing the term $\frac{1}{3}(K^2/R^2)$ with the gravitational influence of isotropic radiation. The density ρ_R and the pressure p_R of this radiation fulfill the equation of state

$$\rho_R = 3p_R. \quad (14)$$

Combining this with the (relativistic!) equation of continuity in a homogeneous substratum

$$\dot{\rho} + 3(\dot{R}/R)(\rho + p) = 0, \quad (15)$$

ρ and p being density and pressure of any gravitating substance (matter and/or field), we get

$$\rho_R R^4 = \text{const} > 0. \quad (16)$$

Consequently the gravitating mass $\mathfrak{M}$ may be considered as composed of matter (with density ρ_M) and radiation (with density ρ_R):

$$\mathfrak{M} = (4\pi/3)R^3(\rho_M + \rho_R). \quad (17)$$

The second term on the right-hand side of (13) then becomes

$$G\mathfrak{M}/R = (4\pi G/3)R^2(\rho_M + \rho_R). \quad (18)$$

The part

$$R^2 \rho_R = \text{const}/R^2 \quad (19)$$

will now permanently compete with the rotation term $-\frac{1}{3}(K^2/R^2)$: Both contain the same power -2 of R and both are the only terms which remain important in case of small R , when compared to all other terms in (13). In order to avoid the singularity, it is necessary to postulate that permanently the inequality

$$(4\pi G/3)\rho_R < \frac{1}{3}(K^2/R^4) = \frac{1}{3}\omega^2 \quad (20)$$

is valid. Here we have designated with ω^2 the square $w_{ij}w_{ij}$ of the angular velocity. Indicating the present moment of the universe with index 1 we have

$$\omega_1^2 > 4\pi G \rho_{R1}. \quad (21)$$

Let us choose $\rho_{R1} = 10^{-33} \text{ g cm}^{-3}$, following Sandage (1961). Equation (21) gives in seconds of arc per century

$$\omega_1 > 2 \times 10^{-5}. \quad (22)$$

This estimate is extremely rough. If its physical foundation is at all correct it says that (21) [or (22)] makes the "big bang" as a singularity impossible. It does not tell us anything about the maximum amount of energy density at the minimum of R .

The second estimate can dispense with relativity. It considers the possibility that the amount of angular momentum could be appreciably larger than indicated by (21). In this case it could be possible that the minimum R_0 of $R(t)$ is reached at such a state of compression that the density of matter and the rotation compensate one another. If R_0 is essentially smaller than R_1 , the present value of R , the inequality

$$\frac{1}{3}(K^2/R^4) > (4\pi G/3)\rho_M \quad (23)$$

should always be valid for $R > R_0$; $\rho_M < \rho_{M0}$; for the minimum $R = R_0$ and the maximum $\rho_M = \rho_{M0}$, however, we have the equality

$$\frac{1}{3}(K^2/R_0^4) = (4\pi G/3)\rho_{M0}. \quad (24)$$

Because of

$$K^2 = \omega_1^2 R_1^4; \quad \rho_{M0} = (R_1^3/R_0^3)\rho_{M1} \quad (25)$$

(in the case of vanishing material pressure, which we assume) we get from (23)

$$\omega_1^2 > 4\pi G(R_0/R_1)\rho_{M1}. \quad (26)$$

This should be understood in the sense that, with given present material density ρ_{M1} , and a given "bottle neck" R_0/R_1 , the possible rotation has to obey (26).

If we assume $\rho_{M1} = 10^{-31}$ g cm⁻³ (Sandage 1961) and $R_0/R_1 = 1/10$, we get

$$\omega_1 > 6 \times 10^{-5}$$

seconds of arc per century. The value of R_0/R_1 is completely arbitrary. But it is sufficiently harmless, leaving the maximum density of the universe only 10^3 times larger than its present value. It is easy for stars or star clusters to pass the minimum, and even single galaxies need scarcely be damaged. Only clusters of galaxies could probably be influenced to a larger extent.

Both numbers $\rho_{R1} = 10^{-33}$ and $\rho_{M1} = 10^{-31}$ may have an uncertainty of a factor 100 or more. Probably both are lower limits. But ω_1 increases only as the square root of the uncertainty. There seems to be no hope of making ω_1 observable by any method of classical astronomy. In principle, a comparison of the constant of precession as determined relative to the extragalactic nebulae with the same constant as determined relative to the invariable directions of the planetary system would give directly the absolute rotation of the whole system of extragalactic nebulae. But the present uncertainty in our knowledge of the precession is 10^4 – 10^6 times too large.

Our present cosmological theories, whether in their classical or in their relativistic form, apparently do not favor a picture of the temporal development of the irregularities in the universe starting with an homo-

geneous state out of which irregularities are slowly formed by gravitational instabilities. On the contrary, states of higher average density are states of higher irregularities; contraction favors temporal concentration, expansion favors temporal dispersion. But there seems to be a chance that a certain amount of angular momentum evenly distributed over the whole cosmos could prevent a state of arbitrarily high density. The amount necessary seems to be astonishingly small. There seems to be not necessarily a contradiction between an age of a star cluster which comes out larger than "the" age of the universe as derived from Hubble's constant. At any rate the present arguments need a careful investigation in the frame of relativistic cosmology which, however, until now seems to be very complicated. Any theoretical considerations of the origin of double and multiple galaxies depend on the result of such an investigation. If singularities of the type of the "big bang" exist always, the theory of all cosmic forms has to take this fact into account. If the possibility and probability of nonsingular world models should be established, the theory of larger cosmic forms could possibly remain uninfluenced by the state of maximum density of the universe.

REFERENCES

- Adams, J. B., Mjolsness, R., and Wheeler, J. A. 1958, *La Structure et l'évolution de l'univers*, Solvay Conference (R. Stoops, Brussels, Belgium), p. 113.
 Bonner, W. 1957, *Monthly Notices Roy. Astron. Soc.* **117**, 104.
 de Vaucouleurs, G. 1958, *Astron. J.* **63**, 253.
 Heckmann, O. and Schücking, E. 1955, *Z. Astrophys.* **38**, 95; 1956, *ibid.* **40**, 81.
 Layzer, D. 1954, *Astron. J.* **59**, 268.
 Lifshitz, E. 1946, *J. Phys. U.S.S.R.* **10**, 116.
 McCrea, W. H. 1955, *Math. Gazette* **39**, No. 330; 1955, *Nature* **175**, 466.
 Neyman, J. and Scott, E. 1959, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin), Vol. 53.
 Sandage, A. 1961, *Astrophys. J.* **133**, 355.
 Schücking, E. 1957, *Naturwissenschaften*, p. 507.
 van Albada, G. B. 1960, *Bull. Astron. Inst. Neth.* **15**, 165.
 Zwicky, F. 1959, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin), Vol. 53.

DISCUSSION

A series of questions were concerned with the problem of replacing the discontinuity (infinite density) with a mild compression in Heckmann's cosmological model. WEYMANN and PAGE stressed the point that irreversible processes such as the conversion of hydrogen to helium and the formation of white dwarfs would then have no limit in the past, as provided in other evolutionary models. LEMAITRE, while agreeing that a mathematical discontinuity of infinite initial velocity is too extreme, emphasized other advantages of his "primeval atom" of very high density, from which cosmic rays may be explained. VON HOERNER and SANDAGE were concerned that the reciprocal Hubble constant, or "age of the universe," would on this theory be within a factor of 2 of separately observed ages of clusters entirely by chance.

HECKMANN explained that his compression ratio of

1000 (maximum density relative to present mean density) was chosen quite arbitrarily; it could be much larger—so that star clusters, at any rate, would be reorganized—but he felt that the approximate coincidence of $1/H$ with ages of clusters need not be particularly significant. He saw no reason to suppose that stars, clusters, and galaxies were not in existence and in the process of formation before the maximum density state in his model, but pleaded ignorance of the consequences. He does not claim that this model explains the thermodynamical history of the universe, and feels that many of the theoretical and speculative concepts must be carefully re-examined. His paper shows that it is “incorrect to conclude from the Hubble law that there must have been a ‘big bang’ in the strict sense.” In answer to GODART he explained that no friction is involved as the shear is zero.

JUST reported that, without rotation, he has found it impossible to avoid the singularity of infinite density under any reasonable assumptions of density and pressure. SCHUECKING added that a negative value of the deceleration parameter $\ddot{R}R/\dot{R}^2$ excludes the steady-state theory and leads to a singularity in the Friedmann models for any arbitrary value of the cosmological constant. With Sandage’s value of the Hubble constant we obtain an age of the universe less than 13×10^9 years. The age of halo globular clusters on the other hand is given by Sandage as 24×10^9 years. If these results can be confirmed, the following alternatives arise: (1)

breakdown of Einstein’s theory of gravitation or (2) introduction of a general rotation of the universe.

In answer to SANDAGE, HECKMANN defined absolute rotation of the universe in terms of observations on a Foucault pendulum. MICHIE added that it is also possible in principle to detect it from observations on light, and HECKMANN explained once more that the constant of precession would be different relative to planetary orbits (defining an inertial reference frame) than relative to extragalactic nebulae (defining a kinematical reference frame). He pointed out that Mach’s principle, denying this distinction, has been brought into doubt by field theory which allows measurement of motion relative to a field as well as relative to any matter in the universe. In answer to PAGE, he made clear that his provisional value of the present rotation ($6''$ in 10^7 years) is beyond detection by any known method. Any effect on the isotropy of density of field galaxies or clusters of galaxies, as suggested by G. BURBIDGE is also too small to detect. Anisotropy in the expansion (different Hubble constant in different directions) would indicate the *shear* that must accompany rotation, but would not prove rotation of the universe since shear can exist without rotation. MCCREA claimed that universal rotation introduces difficulties with boundary conditions in relativistic cosmology that shear can eliminate. He also expected rotational effects to depend upon a cylindrical coordinate, but HECKMANN denied both statements.

Exchange of Galaxies between Clusters and Field

G. LEMAÎTRE
Louvain, Belgium

I HAVE been led to a theory which involves a continuous exchange of galaxies between clusters and the general field of galaxies in which they are imbedded. This theory arose from a study of the instability of the Einstein universe. Such a universe is in equilibrium between gravitational attraction and the cosmic repulsion corresponding to a positive cosmological constant λ in the field equations of general relativity.

My model of the universe is not supposed to have started from the state of equilibrium (as was considered, for a time, by Eddington); it is assumed, rather, to have followed the well-known Friedman equation in which the radius of space started from zero, remained for a period of some 10^{11} years in a state of equilibrium between gravitation and cosmic repulsion, and finally resumed expansion at an accelerated pace as a consequence of the cosmic repulsion. It is realized that this

model, proposed some 30 years ago, may be considered an “old fashioned cosmology.” This is not the place to discuss the strong prejudices that have arisen against it due to reverence of an authority whose influence can only be compared to that of Aristotle in older times.

My cosmology can only be properly discussed when it is coupled with the *primaeva* atom hypothesis involving a beginning of the world, not only as a beginning of space, but also as a beginning of all multiplicity. It thus suppresses metaphysical questions about this beginning—the bottom of space-time and of all physics for which no origin can be conceived.

I also leave aside here any discussion of the first stage of evolution and of the products of disintegration of the *primaeva* atom which may be identified with the cosmic rays. First there must have been formed individual clouds of gas, similar to those which are now observed as

halos of the galaxies. I suppose that such was the state of matter when the universe was hesitating in the state of unstable equilibrium.

For the present discussion, we may confine our attention to a region of space of a size comparable to that part of the universe now accessible to astronomical observation. In such a region, and if fine details are ignored, it is permissible to use the Newtonian approximation of the relativistic field equations and Euclidean geometry. Newtonian mechanics may be safely used with a simple modification that describes the change introduced, even at this level of approximation, by the positive cosmological constant Λ . If ρ_0 is the density in an Einstein universe for the assumed value of Λ and ρ is the real density, the Poisson equation must be modified to read

$$\Delta\Omega = 4\pi G(\rho - \rho_0). \quad (1)$$

One of the effects of the fluctuations of density from place to place is to divide space into attractive regions and repellent regions. The attractive regions will exert a selection effect on the gaseous clouds that fill space and have a distribution of velocities up to a high limit. The clouds that have a velocity not too different from that of an attractive region will remain in or near it and form an assembly from which a galaxy later evolves. The important point is that these protogalaxies do not arise, as in other theories, from some effect due to numerous collisions, but only from a selection effect. Collisions would, of course, reduce the velocities, while a selection produces protogalaxies with the same velocities as the attractive regions. These velocities arise from fluctuations of density in the swarm of the swiftly moving clouds and may therefore be expected to be fairly large.

This brief discussion of the background of the theory does not intend to carry conviction or to replace more detailed expositions given elsewhere. It is probably sufficient to make clear the hypotheses involved in the present discussion of the instability of an Einstein universe filled with particles (the protogalaxies) moving at high relative velocities.

So much for the physics of the problem. In order to get a mathematical solution, we now introduce simplifications of a mathematical nature. These simplifications must define some mathematical model that retains the essential features of the physical problem and at the same time allows an exact solution. Some of these are quite classical; they are essentially the same as those introduced as early as 1915 by Eddington in his classical paper on the "Dynamics of a globular stellar system." The protogalaxies are supposed to move in a gravitational field without interfering with one another, like the particles of a Gibbs ensemble. The gravitational field itself is defined from the density of this ensemble by the modified Poisson equation (1).

In such a case the Liouville theorem applies and asserts that, if the distribution of the particles is given by

a distribution function f , the expression $f dx dy dz du dv dw$ is an invariant differential form, so that its value remains constant as the points move along their trajectories. A great simplification arises in the application of this theorem when, as we shall suppose, the function f is a *step function* that allows only two values: one and zero.

In order to avoid mathematical complications, it is well to assume that f is not strictly discontinuous but that it varies quickly from one to zero in a small domain. Then all the intermediate values are carried along the trajectories, and in the limiting case of a discontinuity itself is carried along the trajectories. In fact, these "limiting trajectories" are the only ones necessary to consider. The other, internal-value trajectories (which specify the way in which interior points are exchanged during the motion) have no relevancy because the value of f , the only thing we are interested in, is precisely the same on the trajectories so exchanged.

The density in space ρ is such that the population of the assembly is simply $\rho dx dy dz$, and it follows that, up to a numerical factor, the space density is given by

$$\rho = \iiint du dv dw, \quad (2)$$

where the integral is extended to the whole domain of existing particles at the point considered and up to the velocities u, v, w of the limiting trajectories.

This special aspect of the Liouville theorem is well known from the theory of the effect of the earth's magnetic field on cosmic rays. Of course, the cosmic-ray case does not involve an assumption—it describes the actual physical situation—while in this theory a definite assumption is used that reduces the generality of the investigation with the hope of obtaining a more definite solution.

Furthermore, we shall also assume spherical symmetry, as a consequence of which the force is central and the angular momentum k is a further invariant for each trajectory. It follows that the transverse velocity is given by

$$\Pi = k/r. \quad (3)$$

We can assume that for each value of Π or k there are only two limiting trajectories P' and P'' that are functions of the distance r to the center of symmetry, the time t , and k . These variables may be replaced with some advantage by two others, both functions of r , t and k :

$$V = \frac{1}{2}(P' + P'') \quad (4)$$

and W such that

$$\Pi^2 + \left[\frac{1}{2}(P' - P'')\right]^2 = W^2. \quad (5)$$

It is then possible to write the fundamental equations of the problem in terms of V and W . For the particular case of Einstein's universe, and even for the Friedman case of uniform disruption of equilibrium by expansion

of the universe without any local condensation, we obtain $W^{\frac{1}{2}}$ depending only on t which is, of course, proportional to $1/R$, R being the radius of the universe occurring in the Friedman equation, and $V = r dR/R dt$ (Hubble's law).

There exist stationary solutions of the equations—solutions which do not depend on time—describing spherical condensations in the Einstein universe. We say “stationary” and not static to emphasize the fact that the situation is not strictly static because the protogalaxies are continually moving through space. They accelerate toward the center of the cluster and pass there with greater velocity and, therefore, with greater density. Their distribution remains constant, however, as in stationary waves, where the amplitude of the wave remains constant in spite of individual velocities in the medium that carries the waves.

The suggestion is made that such standing waves arose while the universe was approximately in the Einstein equilibrium, and have been the cause or one of the causes responsible for the clusters of galaxies. In order to test this hypothesis it will be necessary to investigate how such stationary waves might arise and develop during the equilibrium phase, and how they will be affected by the subsequent expansion. These aspects of the problem are very far from being solved. I wish to report some preliminary results and also to discuss the methods by which further progress might be obtained.

A great number of stationary solutions can be obtained. The elliptical distributions of Eddington (1915) are easily adapted to our particular application of the Liouville theorem; there is no difficulty in including the cosmological term, which can be neglected when the densities are large. (Of course, one cannot go back, as a check, from our general theory to the elliptical distributions of Eddington.)

When the shape of the velocity distribution is constrained to remain roughly spherical, the central condensation of the cluster remains moderate. On the other hand, if the motion is supposed to be exclusively radial, interesting models are obtained with infinite central density, building up a kind of nucleus. These models may be of use in the theory of galactic structure, because the predicted law of density variation is very similar to that obtained by Hubble in 1930 for elliptical galaxies. But they do not seem to be applicable to the present problem of the clusters.

If there were some way to make a choice between the many stationary solutions, it would not be too difficult to work out the effect of the expansion of the universe on any one of them. We shall return later on to this question.

It is first necessary to study how the clustering could arise in the Einstein equilibrium. Some insight into that question can be obtained by studying solutions which differ from the conditions of equilibrium by infinitesimal quantities. This has been done, and it turns out that solutions may be defined formally by power series in

two variables, r and k , the time entering only in an exponential factor $e^{\theta t}$. Solutions may be computed which correspond to each value of θ . In fact, it seems that the general case can be analyzed as a sum of such elementary solutions. If fine details of the initial conditions must be represented, it would be necessary to include imaginary values of θ .

But even if we consider only one particular value of θ , the essential point from a mathematical point of view is that the solution can be defined only if we introduce convenient boundary conditions. It is not sufficient to consider one cluster in infinite space; this does not correspond to the actual case. A cluster does not stand alone; it is surrounded by other clusters; and even if we imagine some indefinite region between the clusters which may be called the field, this field cannot be considered to be an open region extending indefinitely.

To deal with this boundary condition in an exact manner would lead to abandoning spherical symmetry; it would require information on the distribution of the surrounding clusters which is unknown and probably irrelevant for the general features of the theory. But notwithstanding the fact that a boundary condition must be highly idealized to allow a mathematical solution, it must be introduced in a perfectly defined way if we want to obtain a definite solution. The actual solution would finally depend on how realistic a boundary condition has been defined.

The condition to be used is suggested by similar processes in physics—in the theory of molecules, for instance. We must consider a sphere, generally of variable radius, which defines the domain of the condensation under consideration. The surrounding clusters are supposed to be averaged in order to keep the necessary spherical symmetry. Then two conditions are imposed; first, that the boundary sphere moves under the general gravitational force, and second that the number of protogalaxies going out of the boundary would be precisely equal to the number entering it. Mathematically it is convenient to think of the protogalaxies that enter as exchanging their identity with those that leave, and thus to speak of the boundary condition as a “rebounding condition.” This condition must be fulfilled separately for each value of the angular momentum k .

The difficulty is to find the precise value of the radius of the rebounding sphere, which comes out in this problem as a kind of characteristic value allowing the solution of a system of linear equations. Its value is set with precision in order to get a solution. Nevertheless, due to the approximations involved, the precise value is only significant to a general order of magnitude.

Units have been introduced in such a way that $\theta = 1$ corresponds to uniform expansion of the universe—that is, radius of rebound infinite. The time unit has been taken as $\frac{1}{3} T_H$, where T_H is the inverse Hubble constant. The unit of length depends on the value of the velocity $W^{\frac{1}{2}}$ in the Einstein universe; it is the distance traveled with this velocity during unit time.

With this choice of the units, and for $\theta=0.5$ (the case that has been chiefly studied, and the only one hitherto published), the radius of rebound is roughly four. It is difficult to say whether this is to be compared with the size of the clusters or with half the distance between clusters. In any case, it is of the expected order of magnitude, possibly somewhat too large. It increases with the value of θ , and when θ approaches the limiting value, one, the investigation presents special difficulties. This may be of interest in the study of clustering or anti-clustering on a large scale.

Further research will be concentrated on some definite, moderate value of θ . We shall compute about 100 individual trajectories, building up at each stage the corresponding volume defined by these trajectories. There are a number of problems of interpolation to be solved on which only preliminary results have been obtained.

It is expected that it will be possible to follow up the development of such typical condensations while the radius of the universe is still unchanging. It is hoped that this will lead to a stationary solution around which oscillations take place. Then we shall endeavor to suppress arbitrarily these oscillations which physically must be expected to dissipate by some process which has been excluded by the constraints defined above for mathematical simplicity. After this simplification, or some alternative, the radius of the boundary sphere will be allowed to vary according to Friedman's equation, and it should be possible to compute the effect of this expansion on the cluster. If the cluster happens to dissipate in a more or less uniform way, the theory would have no application to real clusters. However, if the cluster dissipates chiefly from the edge, keeping a fairly large central density, it will diminish in size without fading out at the center, and will provide a possible explanation of observed clusters.

Such a theory would explain the positive energy which is found in large clusters, not as a dissipation of the cluster, but as an exchange of the individual members of the cluster with those of the surrounding clusters or those of the general field.

DISCUSSION

In answer to AMBARTSUMIAN, LEMAITRE stated that there are only two adjustable parameters in the theory:

the cosmological constant and the velocity dispersion relative to local rest in the Einstein universe. He admitted that the real world probably requires more than two parameters to represent its many complexities, but wanted to keep this mathematical model as simple as possible. He agreed further with AMBARTSUMIAN that the assumption of a simple expression for the phase density f is not essential, but was made to simplify the mathematics. It should not affect the conclusions significantly.

AMBARTSUMIAN pointed out that the proposed interchange between clusters and field does not allow the ratios between different types of galaxies to be different in the clusters than in the fields. More specifically, as agreed by LEMAITRE in an exchange with SPINRAD, the theory predicts that the ratio of ellipticals to spirals is the same in clusters as in the field—contrary to observation. However, in answer to DE VAUCOULEURS, he pointed out that the velocity dispersion is predicted to be higher in clusters than in the field in proportion to $(\rho-\rho_0)^{\frac{1}{2}}$.

MCCREA argued that the cosmological constant is simply an arbitrary constant in Einstein's field equation analogous to the absolute value of a stress and therefore trivial, since it is the *gradient* of a stress that has observable significance. He asked why this quantity, so trivial in origin, should have fundamental significance in this theory of clusters. LEMAITRE replied that the cosmological constant allows the condition of unstable equilibrium in which density fluctuations can be amplified to lead to the formation of clusters. He considers this unstable equilibrium the most important "tool" for cosmology remaining from the Einstein static universe.

VAN ALBADA asked whether very large clusters can have a place in this theory. LEMAITRE answered that further investigation is necessary, perhaps using new initial conditions such as nonspherical symmetry. In fact, he expects from this approach generally that there should be large scale inhomogeneities in the universe. VAN ALBADA then proposed that the occurrence or absence of large-scale clustering might be used to argue for or against a positive cosmological constant, but LEMAITRE and HECKMANN felt that other models with zero cosmological constant might also lead to large-scale clustering.

Evidence Regarding Second-Order Clustering of Galaxies and Interactions between Clusters of Galaxies

GEORGE O. ABELL

Department of Astronomy, University of California, Los Angeles, California

The evidence for the existence of second-order clusters is reviewed. A statistical analysis of the distribution of the rich clusters identified on the Palomar Sky Survey, the observed extent of 17 typical apparent groupings of galaxies, and de Vaucouleurs' study of the Local Supercluster all indicate a mean linear diameter of the order of 50×10^6 pc for second-order clusters. Typical masses for these systems are estimated at from 10^{16} to 10^{17} solar masses. If the systems are gravitationally stable, they should have internal root-mean-square velocity dispersions in the range $1-3 \times 10^8$ km/sec. The possible influence of second-order clusters upon the observations relevant to the stability of systems of galaxies is discussed.

RECENT investigations of the masses of groups and clusters of galaxies have indicated values for the mass-light ratios of galaxies in those systems that are several times higher than the ratios for galaxies of the corresponding types whose masses have been determined from their rotations or internal motions, or for members of double galaxy systems (e.g., Burbidge and Burbidge 1959; de Vaucouleurs 1960; Noonan 1961; Burbidge and Burbidge 1961). The improbably high mass-light ratios found for many systems of galaxies provide the principal evidence in support of the hypothesis that those systems are unstable. The only other supporting evidence is the weak and rather inconclusive correlation between the magnitudes and velocities of galaxies in several clusters (Neyman and Scott 1961). On the other hand, if groups and clusters are actually expanding they must be very young. A conclusion of such profound importance should be accepted only after all possible effects that might influence the interpretation of the observations have been thoroughly considered and eliminated. It is the purpose of this paper to investigate the evidence for second-order clustering of galaxies, and how superclusters, if they exist, can be expected to affect the relevant observations.

I. DISTRIBUTION OF RICH CLUSTERS

Striking evidence for the existence of second-order clusters is provided by the distribution of the rich clusters of galaxies identified on the National Geographic Society-Palomar Observatory Sky Survey (Abell 1958). Figure 1 shows the surface distribution of a homogeneous statistical sample of 1682 clusters chosen from those in the writer's catalogue. The redshift of each cluster has been estimated by comparing its tenth-brightest member to a step scale of galaxian images that had previously been calibrated against the tenth-brightest members of clusters of known redshift. On the basis of their estimated redshifts the clusters were divided into six distance groups or classes; the type of symbol that represents each cluster plotted in Fig. 1 refers to the distance class of that cluster. Table I gives the mean value of $z (=d\lambda/\lambda)$ for the clusters of each class.

A superficial examination of Fig. 1 shows that the clusters do not appear to be distributed at random over the sky. Rather, there appear to be clumps or groups of clusters; particularly conspicuous, for example, are two concentrations of clusters near $l^I = 8^\circ$, $b^I = -66^\circ$ and $l^I = 11^\circ$, $b^I = -70^\circ$ (old system of galactic coordinates). To test the distribution of the clusters statistically, they were counted in cells formed by a rectangular grid superimposed over Fig. 1. Each grid cell corresponded to an area of 13.2 square degrees in the sky. The frequency distribution $N(t)$ of cells containing t clusters each was compared to the Poisson distribution

$$P(t) = e^{-m} m^t / t!, \quad (1)$$

which would be expected for a random distribution of noninteracting objects. Here the mean number m of clusters per cell was computed from the sample. The probability that the observed distribution is a random sampling from a Poisson distribution, computed with a χ^2 test, was found to be 10^{-61} .

If the area of the grid cells is made very small, so that a cell can contain at most one cluster, $N(t)$ becomes indistinguishable from $P(t)$. The observed distribution may also be expected to approach a random one if the grid cells are made so large that the irregularities in the cluster distribution are smoothed out. $N(t)$ will show the greatest departure from $P(t)$ for cells whose area most nearly corresponds to the scale of the "clumpiness" of the observed distribution. To establish whether a unique scale of the clumpiness exists, the frequency distribution $N(t)$ was determined as a function of cell size separately for the clusters within each distance class (the nearest four distance groups were lumped together to obtain a significant sample size). In each case the

TABLE I. Mean redshifts of clusters in each distance group.

Distance group	Mean $z = d\lambda/\lambda$
1	0.027
2	0.038
3	0.067
4	0.090
5	0.140
6	0.180

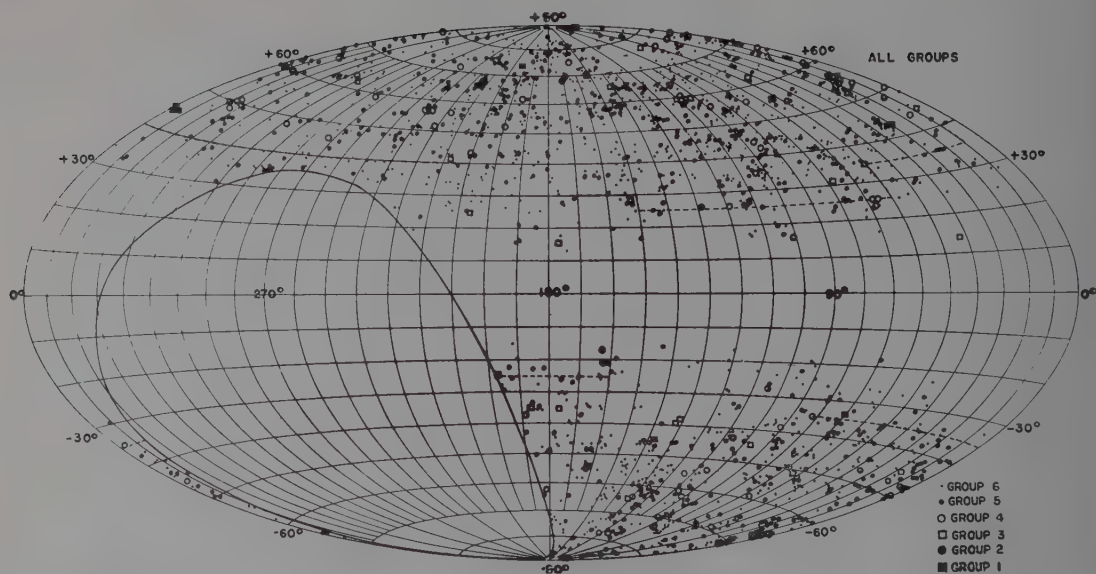


FIG. 1. The distribution in galactic coordinates (old system) of the clusters in Abell's catalogue. Those clusters closer to the galactic equator than the dotted line are not part of the homogeneous statistical sample. The plot is on an Aitoff equal-area projection.

observed and Poisson distributions were compared with a χ^2 test. The computed probability that $N(t)$ is a chance sampling from a population of randomly distributed noninteracting objects is taken as a measure of the departure of the cluster distribution from randomness. The results are exhibited in Fig. 2. The ordinates are the logarithms of the probabilities that the observed frequency distributions are Poisson, plotted in the sense that larger ordinates correspond to greater deviations from randomness. It is seen that for clusters

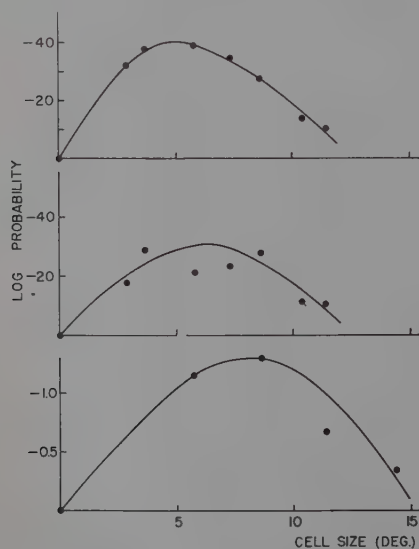


FIG. 2. Probabilities that the observed frequency distributions of clusters among cells of various sizes would be obtained in random samplings from populations distributed with a Poisson law.

in each distance group there is a cell size for which there is a maximum departure from randomness, and that this cell size is smaller (in angular units) for larger distances. The reciprocal of the cell diameter, in deg^{-1} , for which the frequency distribution in each distance group departs most from a random one is plotted against the mean estimated redshift of the clusters within that group in Fig. 3. The observed approximate proportionality suggests that the clumpiness in the cluster distribution has about the same linear scale for clusters of all distances; for a value of the Hubble constant H of $75 \text{ km/sec} \times 10^6 \text{ pc}$ (a value which will be assumed throughout in this paper), that linear size is $58 \times 10^6 \text{ pc}$.

Zwicky (e.g., 1957b) has suggested that the apparent nonrandomness of the cluster distribution may result from intergalactic obscuration. Of course, *galactic* obscuration certainly contributes to the lack of randomness, particularly at low galactic latitudes. In addition, extragalactic absorption may well be present, especially, as Zwicky suggests, within the rich clusters. Neither galactic nor extragalactic obscuration, however, can account for most of the relatively small-scale clumpiness in the observed cluster distribution. If, for example, the apparent clumps of clusters of distance group 5 were really portions of a random cluster distribution seen through "holes" in a galactic or extragalactic absorbing medium, one would also expect to find clusters of group 6 appearing through these same holes, but certainly not between them. Inspection of Fig. 1, however, shows that there is no strong correlation between the positions of clumps of clusters at different distances; on the contrary, many apparent groupings of clusters of group 6 occur in regions comparatively sparse in group 5 clusters, and conversely.

II. DISTRIBUTION OF GALAXIES ON THE LICK ASTROGRAPHIC SURVEY

Shane and his associates have been investigating the distribution of galaxies brighter than $m_{pg} \approx 18$ on the photographs taken for the Lick Astrographic Survey. In their first report (Shane and Wirtanen 1954) they described six clouds of larger size than typical clusters and each of which contains more than one center of condensation. Three of the Shane-Wirtanen clouds (Nos. 4, 5, and 6) correspond to apparent groupings of two or more clusters in the writer's catalogue of those identified on the Palomar Sky Survey. Two of their clouds (Nos. 2 and 3) correspond to single clusters in the writer's catalogue. The other Shane-Wirtanen clusters in the six clouds apparently are not rich enough for inclusion in the writer's cluster catalogue. It should be noted that this catalogue is complete only for unusually rich clusters—those that contain at least 50 galaxies not more than two magnitudes fainter than their third brightest members.

In the second report on the distribution of galaxies on the Lick Survey (Shane 1956), attention is again called to "a considerable number of rich aggregations" of galaxies, "some of which contain several centers of condensation, suggesting multiple clusters."

III. "LOCAL SUPERCLUSTER"

De Vaucouleurs (1953), from an analysis of the distribution of bright galaxies both in direction and magnitude, concluded that the Local Group is part of a large "supercluster," a flattened system centered on the Virgo cluster, and containing also the Ursa Major cloud, and many other smaller groups and clusters of galaxies. More recently he has investigated the distribution of the radial velocities of galaxies in the Local Supercluster (or "Supergalaxy"), and has shown that the data are (at least) consistent with the hypothesis that the entire system is both rotating and expanding (de Vaucouleurs 1958). De Vaucouleurs estimates the diameter of the Local Supercluster to be from 20 to 30 megaparsecs, but this estimate corresponds to a larger value of the Hubble constant than the one used in the present paper; for $H = 75 \text{ km/sec} \times 10^6 \text{ pc}$, the diameter of the system would be about $50 \times 10^6 \text{ pc}$.

R. L. Carpenter (1961) has attempted to verify the existence of the Local Supercluster from the distribution of galaxies brighter than $m_{pg} = 16$ in the northern galactic hemisphere. Carpenter conducted his survey on the blue prints of the Palomar Sky Atlas, and estimated magnitudes (to 0.5 mag.) by comparing the galaxian images with a step scale of one-magnitude intervals. The step scale consisted of a sequence of galaxy images taken from the Atlas prints and whose magnitudes had been measured by Sandage (Humason, Mayall, and Sandage 1956). In a preliminary analysis of his data, Carpenter has considered three strips in the sky, each 18° wide, straddling a 90-deg sector along the

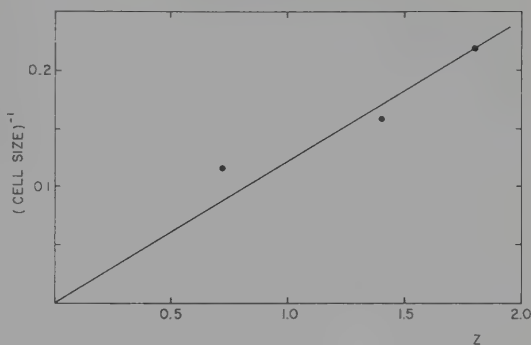


FIG. 3. Reciprocals of the sizes of the cells for which the observed frequency distributions deviate most from a Poisson distribution as a function of redshift.

supergalactic equator, as defined by de Vaucouleurs. There is a highly significant concentration of galaxies along the central strip (along the supergalactic equator) in comparison to the adjacent strips. The excess concentration diminishes, however, at fainter magnitudes; for $m_{pg} \leq 12.5$ the number of galaxies per square degree in the central strip is about three times that in the adjacent strips, but the excess is less than 20% at $m_{pg} = 15.5$. Moreover, in the interval $m_{pg} = 13.5$ to 14.5 , the logarithms of the numbers of galaxies in all three strips increase by far less than 0.6, as would be expected for a uniform distribution of galaxies in space, a circumstance that indicates a thinning out of galaxies beyond a nearby concentration (the Local Supercluster). For $m_{pg} > 14.5$, on the other hand, the increase in the numbers of galaxies in all three strips approaches that expected for a uniform distribution; these numbers, apparently, refer to intrinsically luminous galaxies in the more remote background. Carpenter's preliminary

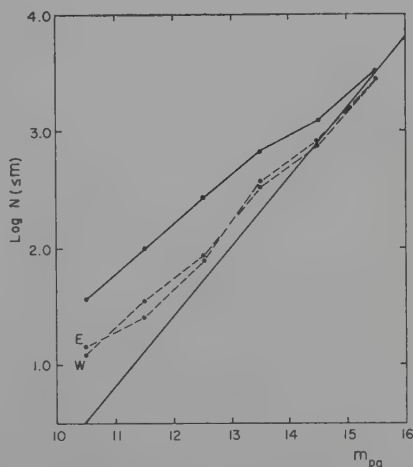


FIG. 4. Numbers of galaxies brighter than various photographic magnitudes. Solid line: galaxies in the strip along the supergalactic equator; dotted lines: galaxies in strips adjacent to supergalactic equator. The heavy straight line has a slope of 0.6. (Adapted from Carpenter.)

TABLE II. Some characteristics of probable second-order clusters.

No.	α (1950)	δ	b	l^I	l^{II}	number of sample clusters	total number of clusters	$cd\lambda/\lambda$ ($\times 10^{-3}$) km/sec	maximum diameter ($\times 10^{-6}$ pc)
1	0 ^h 26 ^m	-11 $\frac{1}{2}$ $^{\circ}$	-73 $^{\circ}$	71 $^{\circ}$	103 $^{\circ}$	10	10	60.2	61.5
2	9 18	+78	+34	102	134	15	19	53.7	74.6
3	10 16	+50 $\frac{1}{2}$	+53	132	164	10	10	49.0	38.7
4	11 39	+54 $\frac{1}{2}$	+60	111	143	6	10	21.4	33.3
5	11 40	+12	+68	221	253	9	9	44.6	32.1
6	11 45	+29 $\frac{1}{2}$	+76	169	201	5	6	8.7	60.2
7	14 5	+27	+73	4	36	26	29	44.6	96.5
8	14 7	+6 $\frac{1}{2}$	+62	316	348	5	6	43.6	45.7
9	14 33	+56 $\frac{1}{2}$	+55	66	98	5	6	43.6	50.8
10	14 39	+30 $\frac{1}{2}$	+66	15	47	11	11	56.2	49.7
11	14 53	+22 $\frac{1}{2}$	+62	358	30	10	14	43.6	55.9
12	15 10	+7	+51	336	8	5	9	24.0	27.4
13	15 16	+29 $\frac{1}{2}$	+57	14	46	8	9	25.0	58.4
14	15 32	+70	+41	74	106	12	12	53.7	47.4
15	16 14	+29	+45	16	48	5	6	10.0	52.0
16	23 6	-22	-66	8	40	23	26	43.6	52.7
17	23 24	-22 $\frac{1}{2}$	-70	11	43	15	19	46.7	47.8

results appear to provide a striking confirmation of de Vaucouleurs' hypothesis of a flattened Local Supercluster. The data just described are exhibited graphically in Fig. 4. (The writer is indebted to Mr. Carpenter for permission to discuss and exhibit these unpublished data.)

IV. EXAMPLES OF SECOND-ORDER CLUSTERS

About 50 apparent groupings of clusters—probable second-order clusters—can be identified in the distribution of clusters in the writer's catalogue (e.g., on Fig. 1). Some of the characteristics of 17 of these groupings are given in Table II. The equatorial and galactic coordinates refer to the approximate center of each configuration. Galactic longitudes are given both for the old system of galactic coordinates (l^I), and for the new IAU system (l^{II}); Fig. 1 is plotted on the old system. The "number of clusters" is the number of clusters in each grouping that belong to the homogeneous statistical sample of clusters that contain at least 50 galaxies not more than 2 mag. fainter than the third brightest members—that is, clusters for which the writer's catalogue is essentially complete. The catalogue contains, in addition, a large number of clusters that contain less than 50 galaxies in the stated magnitude interval—clusters not rich enough to have been completely identified. The "total number of clusters" refers to the number in each second-order grouping that appear in the catalogue, whether or not they are part of the homogeneous sample. The tabulated redshift, in thousands of kilometers per second, is the mean of the redshifts estimated for the member clusters in each second-order group. The "diameter" is the maximum linear diameter, projected onto the plane of the sky, for each group of clusters; it is computed from the observed angular size of the group, the tabulated redshift, and the value of the Hubble constant adopted in this paper.

The mean number of sample clusters per second-order group is 10.6 ± 6.0 (s.d.). It must be remembered, however, that this figure refers only to the number of *very rich* clusters. The total number of clusters and groups of all kinds in a typical second-order cluster might be greater by one or two orders of magnitude. The mean linear diameter for these 17 groups of clusters (corresponding to the assumed Hubble constant) is 52.0 ± 15.9 (s.d.) megaparsecs.

Figure 5 is a reproduction from a portion of a red print of the Palomar Sky Atlas, which shows part of the supercluster No. 4 in Table II.

V. DYNAMICS OF SECOND-ORDER CLUSTERS

Zwicky (1957a, 1957b) has argued that second-order clustering does not exist, and that the absence of superclusters implies that gravitational interactions must not extend beyond the dimensions of a single cluster. As further evidence for the absence of gravitational forces over intercluster distances, he has referred to the low dispersion of velocities between clusters, aside from the apparent velocities associated with the general expansion of the universe. Actually, the only available evidence on the dispersion of velocities between clusters comes from the data published by Humason, Mayall, and Sandage (1956). The spread of velocities of clusters can be determined from the spread of points about the smooth curve in Sandage's relation between $cd\lambda/\lambda$ and m_{pv} ; the root-mean-square deviation of velocities is found to be about 3700 km/sec. From his analysis of the luminosity functions of galaxies in rich clusters, however, the writer has found that most of this dispersion is probably in the magnitudes (both apparent and absolute) of the 10 brightest members of different clusters. The intrinsic dispersion of velocities among the clusters studied by Sandage may well be less than 1000 km/sec. On the other hand, most of the clusters chosen for the purpose of determining the velocity-

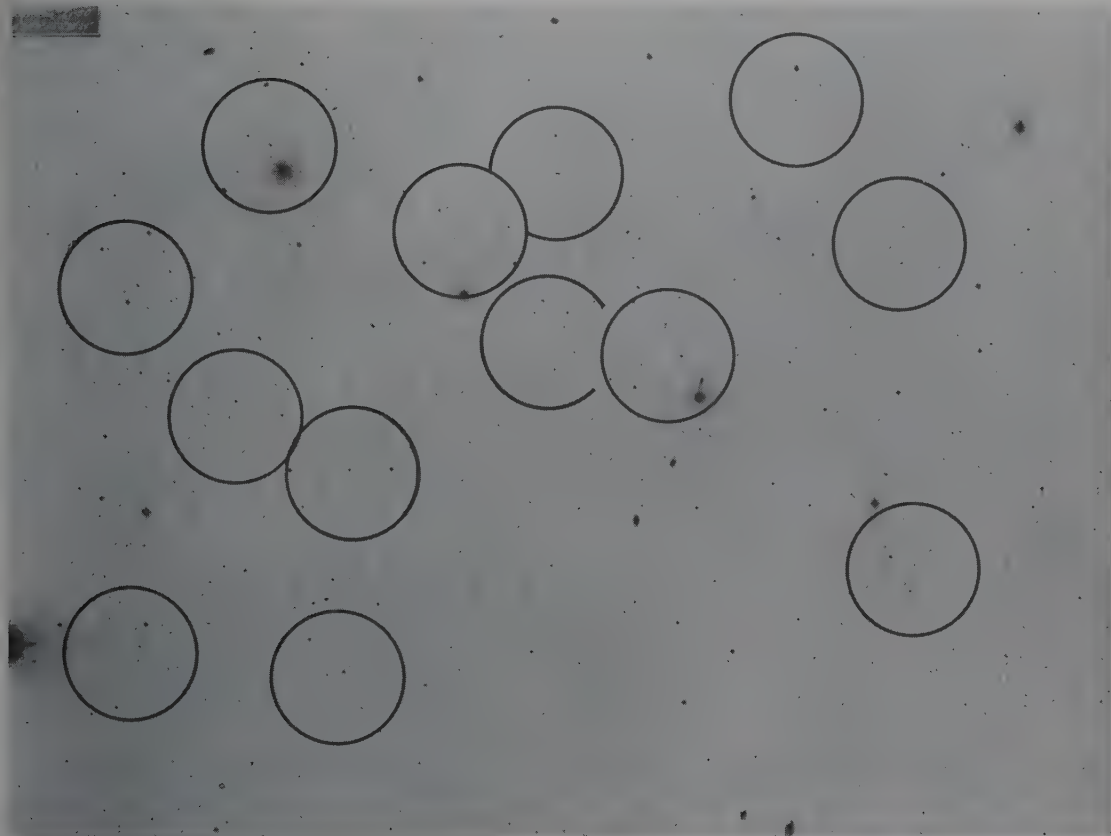


FIG. 5. A portion of the red print of the Palomar Sky Atlas field at $\alpha=11^h 24^m$, $\delta=+54^\circ$. Part of the second-order cluster No. 4 (Table II) appears on the photograph. (Scale: 1 mm=99".) Copyright, National Geographic Society—Palomar Observatory Sky Survey.

distance relation are rich and apparently isolated from other clusters, a selection effect that would tend to *lower* the observed dispersion of velocities between clusters. It is of interest, therefore, to determine what velocity dispersion would be expected between the members in a typical second-order cluster, on the assumption that the system is dynamically stable, and that Newton's laws describe the forces within it.

The mass that is obtained from the application of the virial theorem to a typical rich cluster is of the order of 10^{15} solar masses (e.g., Noonan 1961). Even if the virial theorem does not apply and this mass should be higher than the true value, as some investigators have suggested, it is still the appropriate mass to use in predicting the velocity dispersion within second-order clusters. A lower limit to the mass of a typical second-order cluster, which contains 10 such rich clusters, would then be 10^{16} solar masses. An upper limit to the mass can be estimated by assuming that the lesser clusters and groups in the system add up to an equivalence of 100 rich clusters, in which case we obtain 10^{17} solar masses.

To estimate the diameter of a typical supercluster,

we note the following results: (1) The mean scale of "clumpiness" found from the statistical analysis of the distribution of rich clusters is 58×10^6 pc; (2) the approximate size of the Local Supercluster is about 50×10^6 pc; (3) the mean size of several individual groupings of clusters is 52×10^6 pc. The available evidence suggests, therefore, that the mean size for a typical supercluster (for our assumed distance scale) is in the range 50 to 60 megaparsecs. We shall adopt for a characteristic radius (which corresponds to the mean separation of members in the supercluster) the value $R = 20 \times 10^6$ pc.

If the second-order cluster is in a steady state, the virial theorem predicts for the root-mean-square velocity dispersion (e.g., Chandrasekhar 1942):

$$\langle (V^2) \rangle^{1/2} = 4.6 \times 10^{-2} (M/R)^{1/2} \text{ km/sec}, \quad (2)$$

where M and R are in solar units and parsecs, respectively. On the other hand, the velocity of escape of a body from the edge of a system of mass M and radius R is of the order of

$$V_{\text{esc}} = 9.2 \times 10^{-2} (M/R)^{1/2} \text{ km/sec}, \quad (3)$$

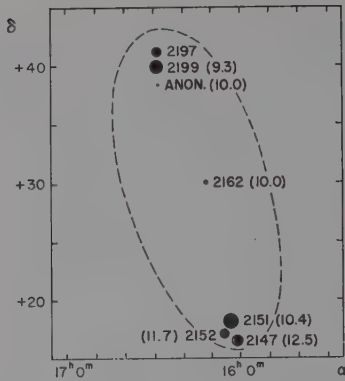


FIG. 6. Second-order cluster No. 15 in Table II. Four-digit numbers refer to catalogue numbers of clusters in Abell's catalogue of rich clusters. Numbers in parentheses are mean velocities of clusters, in units of 10^3 km/sec.

where the same units are used for M and R as in Eq. (2). Whether or not a supercluster is in a steady state, therefore, the velocity dispersion among its member clusters should be within a factor of 2 or 3 of the value predicted by Eq. (2), provided that the system is gravitationally stable. We find, then, the following values of $(\langle V^2 \rangle)^{1/2}$ corresponding to our estimates for the lower and upper limits to the mass of a supercluster:

$$\begin{aligned} \text{for } M = 10^{16}: \quad (\langle V^2 \rangle)^{1/2} &\approx 10^3 \text{ km/sec;} \\ \text{for } M = 10^{17}: \quad (\langle V^2 \rangle)^{1/2} &\approx 3 \times 10^3 \text{ km/sec.} \end{aligned} \quad (4)$$

Zwicky (1957b) estimates that the root-mean-square dispersion of velocities in a second-order cluster should be from 5×10^3 to 10^4 km/sec. However, there may be a misunderstanding about what is meant by a "second-order" cluster. It is possibly significant that Zwicky (1938) has estimated for the mean diameter of a "cluster cell"—the region of space allocated to a single cluster on the assumption that all galaxies are in clusters—the value 7.5×10^6 pc. This figure was based upon Hubble's early distance scale; for $\dot{H} = 75$ km/sec $\times 10^6$ pc, Zwicky's cluster cells would also have a diameter of the order of 50 Mpc. It may be that what we call "second-order clusters," he refers to as single clusters with multiple condensations or "nuclei" of galaxies.

VI. VELOCITY DATA FOR ONE POSSIBLE SUPERCLUSTER

At the time of writing, velocity data are available for only one suspected second-order cluster (except for the Local Supercluster). The group of clusters (No. 15 in Table II) contains six clusters in the writer's catalogue: Nos. 2147, 2151, 2152, 2162, 2197, and 2199. All but cluster 2162 are in the homogeneous sample. In addition there are several dozen clusters and groups that can be recognized on the Sky Survey photographs that probably belong to the system. The group of clusters covers an elongated region centered near: $\alpha = 16^h 14^m$;

$\delta = +29^\circ$. The heaviest concentrations of clusters are at opposite ends of the region, and the identity of the group as a single second-order system is in question. On the other hand, the clusters and groups included all appear at about the same distance, and the entire group seems to be well isolated from other clusters of a similar distance. Moreover, the system has a maximum linear diameter that is typical of those of other suspected second-order clusters.

Cluster 2151 is the Hercules cluster, for which velocities have been given by Humason *et al.* (1956) and by Burbidge and Burbidge (1959). Cluster 2199 is a system centered on NGC 6166; velocities in that cluster have been measured by Minkowski (1961). The writer has obtained velocities for 12 galaxies in other clusters in the system at the 100-inch telescope. The velocity data are summarized in Fig. 6, which is a plot of the rich clusters in the system. Mean velocities, in thousands of kilometers per second, are given beside each point representing a cluster for which two or more velocities are available. There are not enough velocity data to estimate accurately the velocity dispersion of the supercluster, and we cannot draw definite conclusions about the gravitational stability of the system. At least, however, the data are not incompatible with the assumption of gravitational interactions between its members.

VII. DISCUSSION AND SUMMARY

Available evidence appears to be quite conclusive for the existence of second-order clusters with diameters of the order of 50 megaparsecs. Whether these superclusters are dynamically stable systems, however, is not established at present. If they are gravitationally stable, we can expect internal root-mean-square velocity dispersions of the order of $1\text{--}3 \times 10^3$ km/sec, and possibly superimposed upon this a velocity spread due to the general expansion of the universe. If the apparent superclusters are not stable systems, on the other hand, they must represent the regions in space where cluster formation has occurred, and may reflect something of the distribution of the primordial pregalaxian material.

If gravitational forces between members of second-order clusters are absent or negligible, these systems must be expected to be expanding at least with the rate of the general universal expansion. The corresponding spread of velocities across a second-order cluster would then be

$$\Delta V \sim H \times D = 75 \times 50 = 3750 \text{ km/sec.} \quad (5)$$

Since our estimate of the value of D , the linear diameter of a second-order cluster, is proportional to H^{-1} , the derived value of ΔV is independent of the value assumed for H .

The presence of second-order clusters enhances the probability that what appears to be a single cluster

may actually be two or more clusters of the same super system seen in projection. Spurious root-mean-square velocity dispersions, ranging up to nearly 4000 km/sec, could thus be obtained. The possibility is particularly high, for example, in the clusters in Hercules and Virgo, both of which are suspected of being associated with second-order clusters.

It is not likely, however, that the hypothesis of second-order clustering can remove completely the discrepancy between the high mass-light ratios found for galaxies in clusters and for individual objects. In particular, the rich, symmetrical, apparently isolated systems such as the clusters in Coma and Corona Borealis can hardly be multiple systems seen in projection. One must look for other effects—dark intergalactic matter, a grossly incorrect distance scale, expansion of the clusters, etc. The probability of the existence of superclusters, on the other hand, may well account for part of the mass-light ratio discrepancy in the case of many clusters and groups, and should be thoroughly investigated before the hypothesis of the instability of clusters of galaxies is adopted, with all of its ramifications.

REFERENCES

- Abell, G. O. 1958, *Astrophys. J. Suppl.* **3**, 211 (No. 31).
 Burbidge, G. R. and Burbidge, E. M. 1959, *Astrophys. J.* **130**, 629.
 ——. 1961, *Astron. J.* **66**, 541.
 Carpenter, R. L. 1961, Paper presented to June meeting of Astron. Soc. Pacific.
 Chandrasekhar, S. 1942, *Principles of Stellar Dynamics* (University of Chicago Press, Chicago; also, Dover Publications, Inc., New York), p. 201.
 Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astron. J.* **61**, 97.
 Minkowski, R. L. 1961, *Astron. J.*, **66**, 558.
 Neyman, J. and Scott, E. L. 1961, *ibid.* **66**, 581.
 Noonan, T. 1961, "The structure of the Coma cluster of galaxies," Ph.D. thesis, California Institute of Technology.
 Shane, C. D. 1956, *Astron. J.* **61**, 292.
 Shane, C. D. and Wirtanen, C. A. 1954, *ibid.* **59**, 285.
 de Vaucouleurs, G. 1953, *ibid.* **58**, 30.
 ——. 1958, *ibid.* **63**, 253.
 ——. 1960, *Astrophys. J.* **131**, 585.
 Zwicky, F. 1938, *Publs. Astron. Soc. Pacific* **50**, 218.
 ——. 1957a, *Morphological Astronomy* (Springer-Verlag, Berlin), pp. 165–166.
 ——. 1957b, *Publs. Astron. Soc. Pacific* **69**, 518.

DISCUSSION

MRS. BURBIDGE asked about the distances of clusters in the Hercules region and was told that cluster A 2199 and others nearby are all at the same distance as the Hercules cluster as far as can be told from the available magnitude measures.

In answer to PAGE, ABELL said that the root-mean-square deviation of cluster velocities from the Hubble law, assuming no dispersion in absolute magnitude of the tenth brightest member, is about 4000 km/sec. He considers that a large part of this is due to dispersion in absolute magnitude. SCHUECKING noted that this velocity dispersion must be reduced by the factor $1/(1+z)$ to get the velocity dispersion relative to the cluster center at the time of light emission, and possibly by another factor $1/(1+z)$ to correct for change in scale during the light travel time. Of course, for the redshifts considered here ($z = \delta\lambda/\lambda = 0.2$) these corrections are negligible relative to the observational errors.

MILLS observed that his latest survey shows a clustering of radio sources near the south galactic pole, and asked if clusters of galaxies were also concentrated there, and ABELL replied that this is one of the densest second-order clusters, with estimated distance corresponding to 35 000 to 50 000 km/sec redshift.

JUST mentioned work of his underway on a correlation between diameters and distances of superclusters in Abell's catalogue with the purpose of studying whether they are participating in the Hubble expansion. It was noted by HOLMBERG that some of Abell's results confirm an early study of his (*Lund Ann.* **6**, 1937) in which a density curve for the local "metagalaxy" was derived from less homogeneous data.

Statistics of the Double Galaxies, and Their Formation

THORNTON PAGE, CONARD C. DAHN, AND F. FOSTER MORRISON.
Van Vleck Observatory, Wesleyan University

Statistical formulas are developed for the expected number of chance pairs among galaxies of various local surface densities, and it is shown that counts of double galaxies in regions of varying space density of galaxies (in and out of the major clusters) may provide evidence about the process of formation of pairs. Preliminary counts in three regions totaling about 270 square degrees on the Palomar Atlas prints show the expected correlation between density of pairs and the *square* of the density of galaxies as a whole. However, the pairs counted to 18^m5 turn out to be mostly chance pairs, and there is only imperfect evidence that, after correction for such chance pairs, the space density of the remaining physical pairs is proportional to the square of the space density of galaxies. If this fact is confirmed by further counts, it may indicate that double galaxies are formed by capture.

AMONG other things, the study of pairs of galaxies offers promise of information on the masses of galaxies, their luminosity distribution, their formation, and their evolution. Most of these were suggested by Holmberg (1937) in his classic paper and catalogue. More recent work by Page (1952, 1960, 1961) and Holmberg (1954) has been devoted to mass determinations; this paper concerns the question of the formation.

DEFINITIONS: CRITERIA

It is first necessary to define precisely and practically what is to be considered a double galaxy. By far the simplest criterion in practice is the one used by Holmberg (1937):

$$S \leq c(A_1 + A_2), \quad (1)$$

where S is the angular separation between centers, A_1 and A_2 are the major axes or largest angular dimension of the two galaxies, and c is an arbitrarily chosen constant, for which Holmberg used 2.0.

Equation (1) is not sufficient. There is a background of faint galaxies that increases according to Hubble's law of numbers (1936):

$$\log N_1(m) = 0.6(m - \Delta m - K) + C_1, \quad (2)$$

where $N_1(m)$ is the number of galaxies per square degree brighter than magnitude m ; K the correction for redshift, Δm the correction for galactic obscuration, and C_1 a constant. Therefore, there are a very large number of fainter galaxies within a circle of radius cA_1 about any galaxy, and it is necessary to specify a limiting magnitude m_2 or a magnitude difference $m_2 - m_1$ that is not to be exceeded. Because we are interested in "physical pairs"—galaxies with small space separations—it would be well to relate this magnitude criterion to the distribution of absolute magnitudes of galaxies—the luminosity function. But since the luminosity function has not yet been determined on the fainter side of the mean, it provides no sound basis for a criterion. For instance, limiting $m_2 - m_1$ to two or three magnitudes would exclude the well-recognized triple system M31-M32-NGC 205 with magnitudes 4.3, 9.1, 8.9.

An alternative criterion would be a limitation on

$A_2 - A_1$, but there is no evidence to show that the dispersion in diameters of galaxies is less than that in magnitude. From a statistical point of view it may therefore be preferable simply to apply a limit on m_2 , the fainter magnitude in a pair.

In 1954 Holmberg provided a basis for selecting the constant c in Eq. (1). From counts of faint galaxies up to 1° away from selected bright galaxies ($m = 12.1$ to 13.0), and correcting for chance pairs in the manner outlined below, he found a cutoff in separation of physical pairs at 45' (minutes of arc) and a distribution of projected separations that implies a space distribution

$$f(R) = k[1 - (R/R_m)^3], \quad (3)$$

where R is the space separation, $R_m = (2.3/h) \times 10^3$ pc, corresponding to 45' separation at the distance of a 12.6-mag. galaxy, h is the Hubble constant in km/sec/10⁴ pc (near 1.0), and k is a constant. The relation between angular diameters and magnitude (Hubble 1936) shows that, on photographs of the type Hubble and Holmberg were using, $\bar{A} = 6'.0$ at $m = 12.6$; hence Eq. (3) implies that the separation criterion should be $S = 7.5(A_1 + A_2)$, or $c = 7.5$.

However, counts of double galaxies on Palomar Atlas prints soon showed that a criterion as loose as this led to "chaining"; that is, there were often two or more galaxies within $7.5(A_1 + A_2)$ of a given galaxy and others within that separation from them, and so on. For this purely practical reason it was necessary to tighten the criterion to $c = 3.0$.

The remainder of this discussion therefore refers to double galaxies of magnitudes $m_1 < m_2 < 18.5$ (approx) and separations $S < 3(A_1 + A_2)$.

POSSIBLE EXPLANATIONS OF DOUBLE GALAXIES

It is clear that the explanation of any given pair selected according to the above criteria falls into a series of dichotomies; first, it is either a *physical* pair or a *chance* pair. If it is a physical pair, it may be stable or, if it has positive total energy as Ambartsumian (1957) has postulated, it may be flying apart. If it is stable, it may have been formed by capture or by two condensations in a single protogalaxy.

Mass determinations (Page 1952, 1960) have been made on the assumption that double galaxies *are* stable; the results show a large difference between the mean mass of elliptical galaxies and that of spirals, as might be expected on other grounds. Moreover, the mean mass of all galaxies in 52 pairs is consistent with the (many fewer) determinations from internal rotation of nearby single galaxies. In a sense, these results argue for the interpretation of double galaxies as stable, physical pairs.

CHANCE PAIRS

Of course, the existence of chance pairs cannot be denied or ignored. Holmberg (1937) originally argued that they are a small fraction of the total by comparing the observed distribution of close separations $f(S)$, with that expected from a random distribution of galaxies over the sky. This argument led him to the conclusion that less than 13% of the double galaxies brighter than 14th mag. and separated by less than 10' are chance pairs.

However, this reasoning ignores the fact that the 55 000 galaxies brighter than 14th mag. show strong evidence of *clustering*, a fact obvious from Shane's area counts (Shane *et al.* 1959), and also implied by Hubble's counts (1936), which show a normal distribution in $\log N$. (N is the number of galaxies per plate.) A more correct procedure is to compute the number of chance pairs in small regions of the sky from the actual number of single galaxies, the distribution being assumed random in each region.

The basic formula, due to Polya (1919), gives the probability that, if n points are distributed at random on a sphere, none of them will fall within angle θ from an $(n+1)$ st point:

$$p(\theta, n) = \cos^2 \frac{1}{2} \theta \simeq \exp(-\frac{1}{4} n \theta^2). \quad (4)$$

The approximation is very good for n large and θ small; e.g., $n = 3\,500\,000$ galaxies brighter than 18th mag., and $\theta = 0.001$ radian or less is typical of observed separations.

In differential form, the probability of a galaxy's nearest neighbor being between θ and $\theta + d\theta$ from it (by chance) is

$$-d p(\theta, n) = (\frac{1}{2} n \theta) \exp(-\frac{1}{4} n \theta^2) d\theta \quad (5)$$

and the expected number of separations from the closest galaxy between θ and $\theta + d\theta$, over the whole sky is

$$dn(\theta, n) = -(n+1) d p(\theta, n) \simeq (\frac{1}{2} n^2 \theta) \exp(-\frac{1}{4} n \theta^2) d\theta. \quad (6)$$

In an area of one square degree, the expected number of chance separations from the closest galaxy between s and $s + ds$ degrees is then

$$dN_s(s, N_1) = -N_1 d p(\theta, n) = 2\pi N_1^2 s \exp(-\pi N_1 s^2) ds, \quad (7)$$

where $N_1 = n\pi/4(180)^2$ is the total number of galaxies in the square degree, and $s = 180\theta/\pi$ is the separation in degrees. The integral of Eq. (7) is the number of

galaxies from which the nearest galaxy is less than or equal to s ,

$$N_2(s, N_1) = N_1 [1 - \exp(-\pi N_1 s^2)] \simeq \pi N_1^2 s^2. \quad (8)$$

Consider three galaxies, a , b , and c , such that the separations $ab < bc < ac$, and no other galaxies are nearby. Equations (7) and (8) include b as the closest companion to a , a as the closest companion to b , and b as the closest companion to c . That is, the close pair ab is counted *twice*. This consideration leads to a convenient definition of an *unambiguous pair* (in this case ab) as a pair of galaxies of separation s with no other galaxy brighter than the limiting magnitude within s of either member of the pair. The expected number of such pairs separated by s to $s + ds$ is approximately half the number of closest separations given in Eq. (7) multiplied by the probability that no other galaxy be within s , from Eq. (4):

$$dN_2(s, N_1) = \frac{1}{2} dN_s(s, N_1) \exp(-\pi N_1 s^2) \\ \simeq \pi N_1^2 s \exp(-2\pi N_1 s^2) ds. \quad (9)$$

Equation (9) is not rigorously correct because, after a pair is selected, the positions of other galaxies nearby are not independent; that is, Eq. (4) does not apply exactly. In a more refined analysis, the effect of such cross correlation should be included; for present purposes it is appropriate to integrate Eq. (9) to obtain the number of unambiguous pairs of separation less than s ,

$$N_2(s, N_1) = (\frac{1}{4} N_1) [1 - \exp(-2\pi N_1 s^2)] \simeq \frac{1}{2} \pi N_1^2 s^2. \quad (10)$$

The approximation is valid for $N_1 s^2 \ll 1$, under which circumstances N_2 is just half of N_s , and the influence of triples and higher-order groups is negligible.

As was to be expected, the number of chance close pairs of galaxies in a given area of the sky is roughly proportional to the *square* of the total number of galaxies in that area, and it is because N_2 is proportional to N_1^2 , that clustering can cause more chance pairs than Holmberg (1937) estimated for 55 000 single galaxies randomly distributed over the whole sky. [Actually, it appears that Holmberg overlooked the factor-of-two difference between Eq. (10) and (8), and computed twice as many chance pairs as he should have, for a uniform distribution.] For instance, if $n = 55\,000$, $N_1 = 1.3$ galaxies per square degree, and $N_2(0.1, 1.3) \simeq 0.028$ unambiguous pairs per square degree of separation $s \leq 0.1 = 6'$. That is, 97 such pairs are expected over approximately 3400 square degrees of unobscured sky covered by Holmberg's catalog, in which 695 simple pairs are listed. However, if the variance in N_1 is large, as shown below, so that in 10% of the 1-deg squares N_1 is as large as 4, then N_2 is in these cases 0.25, and the total number of chance pairs will be 150 or higher.

ORIGIN OF PHYSICAL PAIRS

Statistically, then, the number of physical pairs can be derived by subtracting the number of chance pairs

of separation s , dN_2 in Eq. (9), from the number counted in a square degree where the total number of galaxies is N_1 . The criterion, Eq. (1), as well as N_1 , depends on magnitude, and it is therefore necessary to consider the data in separate magnitude classes, in each of which the limiting separation S is approximately constant. The counts are grouped, furthermore, in "regions," for each of which $N_1(m)$ has approximately the same value.

After subtracting the chance pairs, the space density of the remaining physical pairs is expected to be correlated with the space density of galaxies as a whole, and the nature of this correlation should provide evidence on the process of pair formation. If most, or all of the pairs of galaxies were formed from condensing clouds of gas, in just the same way as single galaxies, it is to be expected that the pairs form a roughly constant proportion of all galaxies, or that N_2 is proportional to N_1 . Similarly, if the second galaxy of a pair is formed in the "wake" of the first as it moves through an intergalactic cloud as proposed by Sciamia (1954), N_2 is again expected to be proportional to N_1 . Finally, if pairs result from cataclysmic explosions within single galaxies or protogalaxies, as postulated by Ambartsumian (1957), we again expect $N_2 \propto N_1$.

However, if pairs are formed by *capture*, the number of captures will depend on the *square* of the space density of galaxies, and $N_2 \propto N_1^2$. It should be noted that "incomplete captures," or random hyperbolic passages, or collisions are eliminated, statistically, by the procedure of subtracting chance pairs from the counted number of pairs; here we are dealing only with average numbers of stable pairs—galaxies in closed orbits.

It is to be noted, also, that other statistical tests may be possible, using the measured radial velocities of galaxies in pairs. In the case of stable orbits, the orbital velocities are roughly perpendicular to the line of centers of a pair, while in the case of hyperbolic orbits, or of Ambartsumian's postulated explosions, the velocities would be more nearly radial. However, the statistical discrimination between these cases is complicated by the unknown distribution of space separations, masses, and energies among pairs (Page 1960).

COUNTS OF CLOSE GALAXIES

Two of the authors (Dahn and Morrison) have undertaken to locate and measure close pairs and groups of galaxies in selected areas of the Palomar Sky Atlas. Two of the areas were chosen in the Coma and Virgo clusters in order to obtain high N_1 , and another area near $22^h 40^m + 40^\circ 30'$ was chosen for low N_1 . A 1-cm reseau was placed in contact with each Palomar Atlas red, or "E" print, and the number of galaxies brighter than limiting magnitude of about $18^m.5$ were counted in each square. All pairs and groups of galaxies that satisfied the criterion of Eq. (1) were

separately listed, together with coordinates, separation, major diameters, and estimated magnitudes. Altogether, 322 close separations were counted in areas totaling 270 square degrees, 254 of them being unambiguous pairs. In the same areas there were about 8500 galaxies brighter than about $18^m.5$, 38 of them brighter than $14^m.0$.

The magnitude scale was established by comparison with images of galaxies of various types on the red Palomar Atlas prints (both spiral and elliptical) of magnitudes measured elsewhere. (Ames 1930; Holmberg 1958). A rough check indicated that estimated magnitudes are accurate to about $\pm 0^m.5$. No correction was made for edge effect.

The counts in 1-cm squares ($11'.2 \times 11'.2$ in the sky, or about $1/30$ square degrees), are subject to considerable fluctuation; hence they were averaged by fours. D denotes the average density of galaxies (number per square cm) brighter than $18^m.5$ obtained in this way. Values of D varied from 0 in 1300 squares to 20.5 in one square.

The pairs and small separations (30 triples) satisfying Eq. (1) were then each associated with a square; in cases where a pair bridged two squares it was counted as half in each square. D_s denotes the number of close separations associated with each square. All squares with D equal to 0, 0.25, 0.50, 0.75, etc. were grouped in "regions," and the average counted pair density $\bar{D}_s$ was computed for each region. These raw data are plotted as a regression, $\bar{D}_s$ vs D^2 in Fig. 1 and as a log-log plot in Fig. 2.

In this first, preliminary analysis of the counts, no attempt has been made to associate regions of high D with known clusters of galaxies. All the variance was attributed to D_s , and, weights were assigned to each point according to the number of counted squares included in $\bar{D}_s$. Least-squares solutions then gave the following results, together with standard deviations:

$$\bar{D}_s = (0.0279 \pm 0.0043) D^2 - (0.0013 \pm 0.0028) D, \quad (11)$$

$$\bar{D}_s = (0.0265 \pm 0.0031) D^2, \quad (12)$$

$$\bar{D}_s = (0.0288 \pm 0.0018) D^2. \quad (13)$$

In Eq. (13) only the first 13 points were used, omitting

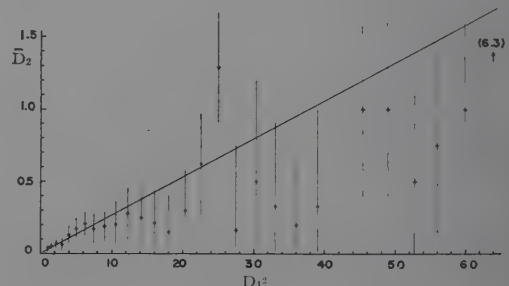


FIG. 1.

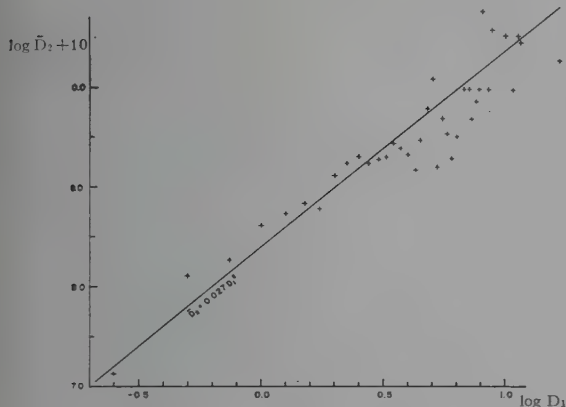


FIG. 2.

the remaining 34 points of high D_1 and low weight that together account for only 2% of the squares counted.

It is reasoned from the appearance of Fig. 2 and the low weight of the second coefficient in Eq. (11), that $\bar{D}_s$ is proportional to D^2 within the accuracy of the counts. Vertical bars through the plotted points on Fig. 1 are proportional to the standard deviation of $\bar{D}_s$ at each point.

Since the *counted* density of close pairs is therefore proportional to the square of the surface density of galaxies and, by Eq. (8) and (10), the expected density of chance pairs is also proportional to D^2 , the difference, representing physical pairs, will also be proportional to D^2 . It remains to calculate the *proportion* of the counted pairs that are due to chance.

Since the selection criterion, Eq. (1), required separations of galaxies to be less than S , which depends on angular diameter, and therefore on magnitude, it was necessary to divide the data into six magnitude classes, as shown in Table I. The geometric mean major diameter of each class of galaxies is related to estimated mean magnitude by the empirical relation

$$\log A_i = 3.6 - 0.2m_i \pm 0.1, \quad (14)$$

and the selection criterion within each magnitude class S_{ij} is obtained from Eq. (1). The selection criterion *between* magnitude classes S_{ij} is obtained in similar fashion.

The proportion of galaxies brighter than $18^m.5$ in each class is defined as $b_i = D_i/D$, and estimated from the interpolation formula

$$\log D(m) = 0.5m + \text{const}, \quad (15)$$

m being the estimated magnitude uncorrected for redshift. Hence

$$\log[(D - D_6)/D] = \log(1 - b_6) = -0.75,$$

$$\log[(D - D_6 - D_5)/D] = \log(1 - b_6 - b_5) = -1.5, \quad (16)$$

and so on, leading to the values of b_i in Table I.

Consider four 1-cm squares on the Palomar Atlas over which the galaxy count is $4D$. Chance pairs in this area may include galaxies within an area a including a strip of width S outside the 2×2 cm² square;

$$a = 4 + 8S + \pi S^2. \quad (17)$$

If there are n galaxies in this "significant area" a the expected number of separations less than S among them is

$$E(4D_s) = 4\pi S^2 n(n-1)/a^2. \quad (18)$$

The expected value of n is aD , and the expected value of the density of chance close separations is therefore

$$\begin{aligned} E(D_s) &= \pi S^2 [D(D-1/a) + \sigma_n^2/a^2] \\ &= \pi S^2 D^2 [1 - 1/aD + (\sigma_n^2/a^2 D^2)], \end{aligned} \quad (19)$$

where σ_n^2 is the variance in n due to fluctuations in D from one square to the next. Equation (19) is to be compared with Eq. (8), with D replacing N_1 .

If the density D is considered in six magnitude classes, $D = D_1 + D_2 + \dots + D_6 = D(b_1 + b_2 + \dots + b_6)$, Eq. (19) can be applied to pairs formed between galaxies of all six classes, so that

$$E(D_s) = D^2 \pi \sum_{ij} S_{ij}^2 b_i b_j \left[1 - \left(\frac{1}{a_{ij} D} \right) + \left(\frac{\sigma_n^2}{a_{ij}^2 D^2} \right) \right]. \quad (20)$$

Equation (20) counts *twice* all chance separations less than S with *both* the galaxies in the 2×2 cm² square, and counts *once* all chance separations with one galaxy in the 2×2 cm² square and one outside. Since the pairs totally within a square were counted once, and the bridging pairs considered to be one-half in each square, Eq. (20) must be divided by two, and the coefficient of D^2 in the relation between D and the chance pairs is

$$\frac{\pi}{2} \sum_{ij} S_{ij}^2 b_i b_j \left[1 - \left(\frac{1}{a_{ij} D} \right) + \left(\frac{\sigma_n^2}{a_{ij}^2 D^2} \right) \right]. \quad (21)$$

When $D \geq 1$, the last term in the bracket of Eq. (20) can be neglected, since $a_{ij}^2 > 20$, and σ_n^2 is of the order of 1. The first term in the bracket produces deviations from $D_s \propto D^2$ of -5% at $D=4$, -10% at $D=2$, -22% at $D=1$ and -60% at $D=0.5$, but this has very little influence on the fit of Figs. 1 and 2. The coefficient of

TABLE I. Magnitude classes.

Class i	Magnitude m	Major diameter A_i (mm)	Max separation S_{ii} (mm)	Proportion of galaxies b_i	Significant area a_{ii} (cm ²)
1	18 ^m .5-17 ^m .0	0.1-0.2	0.85	0.823	4.7
2	17.0-15.5	0.2-0.4	1.69	0.146	5.4
3	15.5-14.0	0.4-0.8	3.36	0.0260	7.0
4	14.0-12.5	0.8-1.6	6.72	0.0040	10.8
5	12.5-11.0	1.6-3.2	13.44	0.00082	20.4
6	11.0-9.5	3.2-6.4	26.88	0.00014	48.2

Eq. (21) is thus determined to be 0.0239 for $D > 4$, 0.0228 for $D = 4$, 0.0217 for $D = 2$ and 0.0200 for $D = 1$.

The difference between these values and the regression of Fig. 1 or Eq. (13) indicates that *physical* pairs form about 20% of the pairs counted to 18^m , and the correlation of physical pairs with surface density of galaxies is

$$D_p = (0.006 \pm 0.002) D^2. \quad (22)$$

Further work is necessary in order to refine this correlation and to relate it with *space* densities in the clusters. Unless the cluster is submerged by background galaxies, the projection of space densities to surface densities of galaxies and pairs of galaxies should not alter the form of Eq. (22).

REFERENCES

- Ambartsumian, V. A. 1957, *Armenian Acad. Sci. Math-Phys. Ser.* 11.
 Ames, A. 1930, *Harvard Ann.* 88, No. 1.
 Holmberg, E. 1937, *Lund Ann.* 6.
 —. 1954, *Medd. Lund Obs. Ser. 1*, No. 186.
 —. 1958, *ibid. Ser. II*, No. 136.
 Hubble, E. 1936, *The Realm of the Nebulae* (Yale University Press, New Haven, Connecticut).
 Page, T. L. 1952, *Astrophys. J.* 116, 63.
 —. 1960, *Astrophys. J.* 132, 910.
 —. 1961, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (to be published).
 Pólya, G. 1919, *Astron. Nach.* 208, 175.
 Sciama, D. W. 1954, *Monthly Notices Roy. Astron. Soc.* 115, 3.
 Shane, C. D., Wirtanen, C. A., and Steinlin, U. *Astron. J.* 64, 197.

DISCUSSION

AMBARTSUMIAN agreed that the close pairs of galaxies have negative energy and are in stable orbits. Pairs with positive energy would increase their separation in a relatively short time and not be recognized as pairs. Therefore Page's closed-orbit model for determination of average masses need not be doubted.

G. BURBIDGE presented a summary of the values of masses of individual galaxies determined from internal motions (Table II) showing that Page's statistical determinations from double galaxies (*Astrophys. J.* 132, 910, 1960) are not inconsistent, except in the values of M/L . PAGE explained this as possibly due to the inaccurate magnitudes used in his determination, over a third of which were early estimates by Holmberg (*Lund Ann.* 6, 1937).

KERR referred to recent 21-cm radio observations of the Magellanic Clouds that show both are embedded in a hydrogen envelope, and asked whether such an intergalactic cloud of material would change Page's derived masses. He also noted that the sharp edge of the hydrogen emission around the two diametrically opposite edges of the Magellanic Clouds suggests that they are moving apart rather than in a circular orbit.

PAGE replied that he had considered the possibility of intergalactic matter in the double galaxies, but that a density of 10^{-26} g/cm³ or more is necessary in order

to change the mean masses in Table II by a factor of 2 downward. The discrepancy in M/L would then be worse for spirals.

ABELL asked whether the selection criterion should not exclude pairs differing widely in radial velocity. When PAGE pointed out the circularity in reasoning that this would introduce, he suggested a limit to the difference in magnitude. PAGE agreed that this would be desirable, but in view of the lack of a definite luminosity function he had included in the present study all pairs with both members brighter than 18^m 5 in order not to impose an arbitrary limitation. He added that the earlier investigation of mean masses was based on much brighter objects, most of them less than 14^m , among which the expected proportion of chance pairs is only a few percent.

VORONTSOV-VELYAMINOV warned that the selection criterion $S \leq 3(A_1 + A_2)$ will include elliptical galaxies that in general are actually overlapping, since the diameters A_1 and A_2 measured on Palomar Atlas prints are underestimates of the actual diameters. For spirals this effect is much smaller, and the constant in the selection criterion should therefore be different for spirals and for ellipticals. PAGE said that the list of pairs counted can be revised to try various modifications of the selection criterion. SCHUECKING pointed out that the selection criterion S is also dependent on distance or redshift, and may need to be modified by a factor of almost 3 at 18^m . No such allowance for cosmological evolutionary effects was made, however.

CHRISTY asked whether the pairs counted could not all be due to chance, and PAGE admitted that the calculation of chance pairs, complicated by the change of selection criterion with magnitude, may need revision. In answer to NEYMAN and SCOTT, he stressed that Fig. 1 is a plot of total pairs counted, from which chance pairs (also proportional to N_1^2) must be subtracted. NEYMAN and SCOTT felt that the conclusions drawn after this subtraction is done need not coincide with the present conclusions.

McCREA pointed out that higher-order systems would also result from capture, modifying the N_1^2 correlation, and PAGE said that only pairs were counted in order to avoid complex mathematics. DE VAUCOULEURS recalled a tabulation published by Holmberg (1940) indicating that the number of galaxies in systems is proportional to $(\frac{1}{2})^n$, where n is the multiplicity ($n=1$ for isolated galaxies, $n=2$ for pairs, 3 for triples, etc.).

AMBARTSUMIAN criticized the correlation of *surface* densities, since a capture process would imply that the *space* densities would be correlated. PAGE claimed that if one could isolate the regions of high density, such as clusters, separate from foreground and background objects, the projection would make little difference, but admitted that the counts for different clusters should be treated separately, with different magnitude limits.

CARPENTER asked how positive energy could be

TABLE II. Summary of mass determinations (in solar units, reduced to $H=75$ km/sec/Mpc).

Galaxy	Type	Method ^a	Reference	$M/10^{10}$	M/L
M82	Irr	l.s.	Mayall, <i>Ann. d' Astrophys.</i> 23 , 344, 1960	1.5	7.4
NGC 55	Irr	HII	de Vaucouleurs, <i>Astrophys. J.</i> 133 , 405, 1961	~4	~6.
NGC 3556	Irr-Sc	l.s.(em)	BBP, <i>Astrophys. J.</i> 131 , 549, 1960	1.4	1.4
LMC	Irr-Sm	HII; 21 cm	de Vaucouleurs, <i>Astrophys. J.</i> 131 , 265, 1960	1.3	4.
NGC 3646	Sb-Sc(p)	l.s.(em)	BBP, <i>Astrophys. J.</i> 134 , 237, 1951	>20-30	>2.5-4.
NGC 2146	Scp	l.s.(em)	BBP, <i>Astrophys. J.</i> 130 , 739, 1959	1.8	3.
M33	Sc	21 cm	Volders, <i>Bull. Astron. Inst. Neth.</i> 14 , 323, 1959	1.8	11.
NGC 157	Sc	l.s.(em)	BBP, <i>Astrophys. J.</i> (in press)	6.0	1.9
NGC 3504	SBb	l.s.(em)	BBP, <i>Astrophys. J.</i> 132 , 661, 1960	0.3-0.9	≤1.
NGC 5055	Sb-Sc	l.s.(em)	BBP, <i>Astrophys. J.</i> 131 , 383, 1960	5.5	2.8
NGC 2903	Sb	l.s.(em)	BBP, <i>Astrophys. J.</i> 132 , 640, 1960	4.9	4.2
M101	Sc	21 cm	Volders, <i>Bull. Astron. Inst. Neth.</i> 14 , 323, 1959	1.4	13.
NGC 5005	Sb	l.s.(em)	BBP, <i>Astrophys. J.</i> 133 , 814, 1961	9.	6.
M31	Sb	21 cm	van de Hulst <i>et al.</i> , <i>Bull. Astron. Inst. Neth.</i> 14 , 1, 1957	34.	23.-10.
M81	Sb	HII	Münch, <i>Publs. Astron. Soc. Pacific</i> 71 , 101, 1959	15.	20.
NGC 3623	Sa-Sb	l.s.(em)	BBP, <i>Astrophys. J.</i> 134 , 232, 1961	20.-30.	10.-20.
Mean	S. Irr	16 Single galaxies	(all from rotation curves)	8.6	7.4
14 pairs	S. Irr	Circular orbits	Page, <i>Astrophys. J.</i> 132 , 910, 1960	2.8±1.3	0.25±0.17
M87	E	dispersion in V	Minkowski (unpublished)	100.	50.
NGC 3115	E	rotation curve	Minkowski and Oort (unpublished)	11.	19.
NGC 3379	E	dispersion in V	Fish and Burbidge, <i>Astrophys. J.</i> 134 , 251	10.	12.
NGC 4111	E	rotation curve	Humason and Oort (unpublished)	1.2	13.
Mean	E	4 Single galaxies		30.	24.
13 pairs	E, S0	Circular orbits	Page, <i>Astrophys. J.</i> 132 , 910, 1960	80. ±13.	71. ±20.

^a l.s. = long slit; l.s.(em) = long slit, emission lines; BBP = Burbidge, Burbidge, and Prendergast. HII = measure of individual HII regions; 21 cm = radio measure, by 21-cm H line.

dissipated in the capture process, tidal distortions either being inadequately minute, or in some cases not observable. PAGE thought the tidal effects of a capture might be expected to damp out in a few rotation periods. VON HOERNER felt that the dissipation of energy is a serious difficulty. He questioned whether an observed correlation between N_2 and N_1^2 necessarily implies a capture process since the unknown process

of forming double galaxies could possibly depend on the square of the density, as in the case of single star formation. In answer to G. BURBIDGE, it was mentioned by HOLMBERG that the relaxation time in a capture process depends on the velocity dispersion among the field galaxies. It was clear that further theoretical investigation is needed of captures of galaxies, and the resulting tidal distortions.

On the Dynamics of the Virgo Cluster

ERIK HOLMBERG

Uppsala Observatory, Uppsala, Sweden

A detailed analysis of the observational material available for the Virgo cluster of galaxies leads to the conclusion that the high redshift dispersion can be satisfactorily explained as a result of orbital motions (especially orbital motions within the subgroups of the cluster), systematic errors in the measured redshifts, and disturbance by optical members. Each of these three sources may be responsible for about one-third of the observed square dispersion. Accordingly, there is no need to introduce any additional dispersion-increasing effects, such as a rapid expansion of the cluster, or a strong gravitational field due to nonluminous intergalactic matter. The results are applicable also to other clusters and agglomerations of galaxies.

A detailed summary of the results, and of the conclusions to be reached, is given in Sec. 6.

1. OBSERVATIONAL MATERIAL

THE present investigation, based on an analysis of all the observational data available, gives the results of a theoretical study of the dynamical properties of the Virgo cluster of galaxies. The observational material refers to the redshifts, magnitudes, and types of the member galaxies; observational data also give information about the transverse extensions of the cluster, and about its distance modulus. It is a well-known fact that previous studies of the dynamics of the Virgo cluster (and of other clusters) have encountered serious difficulties in trying to find a satisfactory agreement between theory and observations, the dispersion in the redshifts of the cluster members seemingly being much too high. The main purpose of this paper is to investigate more closely the possibilities of finding an adequate explanation of the excessive redshift dispersion, that is, an explanation that is uncomplicated and free from any unwarranted assumptions. In the successive sections of the paper, the various dispersion sources will be analyzed separately in some detail.

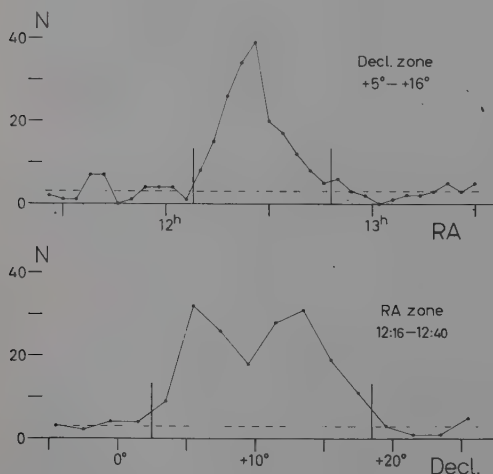


FIG. 1. Area covered by the Virgo cluster. The upper curve gives, for the declination zone $+5^\circ$ to $+16^\circ$, the number of galaxies in successive 4^m intervals of right ascension. The lower curve shows the corresponding distribution in declination for galaxies of the right ascension zone 12^h16^m to 12^h40^m . Both curves based on the Reinmuth catalogue (coordinates reduced to 1950).

In Table I we find, with two exceptions, all the galaxies which have been measured by Humason (Mt. Wilson-Palomar observatories) and which by him are referred to the Virgo cluster; on account of missing data as regards magnitudes, IC 3483 and Anon ($\alpha = 12^h 30^m 1$) have been omitted. The table also includes all the additional objects in the same area which have been measured by Mayall (Lick Observatory). The observed redshifts, as listed in the redshift catalogue (Humason *et al.* 1956), are contained in the last column of the table; in those cases where measurements are available from both Humason and Mayall, the arithmetical mean has been accepted. The second column gives the type of each galaxy, according to the redshift observer. The total photographic magnitudes in the third column have in order of preference, been taken from the catalogues by the writer (1958), by Stebbins and Whitford (1952), by Sandage (1956), and by Shapley and Ames (1932). The magnitudes by the writer (two decimals) and by Sandage are accepted without corrections; the S-W magnitudes have been corrected by $-0^m 1$ (diaphragm somewhat too small), whereas the S-A magnitudes (in brackets) have been reduced to a correct photometric system by a correction formula previously derived by the writer (1958).

Whereas most of the galaxies listed in Table I are part of the main body of the Virgo cluster, a few are located in the "southern extension." For some of the following analyses it is necessary to have access to a representative mean value as regards the radius of the cluster, and we thus have to determine the angular extensions of the main cluster body. For this purpose we may conveniently use the survey catalogue by Reinmuth (1926). As appears from Fig. 1, the statistical distribution curves obtained from this catalogue indicate that the main cluster body extends in right ascension from $12^h 8^m$ to $12^h 48^m$, and in declination from $+2^\circ 5'$ to $+18^\circ 5'$ (1950). The cluster thus seems to cover a more or less elliptical area of the sky with diameters of 10° and 16° , the minor diameter being parallel to the equator. It may be added that the shape of the distribution curves indicate that the space density (number of galaxies per unit volume) is approximately constant throughout the cluster.

As regards the distance of the Virgo cluster, the writer

TABLE I. Galaxies in the Virgo cluster area, for which redshifts and magnitudes are available. The successive columns list the type, the total photographic magnitude, and the observed redshift (km/sec).

NGC	Type	m_t	V	NGC	Type	m_t	V
4179	E	11.7	+1149	4517	Sc	11.10	+1098
4192	Sb	10.89	- 202	4519	Sc	12.22	1123
4212	Sc	11.71	+2047	4526	S0	10.6	357
4216	Sb	10.88	- 34	4527	Sb	11.29	1615
4254	Sc	10.37	+2390	4535	Sc	10.38	1925
4261	E	11.6	2094	4536	Sb	10.94	1819
4267	S0	11.9	1179	4546	S0	11.4	882
4270	E	13.13	2236	4548	Sb	10.86	372
4273	Sc	12.35	2191	4550	E	12.6	280
4281	S0	12.32	2492	4551	E	12.9	908
4293	Sa	(11.1)	693	4552	E	11.0	194
4303	Sc	10.01	1557	4567	Sc	11.98	2206
4321	Sc	10.07	1551	4568	Sc	11.66	2335
4324	Sa	(12.3)	1605	4569	Sb	10.11	896
4339	E	12.5	1173	4570	E	11.8	1640
4343	S0	14.16	614	4578	E	12.3	2201
4350	S0	11.9	1122	4579	Sb	10.32	1679
4365	E	11.0	1130	4594	Sb	9.18	1035
4374	S0	10.21	880	4621	E	11.0	339
4382	S0	10.05	721	4636	E	10.6	858
4387	E	13.0	439	4638	E	12.2	1011
4394	Sb	11.81	+ 720	4643	S0	(11.6)	1324
4406	E	10.10	- 418	4647	Sc	12.05	1379
4421	Sa	11.8	+1628	4649	E	9.88	1248
4425	Sa	12.84	1809	4660	E	12.1	950
4429	S0	11.09	1027	4665	Sa	(11.9)	684
4435	S0	11.86	+ 796	4666	Sc	11.40	1531
4438	Sap	10.92	- 105	4697	E	10.4	1177
4442	S0	11.61	+ 493	4698	Sa	11.56	955
4450	Sb	10.81	1995	4699	Sb	10.2	1370
4458	E	13.0	309	4713	Sc	(11.8)	574
4459	S0	11.5	1042	4742	E	12.5	1176
4461	S0	12.0	1813	4753	S0p	10.7	1256
4464	E	13.5	1104	4754	S0	11.6	1398
4467	E	15.5	1379	4762	Sa	11.0	870
4472	E	9.33	918	4775	Sc	(11.4)	1555
4473	E	11.3	2173	4781	Sc	(11.4)	751
4474	S0	12.7	1458	4856	Sa	11.4	1095
4477	S0	11.4	1195	4866	Sa	12.0	1862
4478	E	12.3	1410	4900	Sc	11.9	958
4479	S0	13.6	753	4941	Sa	12.0	729
Anon	E	15.4	1414	4958	E	11.5	1389
4486	E	9.56	1171	4995	Sb	11.9	+1715
4492	Sa	13.2	1642				
4501	Sc	10.07	+2060				

spiral objects in the cluster. For the third, the distance modulus adopted would lead to an agreement between the luminosity curve of Virgo cluster members and the luminosity curve representing members of the local group of galaxies and the groups around NGC 3031 and NGC 5457. It should be noted that the derived cluster modulus in the first place depends on the distance modulus adopted for NGC 224, and that any change in the latter modulus would lead to a corresponding change in the cluster modulus.

2. VELOCITY DISPERSION DUE TO ORBITAL MOTIONS

For the determination of the radial velocity dispersion corresponding to the orbital motions of the cluster members it is necessary to try to estimate the total mass of the cluster. Such an estimate may be based on the absolute luminosities of the individual members.

For galaxies of the same integrated color index (corrected for internal absorption), or of the same type, there seems to be a more or less constant ratio of mass to luminosity. According to an investigation by the writer (to be published) the mean ratios (solar units) amount to 80 (type E-S0), 20 (type Sa-b), and 2 (type Sc); the two latter values, of rather small importance in the present connection, are still somewhat uncertain. The results are based (a) on nearby galaxies for which rotational masses are available, (b) on the orbital motions of close double objects, and (c) on a theoretical study of the evolutionary changes in total luminosity of the stellar contents of extragalactic systems. The mass-luminosity ratios, being inversely proportional to the distances adopted, are calibrated on the distance scale defined by the Virgo cluster; that is, the results are based on the assumption that the apparent cluster modulus is 30.2 (cf. Sec. 1). It is interesting to note that, for the same distance scale, the result derived by Page (1960) for type E-S0 is practically the same as that obtained by the writer.

Of the galaxies listed in Table I, 52 are to be found within the elliptical cluster area as defined in Sec. 1 (12 suspected optical members omitted; cf. Table V). The distribution of these objects as regards types appears from Table II; the majority belongs to the class E-S0. The third column gives, for each type class, the total absolute photographic luminosity, in solar units; listed values are based on the apparent magnitudes of

TABLE II. Number of galaxies of different types in the main cluster area. Columns 3-5 list the total absolute pg luminosities (solar units), the adopted mass-luminosity ratios, and the resulting total masses (solar units).

Type	No.	ΣL	M/L	ΣM
E-S0	31	169×10^9	80	135×10^{11}
Sa-b	11	68	20	14
Sc	10	86	2	2
Sum: 52		323×10^9		151×10^{11}

has in a previous paper (1958) advanced several arguments pointing to an apparent distance modulus of about 30.2; with a mean galactic absorption in the cluster area of 0.26 mag., the distance of the cluster would thus be 9.7×10^6 pc. One of the arguments is that, with this distance modulus, the absolute total magnitudes of the very brightest spiral galaxies in the cluster would be nearly the same as the absolute magnitude of NGC 224 (dist. mod. assumed to be 24.25). Secondly, the neighbor galaxies NGC 224, NGC 598 (dist. mod. = 24.15), and NGC 3031 (dist. mod. = 27.1) would fit rather nicely into the correlation diagram of total magnitude versus surface magnitude obtained for

TABLE III. Mean-square redshift deviations (square dispersions) in a number of subgroups within the Virgo cluster. Unit = (km/sec)².

Subgroup	Observed square disp.	Corrected square disp.
4374-87, 4425	49×10 ⁴	25×10 ⁴
4435-58-61	59	54
4459-74-77-79	9	12
4464-67-72-92	10	3
4478-86, Anon	2	5
4550-51-52	15	16
4621-38-47-49-60	16	14
Means:	23×10 ⁴	18×10 ⁴

Table I, on a distance modulus of 30.2, and on an absolute pg magnitude of the sun equal to +5.37 (Stebbins and Kron 1957). With the mass-luminosity ratios given in column 4, the luminosities are transformed into the masses (solar units) of the last column. The total mass of all the 52 cluster members thus amounts to 1.5×10^{13} solar units. In a previous investigation (1958), the writer has determined the total apparent pg magnitude of *all* cluster members within the elliptical area defining the main cluster body; the result is $m_{\text{tot}} = +6.0$. With the above distance modulus, the total absolute magnitude thus amounts to -24.2 , corresponding to 673×10^9 solar units. Accordingly, the galaxies of Table II represent only 48% of the total cluster luminosity. If the fainter cluster members not included in the table are assumed to have the same distribution of types, the resulting total mass of the main body of the Virgo cluster would thus be 3.1×10^{13} solar masses.

The determination of the mean square redshift deviation (=square dispersion of the radial velocities) that corresponds to the orbital motions of the cluster members may, as is traditionally done, be based on the virial theorem of classical mechanics (Zwicky 1937). The application of the theorem has to be based on the assumptions (a) that the cluster is a stationary system, (b) that the cluster members are uniformly distributed in space, and (c) that the distribution of space velocities is spherically symmetrical. If the radius of the cluster (assumed to be spherical) is denoted by R , and the cluster mass by $\mathcal{M}_{\text{tot}}$, the virial theorem leads to a square dispersion in radial velocity V equal to

$$\langle V^2 \rangle = (4\pi^2/5R)\mathcal{M}_{\text{tot}}. \quad (1a)$$

The units are the solar mass, the a.u., and the year.

According to Sec. 1, the half-axes of the elliptical cluster area are 5° and 8°; in the present case we may accept the geometrical mean, or 6°3. With a distance of 9.7×10^6 pc, this radius corresponds to 2.2×10^{11} a.u. On the assumption that the extension of the cluster along the line of sight is comparable to the transverse extensions, the latter figure may be adopted to represent the parameter R . With the total cluster mass found above, the virial theorem then leads to a square disper-

sion in radial velocity of about $1100 \text{ (a.u./year)}^2 = 2.5 \times 10^4 \text{ (km/sec)}^2$. In view of the possibility that the cluster may contain a certain amount of intergalactic nonluminous matter, tending to increase the total mass, the result may be rounded off to $3 \times 10^4 \text{ (km/sec)}^2$.

The square dispersion derived by the above procedure is surprisingly small compared to the square dispersion, $39 \times 10^4 \text{ (km/sec)}^2$, of all the redshifts listed in Table I. Seemingly, there is thus a serious disagreement between the "dynamical mass" and the "luminous mass" of the cluster. A more detailed analysis leads, however, to the result that one of the main reasons for this conflict is an incorrect application of the virial theorem.

The theorem, as traditionally applied to clusters (and groups) of galaxies, is based on the tacit assumption that the gravitational force acting on a mass element in the cluster can be represented by a smooth, continuous function of the space coordinates; in the case of a uniform space distribution, the force would be proportional to the distance from the center of the cluster. Local irregularities in the mass distribution are thus not taken into account. However, the gravitational interactions between nearby galaxies cannot be neglected in the present case. An inspection of the Virgo cluster indicates that the space distribution of the members, although more or less uniform from a statistical point of view, is rather irregular, most objects being members of narrow double systems or groups. It is quite clear that the existence of such local concentrations will affect the analysis of the velocity distribution. For a cluster model of given size and given total mass it would apparently be possible to increase the orbital velocities by almost any amount by a suitable arrangement of the members in small groups. If, for instance, the cluster is assumed to be composed of a number of equally large subgroups (radius= r_1 ; population= n_1), and if each of these is divided in a number of subgroups of the second order (radius= r_2 ; population= n_2), and so on, the above equation apparently would have to be replaced by

$$\langle V^2 \rangle = \frac{4\pi^2}{5} \langle \mathcal{M} \rangle \left(\frac{N}{R} + \frac{n_1}{r_1} + \frac{n_2}{r_2} + \dots \right), \quad (1b)$$

the quantity $\langle \mathcal{M} \rangle$ representing the mean mass of the cluster members. The sum of the second and following terms within the brackets could obviously be much larger than the first term.

The complicated structure of the Virgo cluster, the detailed features of which are inaccessible on account of the difficulty in separating the fainter cluster members from the background, apparently cannot be adequately represented by a simple theoretical expression. It seems, however, possible to use the observational data for an estimate of the increase in the velocity dispersion that is caused by the grouping tendencies.

In Table III, the *internal* redshift dispersions have been computed for a number of subgroups (at least

three members) identified in the cluster. The square dispersions, as derived from the redshifts listed in Table I, are given in the second column. The members selected for each group, which are denoted by their NGC numbers (suspected optical cluster members omitted; cf. Table V), are situated quite close together, and it seems likely that they are gravitationally associated. The mean square redshift dispersion amounts to 23×10^4 (km/sec)². If the observed redshifts are corrected for systematic errors (cf. Sec. 4), the mean square dispersion is reduced to 18×10^4 (column 3). Since only part of the systematic errors can be removed by the correction procedure used by the writer (cf. Sec. 6), it seems however likely that the final result will be of the order of 13×10^4 (km/sec)². The velocity dispersion within the individual groups is thus considerably larger than the dispersion derived by means of Eq. (1a). For comparison, it may be noted that an investigation of physical pairs of galaxies outside the Virgo cluster (unpublished paper), with a distribution of types corresponding to that of Table II, gives a square radial velocity dispersion due to orbital motions of 6×10^4 (km/sec)². In view of the fact that the groups of Table III contain 3–5 members, the square dispersion derived above does not seem to be unreasonably large.

Some information about the grouping tendencies in the Virgo cluster can be obtained by a statistical analysis of the distributions of galaxies within circular survey areas centered on cluster members. As centers of the survey areas we may conveniently choose those objects of Table I that are located in the central part of the main cluster area (a border zone with a width of 15' being left out), whereas the Reinmuth catalogue (1926) may be used to supply the data. Figure 2 gives the combined result as obtained from altogether 44 survey areas with a radius of 2°. The total number of galaxies in circular strips, corresponding to distance intervals of 10', ranges from 27 (0'–10') to 190 (110'–120'). For distances larger than about 30' the statistical distribution seems to be a more or less uniform one, and the observed frequencies have accordingly been represented by a straight line. For distances smaller than 30' there is apparently an excess of objects, the difference between the observed number (full line) and the number corresponding to a uniform distribution (dashed line) amounting to 73. We thus arrive at the conclusion that, on an average, each of the 44 selected cluster members has 1.7 physical companions (listed in the Reinmuth catalogue).

The results of the above analyses seem to indicate that the mean square radial velocity dispersion within the subgroups of the Virgo cluster has a numerical value between 6×10^4 and 13×10^4 (km/sec)², the most probable value being approximately 10^5 (km/sec)². The total square dispersion corresponding to all kinds of orbital motions of the cluster members is thus about four times larger than the square dispersion derived from Eq. (1a).

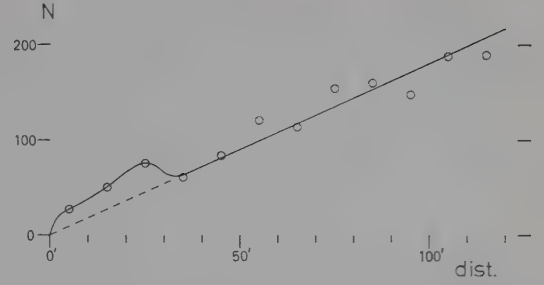


FIG. 2. Grouping tendencies in the Virgo cluster. Distribution of galaxies in circular survey areas (radius = 120'), centered on 44 objects belonging to the redshift list.

3. VELOCITY DISPERSION AS A RESULT OF EXPANSION

An expansion (or contraction) of the Virgo cluster would naturally tend to increase the dispersion in the radial velocities of the member galaxies. It is, however, easily shown that a moderately fast expansion would not change the observed dispersion by any appreciable amount. In fact, an expansion of the cluster at a constant rate from a very contracted size to the present volume during a time interval corresponding to the universal time scale could not be established from the observational data available, the increase in the dispersion being negligible compared to the mean error of the observed dispersion.

In order to demonstrate the effect of an expansion, we will start from the rather natural assumption that the space velocity due to expansion W of a cluster member is proportional to the distance r from the center of the cluster; thus $W = kr$. On the additional assumption that the space density in the cluster is constant, which implies that the statistical distribution of the distances r can be represented by a function proportional to r^2 , we find

$$\langle W^2 \rangle = \int_0^R k^2 r^2 \cdot r^2 dr / \int_0^R r^2 dr = \frac{3}{5} k^2 R^2. \quad (2a)$$

The cluster radius (cluster assumed to be spherical) is denoted by R . On the condition that the distribution of space velocities is spherically symmetrical, the mean square radial velocity is then obtained from the relation

$$\langle V^2 \rangle = \frac{1}{3} \langle W^2 \rangle = \frac{1}{5} k^2 R^2. \quad (2b)$$

In the above expression, the quantity kR represents the outward space motion of an object located at the edge of the cluster, or the maximum expansion velocity. If the radius of the cluster has increased at a constant rate during the expansion, this maximum velocity apparently is equal to $R/(t_1 - t_0)$, where t_1 represents the present epoch, and t_0 the point of time when the expansion started (original size of the cluster supposedly very small). Thus, we arrive at a radial velocity dispersion,

$$\sigma_V = 0.45 R / (t_1 - t_0). \quad (2c)$$

TABLE IV. Mean observed redshifts (km/sec), with mean errors, for different types of members of the Virgo cluster.

Type	No.	Mean redshift
E-S0	41	+1002± 61
Sa-b	21	1276 108
Sc	14	1669 137
All	76	+1200± 54

As in the preceding section, we will assume that the angular radius of the Virgo cluster is 6.3 . With a distance of 9.7×10^6 pc (cf. Sec. 1), this radius corresponds to 1.1×10^6 pc. If $t_1 - t_0$ is put equal to the universal time scale, Eq. (2c) leads to a very small "expansion dispersion"; thus, for $t_1 - t_0 = 10^{10}$ years, σ_V will be 4.9×10^{-5} pc/year = 50 km/sec. If, on the other hand, the cluster expansion is assumed to be responsible for the main part of the high velocity dispersion observed, the expansion time $t_1 - t_0$ will be extremely short. For $\sigma_V = 600$ km/sec, the expansion time would be only 8×10^8 years.

The above numerical analysis, the main results of which are applicable also to other clusters, seems to indicate that, in all probability, the effects of a possible expansion can be entirely neglected in our attempts to find an adequate explanation of the excessive velocity dispersion in the Virgo cluster. An explanation based on the expansion hypothesis would lead to an impossibly short lifetime of the cluster. Generally, the observed frequency of clusters and agglomerations in our extragalactic neighborhood is, according to the writer's opinion, not compatible with the assumption of lifetimes of the order of 10^9 years.

4. VELOCITY DISPERSION RESULTING FROM SYSTEMATIC REDSHIFT ERRORS

A detailed analysis of the redshift data available for members of the Virgo cluster definitely indicates the presence of very peculiar systematic effects. As will be shown below, it seems highly probable that a considerable part of the excessive redshift dispersion can be explained as a result of systematic errors in the observations.

Already a superficial examination of the redshifts listed in Table I reveals the unexpected fact that the measured redshift apparently depends on the type of the galaxy; the redshift increases systematically as the type changes from E-S0 over Sa-b to Sc. According to the means listed in Table IV there is a difference of almost 700 km/sec between classes E-S0 and Sc, a difference that can hardly be explained as a result of accidental deviations. It should be noted that in the computation of the tabulated means, 12 suspected optical members (cf. Table V) have been omitted; the result would however be more or less the same even if the latter objects were included.

The systematic redshift deviations apparently cannot

be explained by assuming that the cluster is rapidly expanding. We would in that case have to make the additional assumption that members of type E-S0 have expansion velocities directed towards the sun, whereas members of type Sc have velocities pointing in the opposite direction. Accordingly, early-type galaxies would be confined to the nearer part of the cluster, and late-type galaxies to the back regions.

According to the writer's opinion, the most reasonable explanation of the above systematic effect is offered by the assumption that the measured displacements of the spectral lines are affected by systematic errors. Since the Mt. Wilson-Palomar and Lick redshift observations represent a rather heterogeneous material, the observing conditions, instrumental equipment, measuring procedure, etc. varying within quite wide limits, the systematic errors may have their origin in a number of different, possibly complex causes. It would apparently not be possible to reveal all these causes simply by a statistical analysis of the catalogued redshift data. In a first approximation, we have to be satisfied if we can find *one* observational parameter that is, in a statistically significant way, related to the hypothetical redshift errors. Since the central surface luminosity of galaxies of different types increases considerably along the sequence Sc over Sa-b to E-S0, the results of Table IV definitely point to the *intensity of the light* (reaching the spectrograph plate) as an important parameter in this connection. The systematic redshift errors would thus be interpreted as a magnitude equation of some kind.

The redshift observations in practically all cases refer to the nuclear regions of the galaxies, and in order to determine the intensity of the light entering the spectrograph slit (width of slit not taken into account) we are thus faced with the problem of determining nuclear surface magnitudes. A reasonably good estimate of the sought magnitude, sufficient at least for statistical purposes, may be obtained if we assume that, for a given type of galaxy, the central luminosity corresponding to the effective aperture of the average spectrograph slit is a constant fraction of the total luminosity of the object. The magnitude would thus be equal to $m_t + k$, where m_t represents the total magnitude and k is a function of the type. The latter relation may be determined by means of the comprehensive photometric material brought together by the writer (1958); we arrive at the approximate results that, on an average, $k_{\text{Sa-b}} = k_{\text{E-S0}} + 1$, and $k_{\text{Sc}} = k_{\text{E-S0}} + 4$. Thus,

$$m' = m_t \text{ (type E-S0), } m' = m_t + 1 \text{ (type Sa-b),}$$

$$m' = m_t + 4 \text{ (type Sc),}$$

where m' , apart from a constant (not needed in the subsequent analysis), represents the sought nuclear magnitude. It should be emphasized that the quantity m' , although it proves to be a valuable statistical tool in the redshift analysis, gives only a very approximate numerical measure of the nuclear luminosity.

We now turn to Fig. 3, where the observed redshifts of the 88 galaxies in the Virgo cluster area have been plotted against the magnitude m' ; it may be recollected that the redshifts are listed in Table I. For galaxies of type E-S0 the magnitude $m' (=m_i)$ ranges from 9.3 to 15.4, whereas for types Sa-b ($m' = m_i + 1$) and Sc ($m' = m_i + 4$) the intervals are 10.2–14.2 and 14.0–16.4, respectively. In spite of the large redshift dispersion, a rather well-defined correlation between V and m' is indicated; on an average, the redshift increases by about 700 km/sec as the nuclear magnitude changes from 10 to 16. If all the 88 galaxies are included, a least-squares solution gives the following result (with mean errors) as regards the mean observed redshift corresponding to a given magnitude:

$$V = 888 + 119(m' - 10). \quad (3a) \\ \pm 115 \pm 36$$

Among the 88 galaxies in the Virgo cluster area we have to expect a certain number of optical members. A separate discussion of possible foreground and background objects is found in the next section; it is shown that 12 of the 88 galaxies must be listed as very doubtful cases. If these objects are omitted (denoted by crosses in the figure) the above relation is changed to

$$V = 905 + 114(m' - 10). \quad (3b) \\ \pm 98 \pm 31$$

In the figure, this relation is reproduced by the straight line. The significance of the result may be judged by means of the open circles, which represent the mean redshifts in five equally large classes; the diameter of each circle is equal to twice the mean error of the class mean.

The above result indicates that the primary sources of redshift error, so far unknown, in some way are correlated with the nuclear magnitude m' . It would naturally be of a certain interest to examine whether, statistically speaking, a better result could be obtained if m' is combined with other observational data, as, for instance, the apparent diameters. Since photometrically measured diameters are not available for all the objects of Table I, we will accept the apparent dimensions as

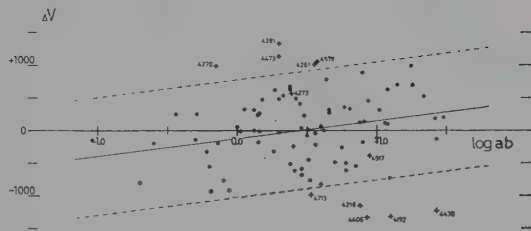


FIG. 4. Relation between redshift residual (as obtained from Fig. 3) and log product of apparent diameters; 12 suspected optical members denoted by NGC numbers. The broken lines represent deviations of ± 900 km/sec from the mean relation.

listed by Reinmuth (1926) in his catalogue of the Herschel nebulae; although these diameters are estimated quantities, they supposedly form a rather homogeneous system. In Fig. 4 the residual redshifts ΔV , or $V - 905 - 114(m' - 10)$, have been plotted against $\log ab$, the quantities a and b representing the apparent diameters in minutes of arc. As may have been expected, the figure indicates a correlation, the mean relation (12 optical members omitted) being $\Delta V = +109 d$; the quantity d stands for $(2.5 \log ab - 2.5 \langle \log ab \rangle)$. We thus arrive at the conclusion that a somewhat more pronounced correlation would have been obtained if, in Fig. 3, the magnitude m' is replaced by $m' + d$. In fact, a least-squares solution gives the result

$$V = 873 + 127(m' - 10 + d). \quad (4) \\ \pm 87 \pm 27$$

Within the limits of the accidental errors, the numerical parameters of this relation agree with those of the previous equation. It may be noted that the inclination factor 127 is almost five times larger than its mean error, which indicates that the result can be accepted as statistically significant.

By the above correction procedure, and by the exclusion of 12 projected objects, the dispersion in the redshifts of the Virgo cluster members has gone down from about 625 to about 450 km/sec, that is, the square of the dispersion has been reduced by 50%.

It should be noted here that the existence of a systematic error (related to m') in the observed redshifts has been established not only for the Virgo cluster but also for other groups of galaxies. We beg to refer to a previous investigation by the writer (1961).

5. DISTURBANCE CAUSED BY OPTICAL CLUSTER MEMBERS

The elimination of projected objects will naturally reduce the redshift dispersion computed for a given cluster. As will be shown below, this source of error cannot be neglected in the case of the Virgo cluster. Since the apparent area covered by a cluster is inversely proportional to the square of the distance, whereas the number of field galaxies per square degree is directly proportional to the third power, the disturbance caused

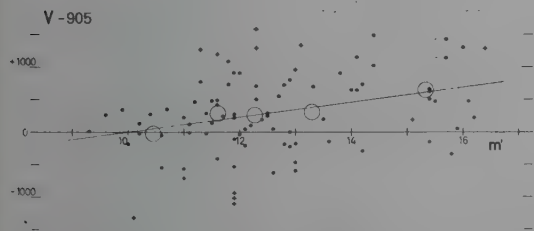


FIG. 3. Relation between observed redshift (red. by 905 km/sec) and nuclear magnitude m' , as derived for 76 nebulae in the Virgo cluster area; 12 suspected optical members denoted by crosses. The mean redshifts in five equally large classes are represented by open circles (radius = mean error).

TABLE V. List of galaxies in the Virgo cluster area suspected of being foreground objects (upper part), or background objects (lower part). The distance moduli contained in the last column are based on the surface magnitudes derived by the writer (dist. mod. of the cluster = 30.2).

NGC	Type	m_t	V_{obs} (km/sec)	Dist. mod.
4192	Sb	10.89	— 202	29.2
4206	Sc	12.69	...	29.4
4216	Sb	10.88	— 34	29.4
4406	E	10.10	— 418	(28.6)
4438	Sap	10.92	— 105	28.6
4517	Sc	11.10	+1098	28.1
Reinm. 80	Sc	12.52	...	29.4
4713	Sc	(11.8)	+ 574	...
4261	E	11.6	+2094	(31.9)
4270	E	13.13	+2236	(31.9)
4273	Sc	12.35	+2191	31.9
4281	S0	12.32	+2492	(31.9)
4473	E	11.3	+2173	...
4578	E	12.3	+2201	...

by projected objects will, however, be more serious for more distant clusters.

Before trying to sort out the background and foreground galaxies that may have been included in Table I, we will, as a preliminary step, analyze the problem from a statistical point of view. In a previous photometric investigation (1958), the writer has determined (a) the statistical distribution function for the apparent magnitudes of galaxies in the general field (outside clusters), and (b) the corresponding distribution for the brighter Virgo cluster members ($m < 13.5$). By combining these data we are able to derive, for each object of a given magnitude in the cluster area, the statistical probability of its being a physical cluster member. The computation gives the result that of the 88 galaxies listed in Table I about 14%, or 12 objects, may be expected to belong either to the foreground or the background. As regards the foreground, we may also approach the problem in the following way. Assuming a cluster distance of 9.7×10^6 pc, and an extension of the cluster along the line of sight comparable to the transverse extensions, we arrive at a foreground volume of about 20×10^{18} pc³. On the other hand, the above magnitude distribution curve referring to field objects, combined with certain reasonable assumptions as regards the mean and the dispersion of the absolute luminosities of galaxies [$\bar{M}_m = -19.3$; cf. Table 6 in Holmberg (1961)], leads to a space density (outside clusters) in the nearer parts of the extragalactic universe of approximately 10^{-18} galaxies per pc³. Accordingly, the total number of galaxies located in front of the Virgo cluster would be about 20. Since only an estimated 40% of these would fall within the magnitude range of the redshift list, the number of objects likely to be picked up by the redshift observers is reduced to 8. Thus, possibly half the number of projected galaxies may belong to the foreground.

With the results of the above analysis as a guide, we

will now try to pick out the individual optical members. As appears from Table V, six objects from the redshift list (plus two objects not in the list) have been referred to the foreground, and six to the background. The selection has been based on (a) the distance moduli as estimated from photometric data (fifth column; only spirals), and (b) the observed redshifts (fourth column). In the photometric investigation referred to above, the writer has shown that there is, for spiral galaxies, a rather pronounced correlation between total absolute magnitude and surface magnitude (corr. for inclination effect). Thus, the latter quantity may be used as an indicator of absolute luminosity, the mean error of the absolute magnitude being about 0.8. On the assumption that the apparent distance modulus of the cluster is 30.2, the absolute magnitudes lead to the individual distance moduli given in the table. As regards the redshifts, we will refer to Fig. 4, where the objects of Table V are denoted by crosses and NGC numbers. In all but two cases, the suspected galaxies fall outside the dotted lines that represent distances from the mean relation equal to twice the dispersion (disp. = 450 km/sec).

A few explanations may be needed to justify the selection of the objects listed in the table. It should be noted that 7 of the 14 objects are located close to the western boundary of the cluster area, a region where the density of cluster members is comparatively low. This is the case with NGC 4192; the position, combined with the low distance modulus, the small redshift, and the large major diameter, definitely indicates the foreground. The same arguments apply to NGC 4216, which very likely forms a double system together with NGC 4206 (sep. = 12'). Another double system, also referred to the foreground, is probably formed by NGC 4406–38 (sep. = 23'), two galaxies that are quite outstanding as regards total magnitudes and major diameters. It seems quite clear that the two spirals, NGC 4517 and Reinmuth 80 (sep. = 17') are also physically connected. The assumption of nonmembership (foreground) is in this case based on the two distance moduli, 4517 fitting quite well into the diagram of Fig. 4; it should also be pointed out that 4517 has a larger major diameter than any other Sc spiral in the cluster area (Holmberg 1958). Since no distance modulus is available for NGC 4713 (not included in the writer's photometric catalogue), the assumption of nonmembership is in this case based on the observed redshift; it may however be noted that this object is located close to the eastern edge of the cluster area. As regards the suspected background objects, NGC 4261–70–73–81 apparently form a physical group; the high distance modulus (for 4273), and the large redshifts, seem to exclude the possibility of cluster membership. Finally, NGC 4473 and 4578 have been referred to the background on account of the large redshifts.

By the exclusion of the above 12 objects from Table I, the observed redshift dispersion is reduced by almost 20%. The result shows that even for a nearby cluster

TABLE VI. Contributions to the square redshift dispersion from different sources.

Source	Square disp.
Optical members	13×10^4 (km/sec) ²
Orbital motions I	3
Orbital motions II	$10 \pm$
Expansion	$0 \pm$
Systematic errors I	6
Systematic errors II	$6 \pm$
Accidental errors	1
Sum: 39×10^4 (km/sec) ²	

the disturbance caused by optical members cannot be neglected. If the Virgo cluster had been located 10 times further out in space, in which case the number of projected objects would have surpassed the number of physical members, a reliable determination of the velocity dispersion in the cluster would apparently not have been possible.

6. SUMMARY AND CONCLUSIONS

According to the numerical analyses of the preceding sections, the excessive redshift dispersion found for the Virgo cluster can be explained as the combined result of a number of different effects. The most important of these are orbital motions (especially orbital motions in the small subgroups within the cluster), systematic errors in the measured redshifts, and disturbance by optical members that are inadvertently included in the material. It seems, in fact, possible to explain the entire amount of the observed dispersion as a result of the three effects mentioned. Accordingly, there is no need to introduce any additional dispersion-increasing effects, such as a rapid expansion of the cluster, or a strong gravitational field due to nonluminous intergalactic matter.

The different results obtained in this investigation have been summed up in Table VI. Since the square of the total (observed) redshift dispersion must equal the sum of the squares of the partial dispersions, it has seemed appropriate to tabulate the squared dispersion values. The dispersion of the redshifts of all the 88 galaxies listed in Table I amounts to about 625 km/sec, and the square of the total dispersion is thus equal to 39×10^4 (km/sec)².

According to the table, the *optical members* included in the material are responsible for one third of the total square dispersion. The tabulated value represents the difference between the square dispersion of all the 88 redshifts and that corresponding to the 76 objects accepted as physical members (cf. Sec. 5). Obviously, the elimination of optical members is a most important step in all analyses of the redshifts of cluster galaxies. The sorting out of projected objects will unfortunately meet with very large difficulties in the case of distant clusters. Even in the Virgo cluster it has naturally not been possible to identify the optical members beyond possibility of doubt.

The tabulated square dispersion corresponding to *orbital motions I* represents the radial velocity dispersion corresponding to the orbital motions (virial theorem) that would be expected if no attention is paid to the grouping tendencies within the cluster, that is, if the local irregularities in the distribution of gravitational potential are neglected. The determination of this dispersion, which is based on a total mass as derived from the absolute luminosities of the individual cluster members, is described in Sec. 2. The small size of the dispersion (compared to the total dispersion) shows that it is practically impossible to derive the total mass of a cluster by means of the observed redshift dispersion.

The square dispersion corresponding to *orbital motions II* represents the orbital motions within the numerous small groups in the cluster. It is, according to Sec. 2, estimated at 10×10^4 (km/sec)². The estimate is somewhat uncertain, the redshift material being too limited to permit a detailed study of the dynamics of the subgroups; there seems however to be no doubt that the dispersion is larger than that corresponding to "orbital motions I."

The square dispersion referring to an *expansion* of the cluster has in the table been put equal to $0 \pm$. If the cluster is assumed to participate in the general expansion of the universe, the resulting radial velocity dispersion would, according to Sec. 3, be considerably less than 100 km/sec. It seems to the writer that, as long as no direct observational evidence is forthcoming, the hypothesis of a rapid disintegration represents an unwarranted complication. As was stated above, the present analysis does not furnish any evidence indicating a disintegration.

The dispersion source listed as *systematic errors I* represents the systematic errors analyzed in Sec. 4. According to Eq. (4) the observed redshifts are affected by systematic errors that, apart from a constant, can be approximated by $+127(m' + d)$. The systematic errors apparently give rise to an increased redshift dispersion, the increase in the square dispersion being equal to $(127)^2$ multiplied by the square dispersion of $(m' + d)$; the result is 5.9×10^4 (km/sec)². Since it has so far not been possible to ascertain the true nature of the systematic errors (Holmberg 1961), it is quite obvious that the numerical results obtained in Sec. 4 only represent a first approximation. Accordingly, the computed increase in square redshift dispersion is certainly too small; if the true radial velocities were known, it would probably appear that the increase is at least twice as large. The probable increase in the square redshift dispersion due to unknown and so far inaccessible systematic errors, assumed to be of the same order of size as the increase corresponding to "systematic errors I," is in the table listed under the title *systematic errors II*.

Finally, it is also necessary to consider those errors in the observed redshifts that are of a purely accidental nature. According to Mayall (1956), a comparison between Lick and Mt. Wilson-Palomar redshifts meas-

ured for the same objects indicates a mean error in one determination of 60–70 km/sec. This result probably represents a minimum value, since the accidental errors of the two lists undoubtedly are to a certain degree correlated. In Table VI (*accidental errors*), the square mean error has been estimated at 10^4 (km/sec)².

Some of the square dispersions listed in Table VI, although presumably representing the best approximations that can be arrived at for the present, are necessarily uncertain and may have to be corrected when further observational data become available. It should however be pointed out that no analysis of the velocity dispersion in a system of galaxies, whether a cluster or a small group, can be considered as complete if not all the dispersion sources listed in the table have been taken into consideration.

Since the sum of the partial square dispersions listed in Table VI is equal to the square of the total (observed) redshift dispersion, there is obviously no need to introduce any additional dispersion sources in order to reach a satisfactory agreement between theory and observation as regards the dynamical conditions in the Virgo cluster. We especially like to point out: (1) that the assumption of an exceptionally large additional mass in the form of intergalactic gas in the cluster (to increase the dispersion corresponding to "orbital motions I") apparently is not supported by the observational data; and (2) that the hypothesis of a very rapid disintegration of the cluster (to increase the dispersion corresponding to "expansion") also seems to be unwarranted. It can in any case be stated that assumptions of this kind, although not disproved by the observations, represent altogether unnecessary complications.

In conclusion, it should be remarked that the inferences reached above as regards the Virgo cluster are applicable also to other clusters of galaxies. The tabulated square dispersions will naturally not be the same. In the Coma cluster, for instance, we may expect the dispersion corresponding to "systematic errors" to be smaller, the redshift observations referring to early-type galaxies of more or less the same apparent magnitudes. On the other hand, the "orbital motions II" will probably be more pronounced on account of the high space density in the cluster. The disturbance caused by optical members will also be much more troublesome than in the Virgo cluster; the relative number of projected objects may very well be so high, and their identification so difficult, that a reliable determination of the redshift dispersion for cluster members proves practically impossible. In order to attain any results approaching significance, a statistical analysis of the dynamics of the Coma cluster (and other distant clusters) has in any case to await the accumulation of a more complete redshift material.

REFERENCES

- Holmberg, E. 1958, *Lund Obs. Medd.* II, 136.
 —. 1961, *Uppsala Obs. Medd.* 138.

- Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *Astr. J.* 61, 97.
 Page, T. 1960, *ibid.* 65, 566.
 Reinmuth, K. 1926, *Heidelberg Akad. Wissensch. Abh.* 13.
 Shapley, H. and Ames, A. 1932, *Harvard College Obs. Ann.* 88,
 Stebbins, J. and Whitford, A. E. 1952, *Astrophys. J.* 115, 284.
 Stebbins, J. and Kron, G. E. 1957, *ibid.* 126, 266.
 Zwicky, F. 1937, *ibid.* 86, 217.

DISCUSSION

It was agreed to postpone discussion of the systematic errors in Humason's and Mayall's radial velocities to IAU Symposium No. 15 (Proceedings, in press). SCOPE pointed out that the studies of NEYMAN and SCOPE (reported above) assigned quite a different cause to the positive correlation between measured velocity and magnitude; that is, this correlation is due to expansion of the cluster rather than systematic error. Furthermore, in about half of the eight clusters and fifteen groups studied the correlation between measured velocity and magnitude is *negative*, interpreted as contraction, and in the other half positive. Such a switch in sign appears inconsistent with any large systematic errors in velocities positively correlated with magnitude. DE VAUCOULEURS asked what would happen to the average masses derived from differential velocities of double galaxies if the large systematic errors are taken into account and HOLMBERG answered that he has recalculated PAGE's masses and found little difference because the difference in magnitude between galaxies in a pair is generally small. PAGE added that his own velocity differences for 30 pairs generally depend on measures of emission lines which would not be expected to suffer the *same* systematic errors as absorption-line measures. Moreover, the error analysis reported in his full paper (*Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Vol. III, in press) showed no inconsistencies between his measures and those of Humason and Mayall.

DE VAUCOULEURS asked how the alleged systematic errors would affect the velocity dispersion in the Coma cluster, for which about 50 velocities are now known. HOLMBERG replied that, again, the dispersion in magnitude is small in the Coma cluster, so the systematic error in V would be minimal, but that foreground objects would undoubtedly account for the apparent instability. To this BURBIDGE responded that, if the Coma cluster is a member of a supercluster, foreground objects should also be members, and the stability of the supercluster comes into question.

SPINRAD and SCHMIDT asked how the value $M/L = 0.8$ was calculated for elliptical galaxies from the theory of star formation. HOLMBERG said that the study is as yet unpublished; he uses Salpeter and Schmidt's distribution of initial masses and ageing process, and derives M/L as a function of color index. He admitted that there are difficulties in correcting for interstellar absorption in spirals, and other difficulties in the case of ellipticals.

Recent Studies of Clusters and Superclusters

G. DE VAUCOULEURS

Department of Astronomy, University of Texas

1. SUPERCLUSTERING

THE question of the stability or instability of groups and clusters of galaxies is closely related to the problem of the existence or nonexistence of second-order clustering. More precisely, we are concerned with the relative sizes of ordinary clusters and the large-scale density fluctuations in the universe. Clusters are isolated, i.e., are imbedded in a statistically uniform distribution of so-called "field" galaxies and only known forces determining their stability are the collective gravitational attraction of the cluster members and possibly the presumed universal cosmic pulsion, if gas and radiation pressure can be neglected at the present time.

If, on the other hand, clusters are not isolated and independent units, but are merely condensations in larger systems or superclusters, their dynamics may be compared to that of star clusters in the general gravitational field of the Galaxy; in particular, if the supercluster is centrally condensed and in a state of differential rotation and expansion shearing and tidal forces may be operative which tend to reduce the stability of subunits. It should be well understood that the analogy with the Galaxy is merely illustrative and that the over-all central condensation, general rotation, if any, and possible expansion refer only to a smoothed-out model. Because of the smaller population ($\sim 10^4$ – 10^6 galaxies per supercluster vs $\sim 10^{10}$ – 10^{11} stars per galaxy) and larger random velocities (~ 100 – 1000 km/sec vs ~ 100 km/sec) the internal structure of the supercluster is expected to be essentially irregular and to exhibit considerable subclustering. However, it is not a complete chaos and its mean density is still higher than the over-all average density of the observable region.

This, in fact, is the main difference between the two viewpoints; if clusters are the largest organized agglomerations of matter in the universe, the large-scale distribution of matter can be considered as already statistically uniform and isotropic in volumes of space of the order of 10 Mpc³, since typical cluster diameters are of the order of 1 to 2 Mpc. The distance scale used is such that at large distances $H = 150$ km sec⁻¹ Mpc⁻¹. If on the other hand, the largest recognizable agglomerations of matter are superclusters with typical diameters of the order of 20 or 30 Mpc, the presumed uniform and isotropic large-scale distribution will not emerge until volumes of space of the order of 100 Mpc³ are considered. If so, considerable nonuniformity and anisotropy will prevail in regions of radius less than 10 Mpc, as indeed is actually observed.

The argument about simple clustering vs super-

clustering is to some extent based on a misunderstanding and reminds one of the futile debates of the 1930's between the concepts of a general field and of extensive clustering. It is largely a question of scale, so that qualitative pronouncements are almost meaningless. The concept of a general field of isolated galaxies was the product of the small-field Newtonian telescope; the concept of a cluster field arose from the coverage of the Schmidt telescope and wide-field astrograph. In the first case we were dealing with characteristic volumes of the order of 0.5 Mpc³, in the second we are considering volumes of the order of 5 Mpc³.

Now we have several independent lines of evidence pointing to the reality of large-scale irregularities of the distribution of groups and clusters of galaxies in volumes of the order of 50 Mpc³. This evidence comes from:

(1) the analysis of the distribution of galaxies brighter than the 14th mag. in both hemispheres (de Vaucouleurs 1956a, 1960a) [additional supporting evidence from counts to the 16th mag. in the northern galactic hemisphere has been recently reported by Carpenter (1961)];

(2) the large transverse gradients disclosed by Harvard counts (Shapley 1938) to the 17th mag. in the southern galactic hemisphere;

(3) the statistical analysis of the Lick counts of individual galaxies to the 18th mag. in the northern hemisphere (Shane and Wirthanen 1954; Neyman, Scott and Shane 1956);

(4) the statistical analysis of the Palomar counts of distant clusters to the 20th mag. (Abell 1958).

It is remarkable that all four approaches lead to essentially the same characteristic length $2R \approx 30$ Mpc (on the present distance scale) for the maximum size of the large-scale irregularities.

There is at present no evidence for clustering of the third order, either because any such supersuperclustering would be on a scale of the same order of size as the observable region (available counts stop at $m = 18$ to 20 or $2R = 200$ to 500 Mpc) or perhaps because such a region would be too large for gravitational instability to be significant on the time scale available (van Albada 1960).

2. CLUSTER TYPES

Our data are still too incomplete to permit a complete, detailed classification of clusters of galaxies; it is clear, however, that groups and clusters of different types exist which can be distinguished by differences in size, structure, and galaxy population. We recognize the following types.

A. Clusters of Spirals and Irregulars

(1) Small, low-density groups of spirals and irregulars (with occasional dwarf elliptical companions) having typical diameters of the order of 0.5 Mpc; the total population is a dozen or less (excluding the very faint dwarfs, which are invisible, except in the nearest systems, and presumably contribute very little to the total mass); the velocity dispersion is of the order of 200 km/sec. Very often the center of the group is empty, most of the bright members being scattered along its periphery. Examples are the Local Group (Humason and Wahlquist 1955; Kahn and Woltjer 1959), the Sculptor group (de Vaucouleurs 1959), the NGC 1023 group, and several of the nearby groups in Leo, Ursa Major, etc. These groups are clearly not in stable statistical equilibrium; they are comparable to expanding associations of stars and have expansion ages of the order of 10^9 years or less (de Vaucouleurs 1959, 1960b).

(2) Large elliptical clouds of spirals and irregular galaxies with very few or no ellipticals or lenticulars are characterized by a nearly uniform surface density out to the sharply defined edge of the system; there is no central condensation and occasionally an actual vacancy in the center. Diameters are of the order of 2 or 3 Mpc, with minor axes perhaps one third of the diameters, the total population in the first three magnitudes is a few dozen, the velocity dispersion is of the order of 300 km/sec. Examples are the Ursa Major cloud (Morgan 1959), the Canes Venatici cluster (van den Bergh 1960a), the cloud in Grus (de Vaucouleurs 1956b) and the "S" component of the Virgo cluster (de Vaucouleurs 1961; cf. Sec. 3).

B. Clusters of Ellipticals and Lenticulars

(3) Small, dense clusters of elliptical and lenticular galaxies are characterized by a compact nuclear concentration surrounded by an irregular "atmosphere" of much lower density; the diameters are of the order of 1 Mpc; the total population is less than a hundred in the first three magnitudes; the velocity dispersion is of the order of 500 km/sec. Examples are the core of the Fornax cluster (de Vaucouleurs 1956b; Hodge 1960) and the "E" component of the Virgo cluster (de Vaucouleurs 1961; cf. Sec. 3).

(4) Large globularlike clusters of the Coma type are populated predominantly or exclusively by elliptical and lenticular galaxies; the diameters are of the order of several megaparsecs, the total population is several hundreds in the first three magnitudes, the velocity dispersion is of the order of 1000 km/sec. The density distribution in these clusters is closely similar to the density distribution in globular (E0) galaxies (de Vaucouleurs 1948). Examples are in Coma, Perseus, Pegasus, Cancer, Corona Borealis, Hydra (Zwicky 1957).

C. Clouds of Mixed Types

(5) Large ellipsoidal clouds or cloud complexes of galaxies of mixed types on a scale somewhat larger than the clouds of spirals and with a moderate degree of central condensation form the transition with the large-scale superclusters. These huge complexes often show a substantial degree of subclustering and type segregation (Sec. 3). Typical diameters are of the order of 10 Mpc with an axis ratio of the order of 4/1. The total population may be of several hundreds in the first three magnitudes, the velocity dispersion is of the order of 700 km/sec. Examples are the Hydra cloud (de Vaucouleurs 1956a, b), the multiple Hercules cluster (Burbidge and Burbidge 1959), the Leo cloud complex and the belt of galaxies centered in Fornax forming the so-called "Southern Supergalaxy" (de Vaucouleurs 1956a).

(6) Superclustering on the largest scale has typical diameters of the order of 20 or 30 Mpc, with axis ratios of perhaps 4/1, at least in some cases. The total population may be several thousands in the first three magnitudes, the velocity dispersion is of the order of 1000 km/sec. Examples are the Local Supercluster (de Vaucouleurs 1960a), the large supersystem centered in Hercules indicated by the Lick counts (Shane and Wirtanen 1954), the elongated complex of clouds and clusters stretching across Perseus and Andromeda (Tombaugh 1937), the huge elongated cloud of galaxies fainter than $m=16$ extending from Dorado towards Fornax and still incompletely covered by the Harvard counts (de Vaucouleurs 1956a).

A great deal of the confusion in the discussion of clustering can be avoided if one takes into account the great variety of scale, structure, and composition among galaxy clusters.

3. STRUCTURE OF THE VIRGO CLUSTER

Because of its location in the general direction of the center of the Local Supercluster the Virgo cluster has been considered as its probable nucleus and the center of its presumed general rotation and differential expansion (de Vaucouleurs 1958, 1960a). This location also raises the possibility that the cluster may be not a single isolated unit but, at least in part, an appearance caused by chance superposition of two or more independent clusters and groups on the same line of sight. A discussion of the surface distribution, morphologic types, apparent magnitudes, diameters, and radial velocities of over 200 galaxies generally brighter than the 14th mag. between 12^h and 13^h , and 0° and $+20^\circ$ suggests that such may well be the case.

The evidence recently presented (de Vaucouleurs 1961) need not be repeated here; it rests primarily on the contrasting surface and velocity distributions of elliptical-lenticular galaxies and of spiral-irregular galaxies in the area. In particular the Virgo cluster

TABLE I. Masses and densities of groups and clusters of galaxies.

System	Distance Δ (Mpc)	Mean radius R (Mpc)	$\log R$ (cm)	Population $\log N_s^a$	Total mass $\log N \mathfrak{M}^*$ ($\odot$)	Mean mass per galaxy $\log \mathfrak{M}^*$ ($\odot$)	Mean density $\log \rho^*$ g cm^{-3}	Notes
Seyfert	29	0.008	22.4:	0.7	11.2:	10.5:	-23.3:	b
Stephan	45	0.024	22.80	0.7	12.7	12.0	-22.9	e
Virgo E (core)	7.5	0.12	23.6	1.3:	13.4	12.1	-24.65	b
Fornax (core)	10	0.17	23.7	1.0:	13.0	12.0	-25.45	b
Sculptor	1.8	0.25	23.95	0.8	13.05	12.3	-26.1	d
G 5846	12	0.35	24.04	1.3	13.50	12.2	-25.9	e
Pegasus	26	0.45	24.14	1.7:	13.45	11.7	-26.3	b
G 1023	7	0.5	24.16	1.0:	12.8:	11.8:	-26.2	b
Fornax	10	0.5	24.19	1.5	13.5	12.0	-26.4	b
Canes	4.5	0.75	24.36	1.5	13.48	12.0	-26.9	f
Virgo E	7.5	0.75	24.36	2.0:	14.2	12.2	-26.2	g
Hercules	72	0.85	24.47	1.9	13.58	11.7	-27.15	h
Ursa Major	13	1.2	24.57	1.7	13.27	11.6	-28.3	b
Virgo S	12.5	1.25	24.58	2.0:	<14.5	<12.5	<-26.5	g
Coma	45	3.5	25.03	2.7	14.70	12.0	-27.15	i
Local	...	10.:	25.5	4.-?	14.7	10.7	-29.1	j, l
Supercluster					(16.0)	(12.0)	(-27.8)	m
Harvard	33:	33:	26.0	4.6	(15.3)	(10.5)	(-30.2)	k, n
counts					(16.6)	(12.0)	(-28.7)	m
Lick counts	125:	125:	26.6	6.4	(16.9)	(10.5)	(-30.2)	b, n
					(18.4)	(12.0)	(-28.7)	m

^a Within 3 mag. from brightest.^b This paper.^c Burbidge, G. R. and Burbidge, E. M. 1959a, *Astrophys. J.* **130**, 15.^d de Vaucouleurs, G. 1959, *ibid.* **130**, 718.^e —, 1960, *ibid.* **131**, 595.^f van den Bergh, S. 1960, *ibid.* **131**, 565.^g de Vaucouleurs, G. 1961, *Astrophys. J. Suppl.* **VI**, No. 56, 213.^h Burbidge, G. R. and Burbidge, E. M. 1959b, *Astrophys. J.* **130**, 629.ⁱ Schwarzschild, M. 1954, *Astron. J.* **59**, 273; Zwicky, F. 1937, *Astrophys. J.* **86**, 217; 1957, *Morphological Astronomy* (Springer-Verlag, Berlin), pp. 132,^j de Vaucouleurs, G. 1958, *Astron. J.* **63**, 253.^k —, 1960, *Astrophys. J.* **131**, 588.^l From rotation.^m For assumed value of $\mathfrak{M}^*$.ⁿ For assumed value of $\mathfrak{M}$.

itself seems to consist of two overlapping, but separate clouds, each about 12° in diameter:

(1) a concentrated cluster of elliptical and lenticular galaxies centered at $C_E(12^h 26^m 5, +13^\circ 2)$ with a mean radial velocity $\bar{V}=950 \pm 70$ p.e. km/sec and a line-of-sight velocity dispersion $\sigma_E \approx 550$ km/sec, at a distance of the order of 7–8 Mpc;

(2) a diffuse, ellipsoidal cloud of spiral and irregular galaxies centered at $C_S(12^h 27^m 5, +13^\circ 9)$ with a mean velocity $\bar{V}_S=1450 \pm 120$ p.e. km/sec and a velocity dispersion $\sigma_S \approx 750$ km/sec, at a distance of the order of 11–12 Mpc.

The difference between the systemic velocities of the E and S clouds computed in various ways is from 3 to 4 times its probable error, and the velocity distributions in each cloud are widely different.

The mass of either cloud derived from velocity dispersion, $N\mathfrak{M}^* \approx 10^{14} \odot$, and the mean mass per galaxy, $\mathfrak{M}^* \approx 10^{12} \odot$ are not significantly reduced compared with previous estimates for both clusters taken together (Oort 1958, de Vaucouleurs 1960b, van den Bergh 1960b).

4. MEAN DENSITIES OF GROUPS, CLOUDS, CLUSTERS, AND SUPERCLUSTERS

According to classical mechanics the stability or otherwise of groups and clusters depends by the virial theorem on whether the mean density ρ is greater or smaller than

$$\rho^* = 3\sigma_v^2 / 4\pi R^2 G,$$

where σ_v is the space velocity dispersion and R is a suitably defined mean radius. As is now well-known estimates of ρ^* through the virial theorem are systematically from one to one-and-a-half order of magnitude larger than values derived from galaxy counts and plausible values of galaxy masses (or mass-luminosity ratios) derived from internal rotation (de Vaucouleurs 1960b). The average logarithmic galaxy mass (in solar units) is $\log \mathfrak{M} \approx 10.5$ by the latter method, while it is $\log \mathfrak{M}^* \approx 12.0$ by the former (on the present distance scale). Attempts to explain this difference by observational selection, measuring errors, inclusion of non-members or multiple clustering have not been successful; attempts to account for it by invisible matter seem artificial. Some basic assumption must be wrong and the most likely to be in error is that the clusters are in

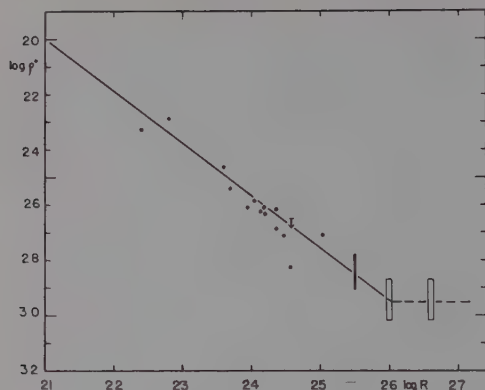


FIG. 1. Density-radius for groups, clusters and superclusters of galaxies (cgs units for $H=150 \text{ km sec}^{-1} \text{ Mpc}^{-1}$). Dots: apparent density ρ^* derived from virial theorem; see text and Table I.

stable statistical equilibrium; except perhaps for the large symmetrical globular clusters of the Coma type, the evidence for stable statistical equilibrium is weak and the evidence for instability and expansion is strong.

Table I gives a summary of estimates of total mass and mean density for a number of groups and clusters. A plot of $\log \rho^*$ vs $\log R$ indicates a linear relationship between density and radius

$$\log \rho^* = -1.9(\log R - 10.5 \pm 0.5). \quad (\text{cgs units})$$

The larger the region of space considered, the lower the average density ρ^* (incidentally this relation can be extended some 23 orders of magnitude upwards to fit average stellar densities and radii). The form of this relation does not depend on the availability of σ_0 ; it can be established from simple galaxy counts (Carpenter 1931, 1938). There is no indication of a constant density level being reached in the observed range. If a constant "mean density" of matter in the universe exists it must be reached only for $\log R \geq 26$, i.e., when a region at least one order of magnitude larger than the Local Supercluster is surveyed. Conversely this means that the universe in our vicinity and at least out to $R \approx 40 \text{ Mpc}$ is a region of above-average density. In other words, regions much larger than single clusters have mean densities much greater than the mean density of the universe; presumably the latter emerges only when very large volumes of space, of the order of 100 Mpc^3 , are surveyed. This is another expression of the presence of large-scale superclustering in our neighborhood. Note, however, that there is nothing peculiar or geocentric in the concept of the Local Supercluster and the conclusion above does not depend on our location in it; the same conclusion would hold for any observer located in any supercluster, for instance

the same density out to $\log R = 25$ can be derived for the large supercluster centered in Hercules.

The prevalence of superclustering on the scale considered raises the possibility that the velocity dispersion in subunits may not be determined primarily by the gravitational potential of each individual group or cluster, but perhaps by the potential of the larger system as a whole. Alternatively, it might be related to the physical mechanisms leading to the formation of galaxies and groups of galaxies in the larger system (cf. stellar associations). In either case the masses derived from the velocity dispersion through the virial theorem will be illusory and larger than the actual masses.

REFERENCES

- Abell, G. O. 1958, *Astrophys. J. Suppl.* **III**, No. 31, 270.
 Burbidge, G. R. and Burbidge, E. M. 1959, *ibid.* **130**, 629.
 Carpenter, E. F. 1931, *Publs. Astron. Soc. Pacific* **43**, 247.
 ———. 1938, *Astrophys. J.* **88**, 344.
 Carpenter, C. 1961, paper read at June meeting of Astron. Soc. Pacific.
 Hodge, P. 1960, *Publs. Astron. Soc. Pacific* **72**, 188.
 Humason, M. L. and Wahlquist, H. D., 1955, *Astron. J.* **60**, 254.
 Kahn, F. D. and Woltjer, L., 1959, *Astrophys. J.* **130**, 705.
 Morgan, W. W., 1958, *Publs. Astron. Soc. Pacific* **70**, 372.
 Neyman, J., Scott, E. L., and Shane, C. D., 1956, *Proc. Third Berkeley Symposium on Math. Stat. and Prob.* 97.
 Oort, J. H. 1958, *La Structure et l'évolution de l'univers*, Solvay Conference (R. Stoops, Brussels, Belgium), p. 163.
 Shane, C. D. and Wirtanen, C. A. 1954, *Astron. J.* **59**, 285.
 Shapley, H. 1938, *Proc. Natl. Acad. Sci.* **24**, 282 and 527 = *Harvard Rept.* 150, 154.
 Tombaugh, C. W. 1937, *Publs. Astron. Soc. Pacific* **49**, 259.
 van Albada, G. B. 1960, *Bull. Astron. Inst. Neth.* **15**, Nr. 500, 165.
 ———. 1960b, *Monthly Notices Roy. Astron. Soc.* **121**, 387.
 de Vaucouleurs, G. 1948, *Compt. rend.* **227**, 586 = *Contrib. Inst. Astrophys. Paris*, Ser. A, No. 27.
 ———. 1956a, in *Vistas in Astronomy* (Pergamon Press, London), Vol. 2, p. 1584.
 ———. 1956b, *Mem. Commonwealth Obs.* **III**, No. 13.
 ———. 1958, *Astron. J.* **63**, 253.
 ———. 1959, *Astrophys. J.* **130**, 718.
 ———. 1960a, *Soviet Astron.* **3**, 897.
 ———. 1960b, *Astrophys. J.* **131**, 585.
 ———. 1961, *Astrophys. J. Suppl.* **VI**, No. 56, 213.
 Zwicky, F. 1957, *Morphological Astronomy* (Springer-Verlag, Berlin).

DISCUSSION

CARPENTER pointed out, and DE VAUCOULEURS agreed, that clusters of low density ($< 10^{-26} \text{ g/cm}^3$) and small radius ($r < 10^{23} \text{ cm}$) would not be detectable; therefore, the curve in Fig. 1 should be considered as an envelope or upper limit of a relation between density and radius, as he had shown in 1938 (see *Astrophys. J.* **88**, 344, 1938 and **131**, 593, 1960).

In answer to ABELL, DE VAUCOULEURS agreed that "Type A Clusters" consist mostly of spirals and irregulars, but also contain dwarf elliptical galaxies.

Summary of the Conference

EVEN if the Conference did not establish whether all systems of galaxies are stable, or whether at least some of them are expanding, it has raised a wide variety of questions. This summary is an attempt to sort out these questions and to connect the various ideas expressed at the Conference; it may gain a certain perspective on the proceedings, even though our impressions are undoubtedly subjective. In this process of abstracting and re-ordering, the problem of the stability of systems of galaxies may emerge in better clarity.

We begin by calling attention to the important new observations presented by the Burbidges and Minkowski.

The analysis of the evidence for and against stability, presented in an extensive review by Ambartsumian and several other authors, suggests a subdivision of the systems of galaxies into several classes, for which the question of stability may well have different answers. At least four different classes appear necessary. A tentative classification, with even more tentative conclusions, is as follows:

- (i) Close pairs of galaxies: apart from optical systems, probably stable.
- (ii) Small groups with trapezoidal configuration (Stephan's Quintet): prime suspects of expansion, particularly those containing peculiar objects, with jets, "projectiles," etc. In some cases, the suspected expansion is of an explosive character.
- (iii) Loose irregular clusters (Virgo): some suspicion of instability, not violent.
- (iv) Compact regular clusters (Coma): indications of stability, negative total energy.

We wish to emphasize the tentative character of the conclusions on stability, as representing our own impressions gained at the Conference. In addition, there are several difficulties with these conclusions, particularly (ii), (iii), and (iv), described below.

One of the main difficulties, mentioned by most of the speakers concerned with empirical evidence, is the assignment of particular galaxies to specified systems. Is NGC 7320 a physical member of Stephan's Quintet, or is it a foreground object? If the former, then, in the words of Mrs. Burbidge, it is "literally exploding away from the other members" and the explanation "falls outside conventional ideas on the dynamics of systems of galaxies." In a number of papers, efforts were made to reinforce confidence in the assignment of objects to membership in a particular group by probability calculations. The weakness of this argumentation was pointed out by Joseph Bertrand and Emile Borel long ago; as a result we find the intuitive feelings of experienced observers to be more persuasive than elaborate probability calculations. Moreover, the studies of forms and projected configurations of small groups may

be said to show primarily our ignorance of the processes whereby galaxies are formed and set in motion. Without such an understanding, many participants felt, we cannot speak meaningfully of the low probability of the chance occurrence of a given configuration.

In addition to the classification of systems of galaxies, it appears desirable to classify also the kinds of instability that these systems may possess. Here we wish to distinguish three categories:

- (a) Explosive expansion,
- (b) Mild expansion, perhaps simply the general expansion of the universe, and
- (c) Contraction.

To these should be added two distinct categories of stability:

- (d) Ordinary, dynamical stability to which the virial theorem applies, and
- (e) Stability of form, with exchange of galaxies between clusters and the field, to which the virial theorem does not apply.

Category (a), mentioned in the above quotation from Mrs. Burbidge, is the core of what is now known as the Ambartsumian hypothesis. Vorontsov-Velyaminov presented a point-by-point formulation of this hypothesis, involving violent phenomena within nuclei of the giant galaxies, occasionally radio emissions, and jets and projectiles visible on photographs. While many details remain obscure, Vorontsov-Velyaminov is inclined to believe that, in order to explain the many peculiarities observed by him for multiple galaxies, it is necessary to admit the existence of some unexplored forces in addition to gravitation.

Instability of category (b) suspected for loose clusters and clouds, if it exists at all, is visualized as a relatively slow process, not comparable to an explosion—perhaps a phase in the evolution of clusters.

The possibility of category (c), predicted by the dynamical study of van Albada, is also visualized as a relatively slow process in the evolution of a cluster under gravitational forces.

Category (e), investigated by Lemaître, implies that the members of a cluster had no common origin, that clusters are maintained by the motions of galaxies in the general field, and that their cause is to be found in the earlier history of the expanding universe.

The evidence adduced to substantiate the various opinions is essentially of the following five categories:

- (1) Radial velocities of individual members of the systems studied,
- (2) Configuration of the systems,
- (3) Association between clusters and radio sources,
- (4) Discrepancy between the masses of galaxies

estimated from internal rotations and also from double galaxies, and those obtained from the application of the virial theorem to clusters.

(5) Correlation between radial velocities and luminosities in groups and clusters of galaxies.

Category 1 may be exemplified by differences in radial velocities of the order of 6000 km/sec or more observed for apparent members of Stephan's Quintet by the Burbidges, and of the triple system IC 3481, Anon, and IC 3483 by Zwicky.

The configuration of the system (category 2) is very relevant for asserting stability of close pairs and compact clusters and for asserting instability of trapezoidal multiples. In the case of pairs the argument is that if a pair of objects is disintegrating it cannot remain a close pair for an appreciable length of time. Thus, unless most or all of the close pairs were of unusual form suggestive of very recent creation, objects of this class are most likely stable. On the other hand, the formation of pairs by capture seems unlikely for dynamical reasons, and Page's preliminary statistics, if confirmed, will probably be explained by another formation process.

These arguments do not apply so clearly to the small groups of trapezoidal configuration that are found frequently among galaxies and infrequently among stars. Such groups are expected to lose members through perturbations even if they were not formed by some kind of explosion. Formation by capture is somewhat more likely than in the case of pairs. Nevertheless, the question of their age remains a problem as indicated below.

Van den Bergh gives evidence that the extragalactic radio sources are associated with ellipticals in the rich clusters of galaxies (category 3). Tentative statistics show that the probability of a radio source occurring in a cluster is independent of the number of member galaxies, which argues against collisions of galaxies causing the radio sources. In fact, Minkowski now finds no evidence that radio sources are specifically associated with double galaxies as formerly thought. Interestingly enough, more and more of the radio sources are found by Kerr and other to be double themselves, and as the precision of locating them improves, it appears that the radio emission often originates from regions symmetrically on either side but quite far from the optical image of a galaxy. This adds weight to the evidence for intergalactic material and may later provide a clue to the process of formation of galaxies.

Category 4 is the most frequently mentioned evidence of instability of systems of galaxies: Masses estimated from the virial theorem exceed those estimated otherwise by a considerable factor. Both types of estimate have been subject to doubt, but van den Bergh's review of Page's masses of the double galaxies, and the summary of individual masses determined from rotations, tend to strengthen confidence in these estimates and to

throw suspicion on those obtained by application of the virial theorem.

Several possible explanations of this mass discrepancy were discussed at the Conference and are covered in more detail below. The magnitude of the discrepancy is emphasized by van den Bergh, whereas Holmberg seeks to explain it away as a combination of observational errors, misidentifications and misapplication of the virial theorem. Nevertheless, many of those present consider that it may be real and due to invisible intergalactic material in the clusters, totaling 90 to 99% of their mass. If these possibilities are excluded, however, the discrepancy in mass indicates positive total energy and instability of the system involved.

Category 5 is based on the consideration that, if a system is stable, its particles must approach the center with any given velocity as frequently as they recede from it with that velocity. That is, the correlation between the radial velocities of galaxies and their distances from the observer would be zero for stability of this type, positive for expansion, and negative for contraction. In principle, apparent magnitude can be substituted for distance in this correlation, but if there is no intergalactic absorption the apparent magnitude is a very poor distance indicator. However if we observe a significant correlation between magnitude and radial velocity, this is indicative both of instability and of intergalactic absorption—the latter adding further evidence of intergalactic material. Several types of error may invalidate such a conclusion, as noted below.

Returning now to the question of the stability of the four different categories of systems of galaxies, the application of the five types of evidence just enumerated leads tentatively to the following conclusions:

(i) The stability of close pairs is generally accepted; Page's calculations confirm that only a small fraction of bright pairs can be optical, although this proportion rapidly increases with faintness.

(ii) Small groups of galaxies are the chief suspects of instability for the reasons of large dispersion in radial velocity, of unstable configuration, of the large masses computed from application of the virial theorem, and of positive correlation between apparent magnitude and radial velocity. This conclusion is based on the unavoidable assumption that objects assigned to the given systems are physical members. In a number of instances, this assumption was vigorously questioned at the Conference, although it was not clear that dropping one or two "outliers" would always correct the situation.

(iii) Analysis of motions in the Virgo cluster gives excessive mass from an application of the virial theorem, and a mild indication that the magnitude-redshift correlation is positive. However, a strong counterargument is the evidence for subclustering suspected by Abell, de Vaucouleurs, and Holmberg. According to de Vaucouleurs, the Virgo cluster is in reality a superposition of at least two clusters, one composed of spirals.

and irregulars and the other of ellipticals and S0 galaxies. The two clusters have substantially different radial velocities, and the application of the virial theorem cannot give correct estimates of the mass. Also, this combination of systems creates a spurious luminosity-redshift correlation even if the separate systems are stable.

According to Holmberg, the large radial velocity dispersions within the Virgo cluster has nothing to do with expansion. He points out first that the Virgo cluster contains a large number of small subsystems, and that the motions within these subsystems should be counted separately. (Some of these subsystems are small groups strongly suspected by Markarian of explosive instability.) Second, Holmberg suspects the presence of a substantial number of foreground and background objects, possibly 12, not belonging to the cluster. Finally, the difference between radial velocities of the spirals and irregulars, on the one hand, and of the elliptical and S0 galaxies, on the other, is attributed by Holmberg to systematic errors in measuring radial velocities. After allowing for the subsystems, the 12 nonmember galaxies, and the presumed observational errors, Holmberg arrives at an M/L ratio derived from the application of the virial theorem which agrees with that estimated by other methods. His argument, in which no appeal is made to the hypothesis that the cluster contains a large amount of invisible matter, may also be applicable to other clusters of galaxies.

Two of Holmberg's points call for careful investigation: first is the suspected systematic error in measured radial velocities which would affect a number of other studies; the other is the elimination of suspected outliers. Four of these "outliers" are also suspected by de Vaucouleurs, who adds two others not suspected by Holmberg.

(iv) Our assessment that there is less reason to suspect instability of compact clusters such as the Coma cluster is based on several considerations: First, with reference to the configuration of the Coma cluster, even Ambartsumian felt that "it might be conjectured that perhaps in the Coma cluster a negative total energy can still occur . . ." The symmetry of this cluster and the time necessary for this to be achieved were also mentioned frequently by other speakers.

It is true that applications of the virial theorem, such as van den Bergh's, yield excessive M/L ratios for the Coma cluster, but these findings should be considered in conjunction with Oort's similar calculations which led to lower values of M/L . Also, the theoretical study of van Albada indicated that the evolution of a stable cluster, subject to gravitational forces alone, is a complicated process during which the usual application of the virial theorem may lead to biased results. In one phase of the evolution, the theory predicts contraction, and the virial theorem would give estimates of the mass exaggerated by a factor as large as 3. Slight indications

of a contraction of the Coma cluster were found empirically by Neyman and Scott.

Both Abell and de Vaucouleurs feel that superclustering is established beyond doubt, and that the dimensions of second-order clusters (clusters of clusters of galaxies) are 30 to 60 Mpc. This means that clusters cannot be treated as isolated systems embedded in an isotropic, homogeneous medium of field galaxies, and the virial theorem must be modified. An estimate of the magnitude of this effect in accounting for the apparent instability of clusters has not yet been made.

Another theoretical possibility of obtaining exaggerated masses through the application of the virial theorem to essentially stable clusters of galaxies was presented by Lemaître. According to him, clusters may be stable in form under a continuous exchange of galaxies, some evaporating from the cluster and some others entering it from the surrounding field. This model, which has yet to be shown rigorously to develop from reasonable conditions during a phase of instability in an evolutionary cosmology, was criticized because observed clusters appear to be constituted of a mix of galaxy types different from that of the field in which they are embedded. This difference is cited as evidence that the galaxies in a cluster had a common origin, though it must be admitted as slender evidence; morphological types are uncertain in many clusters, and the mix of field galaxies is indeterminate, in principle, without further knowledge of the luminosity function and its variation among morphological types. Ambartsumian's argument that clusters must have a common origin because stable clusters could not be built by capture, does not apply to Lemaître's model, where galaxies in a cluster are not captured.

The previous discussion was concerned with the dynamical aspects of particular systems of categories (i) to (iv); we now turn to how the hypothesis of widespread instability fits into the general picture provided by the rest of astrophysics.

Limber, in his discussion of the local system and the M81 group, which are better explored than other systems, found it extremely difficult to interpret available information either in terms of positive energy and disruption or in terms of stability. He concludes that the reconciliation of all the data will require "a significant revision of parameters that we had believed were reasonably well tied down," perhaps admitting a large mass of unseen intergalactic matter, or some unknown process communicating large kinetic energies to newly formed galaxies. One particular difficulty with the hypothesis of expansion is that it implies an age of order of 2×10^8 years for the M81 group; it is difficult to believe that M81 can be nearly this young.

The same time-scale argument against the hypothesis of widespread instability of clusters and groups was put forward by van den Bergh: The typical age of a supercompact cluster appears to be of the order of 10^8

years—considerably less than the typical age attributed to the apparently normal giant elliptical galaxies contained in such a cluster. Furthermore, if a large proportion of the clusters are rapidly disintegrating, then the relative number of field galaxies would be much greater than is observed. Also there would have been among the field galaxies many more ellipticals than observed, since ellipticals are predominant in clusters. Finally, the redshift dispersion of field galaxies, many of which would then represent high-speed escapees from clusters, would have been much higher than observed.

According to the Burbidges, if groups and clusters are predominantly unstable the major problem is that of the time scale. Can the members of expanding groups have evolutionary ages as small as the expansion ages of 10^8 to 10^9 years? Is the age of the universe, as currently estimated from the Hubble constant, sufficient for the development of the Coma-type clusters? Is it, indeed, sufficient for the oldest galactic star clusters? According to the Burbidges, no definite evidence exists against a positive answer to the first question. An indirect answer to the second and third questions was provided in Heckmann's paper.

In this paper Heckmann takes another look at the concept of the "big bang" and, with it, the whole question of the age of the universe. By the established routine of thought, the "big bang" appears to be an inescapable consequence of a well-founded theory. Contrary to this, Heckmann finds that it is only a special feature of models he considers oversimplified; by introducing a general rotation (of unobservably small magnitude at the present epoch) he finds that the discontinuity in density from which the "primeval atom" and "big bang" theories are observed would be smoothed out to a compression of only a few powers of 10. That is, condensations, which may easily originate during the period of contraction prior to the state of maximum

density of the universe, might go unaffected through that phase of evolution. As a result, clusters of stars and of galaxies may exist that are older than the inverse Hubble constant. However, this may in itself raise a physical problem for any evolutionary cosmology in such a rotating universe: If there were no high compression of matter at a finite time in the past, what has limited the "ashes" of processes that are essentially irreversible? Why isn't *all* hydrogen used up in fusion reactions that have gone on for an infinite time? Why aren't *all* stars white dwarfs or older?

It is just possible that the time-scale arguments are particularly relevant with reference to the prime suspects of explosive expansion. Are the interacting galaxies all young? Could it be that the distant young galaxies, as argued by Shklovskii, are frequently misclassified as old ellipticals?

It must be admitted that more questions have been raised than settled. To the question of the stability of clusters and groups of galaxies, about which this Conference was organized, we have added the following, among others:

What is the evidence that members of a cluster had a common origin?

Can the age of a galaxy be as little as 10^8 years?

Are nongravitational forces necessary to understand the dynamics and configurations of small groups of galaxies?

In what way are the extragalactic radio sources associated with individual galaxies or with clusters?

What is the mechanism by which the galaxies were formed, and how does it account for clustering?

JERZY NEYMAN
THORNTON PAGE
ELIZABETH SCOTT

FURTHER REPORTS OF OBSERVATORIES

Flower and Cook Observatory, University of Pennsylvania, 1960-61

Personnel. Dr. William M. Protheroe was promoted to Associate Professor of Astronomy, effective 1 July, 1961. During the 1960-61 academic year, Dr. Protheroe also assumed the position of Vice-Dean of the Graduate School of Arts and Sciences.

Frank M. Bateson was appointed Research Associate. He will direct a survey of New Zealand to locate and to test sites for an astronomical observatory.

Miss Eva Novotny was appointed Instructor of Astronomy, effective September 1961.

Dr. Ernest G. Reuning was awarded the Ph.D. degree in February 1961. Dr. Reuning accepted a position as Assistant Professor of Astronomy at the University of South Florida.

Stanley Sobieski and Mrs. Beverly B. Bookmyer each received the M. S. degree in June 1961. James Wanner received the M. S. degree in February 1961.

William Klepczynski and Ronald Weiner each received the B.A. degree. Klepczynski accepted appointment at the United States Naval Observatory.

Kwan-Yu Chen and Sobieski served as Research Assistants. Kyong Chol Chou held the Pawling Fellowship. Mrs. Bookmyer held the Moore Fellowship. Henry Fliegel held a National Science Foundation Fellowship. Fliegel spent the spring term at the Lowell Observatory making photoelectric observations of eclipsing variables. Sobieski spent the summer of 1961 and Chen and Wanner spent the summer of 1960 at the National Radio Astronomy Observatory.

Scientific Programs. Dr. Charles P. Olivier continued to serve as president of the American Meteor Society. He prepared an Annual Report of Meteors published by the American Meteor Society which appeared as Flower and Cook Observatory Reprint No. 119.

Dr. Leedert Binnendijk used the 28-inch reflector to continue photoelectric observations of W Ursae Majoris systems in two wavelength regions. Light curves and orbital solutions of AH Virginis and AK Herculis were completed and published (*Astron. J.* **65**, 358, 1960 and **66**, 27, 1961). The light curves and orbital solution of ER Orionis were completed and observations of FG Hydrae were begun.

Dr. John E. Merrill continued to serve as Secretary of Commission 42 of the International Astronomical Union. Dr. Merrill also derived simplified methods of reaching approximate solutions of light curves of eclipsing variables.

Dr. William Blitzstein began the study of the instrumentation needed to convert the Pierce photometer into a fully automatic recording unit by modifying it so that it will punch all pertinent data on cards to be reduced by an electronic computer. Miscellaneous experiments were performed on the photometer as part of a systems

analysis aimed at improvement of techniques and increased accuracy of observations. As consultant at the Radio Corporation of America, Dr. Blitzstein also participated in radar experiments which received returns from Venus and from meteor trails.

Dr. Protheroe and Chen wrote an analysis and discussion of their scintillation studies. This was published as "The correlation of stellar shadow band patterns with upper air winds and turbulence."

Dr. Reuning continued his photoelectric observations in the infrared region, concentrating chiefly on Algol. A lead-sulfide cell was used together with a Wratten 87c filter which excluded all radiation shorter than 8700 Å. A light curve is nearly completed; it shows a well-defined secondary of 0.14 mag. One lunar eclipse was also observed. The infrared work is being continued by Chen.

Dr. F. B. Wood continued his work on close eclipsing systems. He and George McCluskey completed the discussion of V Tucanae, based on observations obtained at the Mount Stromlo Observatory, and the paper is in press. Dr. Wood prepared a chapter discussing empirical data concerning eclipsing systems to appear in volume 3 of the *Handbook of Astrophysics*, edited by K. Aa. Strand. He discussed evolution of close double stars in an article in volume 4 of *Vistas in Astronomy*, edited by Arthur Beer. Survey articles on Photoelectric Photometry in Astronomy and on Close Double Stars appeared in *Science* and in the *Leaflets of the Astronomical Society of the Pacific*. With the aid of translations by Chen, he gave a paper on *Astronomy in Communist China* at a symposium on *The Sciences in Communist China* organized by the American Association for the Advancement of Science.

Igor Jurkevich began an analysis of the observations of VW Cephei taken during the recent international cooperative program organized by Dr. K. K. Kwee of the Leiden Observatory, under the auspices of Commission 42 of the I.A.U. Dr. Kwee also kindly supplied many earlier observations of his own. Since more than 13 000 observations are involved, obviously the use of an electronic computer is needed. Preliminary tests, using an IBM 7090, are very encouraging. For example, for a set of observations consisting of 200 points, simultaneous solution for both the period and the time of minimum requires somewhat less than 0.1 sec of machine time. In addition, automatic methods have been employed to produce the light curves from the listed observations.

Fliegel while at Lowell Observatory made two-color photoelectric observations of AG Vir, NQ Her, and BD +35°4496, using the 42-inch reflector.

Chou used the 15-inch telescope for photoelectric observation of eclipsing systems. Photoelectric light curves were completed including β Per (three colors)

and RZ Dra and BX Peg (each in two colors). Two-color observations were also made on BX Aur, W UMa, V 451 Oph, and VV Cep. Chou also performed experiments on the resolving time of the old and new amplifiers of the Pierce photometer, under the direction of Dr. Blitzstein. Other observers included Wanner and Fliegel.

At the National Radio Astronomy Observatory Sobieski, working with Dr. C. R. Lynds, made a search for radio emission from optically peculiar galaxies with the 85-ft Tatel radio telescope at 10 cm, and also prepared a radio contour map of the region of the Perseus cluster of galaxies. Sobieski also investigated the wavelength dependence of polarization for Mars using the 8-inch Clark refractor for ultraviolet, blue, and yellow measures, and the 28-inch reflector with the lead sulfide photometer for measures in the infrared (1.67μ). Preliminary results indicate that the wavelength dependence follows a $1/\lambda$ law in the three shorter wavelength regions, but falls well above this in the infrared. The percent polarization at $\lambda_{\text{eff}} = 0.535 \mu$ was approximately equal to that of $\lambda_{\text{eff}} = 1.67 \mu$.

James Gleim, under the supervision of Dr. Blitzstein, concluded tests of the Pierce photometer, comparing observations made simultaneously of the variable and comparison stars and those made in the conventional manner. Systematic differences in the two methods were too small to be detected and must be of the order of 0.001 mag. or less.

Mrs. Bookmyer used the 28-inch reflector to obtain two-color photoelectric light curves of SW Lacertae. She also made period studies of AK Her, W UMa, and AM Leo (*Astron. J.* **61**, 24, 1961).

Chen used the same telescope with the lead sulfide photometer. Objects observed included ζ Aur and ϵ Aur as well as Algol. Chen also worked on the Beam calculation of the Little-Big Horn while at Green Bank.

Wanner derived a contour map of IC 443 at 1400 Mc from observations with the 85-ft Tatel telescope at Green Bank. His work was published in *Publs. Astron. Soc. Pacific* **73**, 143, 1961.

William Klepczynski began an analysis of the light curve of RS Lep and Ronald Weiner began a similar treatment of that of ST Car. Both used the two-color observations obtained by Dr. Wood in 1957-58 at the Mount Stromlo Observatory of the Australian National University.

Equipment. Only minor changes were made in the equipment. New amplifiers and discriminators with lower resolving times were installed in the Pierce photometer. The 28-inch mirror and Cassegrain secondary were realuminized.

Site Testing Program. A site survey of New Zealand was begun by Frank M. Bateson with the support of the National Science Foundation. Two 5-inch refracting telescopes complete with mountings and drives were loaned by the U. S. Naval Observatory. After preliminary study of the weather patterns and other availa-

ble material, Bateson personally inspected every likely site. Four were selected as the most promising. On one of these a road, telescope shelter, and living accommodations were constructed with the generous assistance of the local groups and individuals. Operation began on June 1 at the Black Birch Site Testing Station near Blenheim and is continuing. Preliminary results are quite favorable.

Support of various phases of the work by the National Science Foundation and the Office of Naval Research is gratefully acknowledged.

FRANK BRADSHAW WOOD, *Chairman*

Addendum to NASA Report for Reports of Observatories

Operational Program. The Jet Propulsion Laboratory conducted radar experiments almost daily from 10 March 1961 to 10 May 1961 yielding more than 200 hr of valid recorded data. The principal result of the experiment was the determination of a new value for the astronomical unit which, after two months of data reduction and analysis, is $149\,598\,500 \pm 500$ km. Spectral analyses of the signal indicated that Venus is either a highly specular reflector or rotates at an extremely slow rate. Relative to a polished conducting sphere of equal size, Venus has a reflectivity of 10 to 12% at 2388 Mc compared to 2% for the moon.

When the polarization of the transmitting antenna was reversed, the Venus-reflected signal was approximately 12 db lower than the signal with normal polarization, as had also been found for the moon. Were Venus a polished conducting sphere, the mismatched polarization would have produced a signal which was about 20 db lower. This probably indicates that the surface roughness of Venus is quite comparable to that of the moon when the roughness is commensurate with the wavelength of the radio-frequency signal.

The development of instrumentation for planetary surface measurement has been initiated. Among these are a gas chromatograph, an abbreviated microscopic system, and a resperometer, all intended to sample the Martian soil and atmosphere for organic and biological materials.

An instrument is being developed for measuring the lunar atmosphere and an x-ray diffractometer to analyze the surface of the moon and planets.

Theoretical and Laboratory Studies. R. Marshall in collaboration with R. H. Bieri has completed an isotope analysis of potassium extracted from a Canon Diablo iron meteorite. Marshall has found that the isotopic composition in iron micrometeorites indicates a composite origin for the elements making up these meteorites. In the stone meteorites, and apparently in the earth, the original group of elements have been well mixed.

L. Trafton is working on an improved model of the atmosphere of Jupiter. D. Jones has devised a theoretic-

al model for the ionosphere of Venus that would explain the 600°K radio brightness measurements. This model requires a weak magnetic field and a very high electron density of the ionosphere.

Z. Kopal is developing a theory of the thermal and stress history of the moon, Mercury, and Mars with particular regard to the dynamical interaction of the solid mantles of these bodies and possible liquid cores.

Visiting Professor G. de Vaucouleurs is preparing an improved map of the planet Mars.

NANCY G. ROMAN, *Chief,*
Astronomy & Astrophysics
Satellite & Sounding Rocket Programs

National Bureau of Standards, Washington, D. C.
and Boulder, Colorado

Emphasis has continued on the development of a general interdivision and interlaboratory program on measurement and standards in plasma and astrophysics.

Atomic Spectra. At the Washington Laboratories, work has centered on the observation, description, and analysis of atomic spectra, with special attention to selected spectra of the rare-earth elements in the lanthanide group. Observation of these spectra with suitable sources is urgently needed to secure the data for volume IV of *Atomic Energy Levels*.

The spectra of Ce II and Ce I have been observed from 4800 to 5900 Å, with an ac arc, hollow cathode, and an electrodeless lamp as sources. Measurements have been completed in this range. These sources have provided the most complete cerium spectra ever obtained. [1].

The spectra of Pr II and Pr I have been observed and measured from 2600 to 9000 Å. Preliminary observations in the range 2200 to 2600 Å have been made, and Zeeman patterns have been measured from 3500 to 7000 Å [2]. More than 1500 lines have been classified in Pr III, in collaboration with Dieke and his staff at Johns Hopkins. The regularities include terms and levels from four configurations [3]. Theoretical work on Pr III has been of great assistance in the interpretation of known levels and in predictions of the positions of missing levels [4].

Approximately 100 classified lines are known in Dy II, but further observations are needed to extend this work [5].

The analysis of Yb II is nearly finished [6]. In Yb I there are about 2000 observed lines, 70 new levels, and approximately 330 classified lines [7].

The analysis of Br I is essentially complete. Practically all of the 1300 lines observed in the range from 2000 to 13 000 Å [8], are classified. All leading configurations are known, and extended series have been observed [9]. In Br II improved values of previously known energy levels have been reported, together with observed magnetic dipole transitions in the $4p^4$ ground configuration [10].

The analysis of Hf I is nearing completion. There are 66 new energy levels, and a revised interpretation is in progress [7].

The analysis of Ta II has been completed from observations in the interval 2000 to 7813 Å, including extensive Zeeman data. The leading configurations are known and 1890 lines are classified [11]. Here again theoretical investigations have been of great assistance in the interpretation of the spectrum [12].

More general theoretical work has also been carried out on the parameters α and β in the spectra of the iron group [13].

Wavelength Standards. The wavelengths of the 2537 and 3132.7 Å lines emitted by a sealed-off Hg¹⁹⁸ atomic beam, excited by electron bombardment, have been measured very precisely in terms of the standard line of Kr⁸⁶, 6057 Å [14]. The hyperfine structure and isotope shifts in the 2537 Å line of natural mercury have been measured with high precision by using interferometric techniques on the absorption spectrum of an atomic beam [15]. Work is continuing on the measurement of the wavelength passed by a Hg¹⁹⁸ Zeeman filter at 2537 Å in terms of the standard line of Kr⁸⁶, 6057 Å. The wavelengths of Hg¹⁹⁸ lines emitted by an electrodeless lamp are being remeasured relative to the Kr⁸⁶ standard.

Spectroscopic Tables. The NBS *Tables of Spectral-Line Intensities* are in press [16]. The temperature and ionization in the copper arc used as the light source have been determined. Work is in progress on the calculation of gf values from the intensities.

Section 3 of NBS Circular 488, "An Ultraviolet Multiplet Table" has been completed. This circular will be concluded with two Finding Lists, which are also in press. Section 4 is the Finding List for Sections 1 and 2 of the Multiplet Table, Hydrogen through Niobium ($Z=1$ to 41). Section 5 is similarly arranged and is the Finding List for Section 3 of the Multiplet Table, Molybdenum through Lanthanum, ($Z=42$ to 57) and Hafnium through Actinium ($Z=72$ to 89) [17].

Press copy is being prepared for the current revision of the 1928 edition of the solar spectrum table. The measured solar equivalent widths furnished by Minnaert and his staff extend from $\lambda 3061$ to $\lambda 8770$ Å, and have been completed [18].

Transition Probabilities and Cross Sections. A center has been established at the Bureau to collect all references in the literature on transition probabilities and collision processes. A critical evaluation of the published data on transition probabilities for the first 10 elements is nearing completion and will be published within a year. Bibliographies on selected elements are available on request. A similar program relating to collision processes is in progress. These references are recorded on IBM cards with notes indicating the process involved. Collaboration in keeping the reference catalogues up to date is invited.

The hydrogen plasma has been studied in a well-

stabilized dc arc to test the applicability of this technique to the measurement of atomic transition probabilities. The measurements were found to be reliable in temperature to about 1% and in electron density to 10%. The shape of the Balmer lines was found to be in very good agreement with the Stark broadening theory of Kolb, Griem, and Baranger. The transition probability of the O I multiplet at 6157 Å has been studied [19], and agrees well with Jürgen's measurement.

Considerable progress has been made in the study of negative ion absorption spectra. The threshold for photodetachment of electrons from carbon negative ions has been measured [20] and is at $0.99 \pm 0.02 \mu$. The carbon negative ion absorption coefficient has been measured from threshold to about 4000 Å. Absorption coefficients have been calculated [21] for negative ions of atomic oxygen, carbon, chlorine and fluorine from threshold to four rydbergs. Theoretical studies of the H^- bound-free absorption coefficient are being continued with the objective of obtaining very accurate values. Instruments are being developed for the study of elastic and inelastic electron scattering by atoms and molecules.

Astrophysics. A large part of the astrophysical research has been carried on at the Boulder Laboratories. Here the program of research on the physics of stellar atmospheres continues, with primary emphasis on the stellar atmosphere as a laboratory for the study of gaseous physics under conditions not easily reproducible terrestrially. The two principal directions of current interest lie in the study of nonequilibrium thermodynamics in the presence of a radiation field, and of aerodynamic motions under the environment characteristic of stellar atmospheres [22] [23].

In collaboration with the Sacramento Peak Observatory, investigations have continued on the physical state of solar prominences and on the physical structure of solar flares [24].

Solar research has also been carried on in collaboration with the University of Colorado. A monograph on *The Physics of the Solar Chromosphere* has been published [25]. Studies of the ultraviolet rocket solar spectra have continued as a project on which this Bureau and the Universities of Colorado and Paris are collaborating.

Close collaboration with the University of Paris has been maintained in theoretical investigations, with special stress on nonequilibrium thermodynamics in the presence of a radiation field.

Research on the aurora, airglow, and related subjects has continued and calculations of the total integrated starlight have been carried out [27]. A series of theoretical investigations of the ultraviolet coronal lines has been initiated [26].

An extensive bibliography through June 1961 of research at the Bureau that is of possible interest to astrophysicists, may be found in a current Technical Note [28].

This report has been prepared with the cooperation of L. M. Branscomb, R. N. Thomas and the staff of the Spectroscopy Section. Their cooperation is gratefully acknowledged.

REFERENCES

- [1] Martin, W. C. and Corliss, C. H. unpublished material (June 1961).
- [2] Zalubas, R. unpublished material (May 1961).
- [3] Sugar, J. unpublished material (May 1961).
- [4] Trees, R. E. unpublished material (May 1961).
- [5] Corliss, C. H. unpublished material (May 1961).
- [6] Meggers, W. F. and Racah, G. unpublished material (May 1961).
- [7] Meggers, W. F. unpublished material (May 1961).
- [8] Tech, J. L. and Corliss, C. H. *J. Research Natl. Bur. Standards* **65A**, 159 (1961).
- [9] Tech, J. L. unpublished material (June 1961).
- [10] Martin, W. C. and Tech, J. L. *J. Opt. Soc. Am.* **51**, 591 (1961).
- [11] Kiess, C. C. unpublished material (June 1961).
- [12] Trees, R. E. *Phys. Rev.* **123**, 1293 (1961).
- [13] Trees, R. E. and Jørgensen, Chr. K. *Phys. Rev.* **123**, 1278 (1961).
- [14] Barger, R. L. and Kessler, K. G. *J. Opt. Soc. Am.* **51**, 827 (1961).
- [15] Schweitzer Jr., W. G. *J. Opt. Soc. Am.* **51**, 692 (1961).
- [16] Meggers, W. F., Corliss, C. H., and Scribner, B. F. *Natl. Bur. Standards Mono.* 32, in press (1961).
- [17] Moore, C. E. *Circ. Natl. Bur. Standards* 488, Secs. 3, 4, 5, in press (1961).
- [18] Minnaert, M. and Houtgast, J. *Rech. Astron. Obs. Utrecht* **15**, 168 pp. (1960) and unpublished material (1961).
- [19] Wiese, W. L. and Shumaker, J. B., Jr. *J. Opt. Soc. Am.* **51**, 937 (1961).
- [20] Seman, M. S. and Branscomb, L. M. *Phys. Rev.*, in press (1961).
- [21] Copper, J. W. and Martin, J. B. private communication (1961).
- [22] Pottasch, S. R. and Thomas, R. N. *Astrophys. J.* **132**, 195 (1960).
- [23] Thomas, R. N. and Zirker, J. B. *Astrophys. J.* **133**, 588 (1961).
- [24] Jefferies, J. T. and Orrall, F. Q. *Astrophys. J.* **133**, 946, 963 (1961).
- [25] Thomas, R. N. and Athay, R. G. *The Physics of the Solar Chromosphere*, Interscience Press, 422 pp. (1961).
- [26] Pecker, C. and Thomas, R. N. *Compt. rend.* **252**, 3000 (1961); Cloudcraft Symposium (1961).
- [27] Roach, F. E. and McGill, L. R. *Astrophys. J.* **133**, 228 (1961).
- [28] See "Astrophysical and plasma physics research at the NBS," *Natl. Bur. Std. Tech. Note* 116, PB 161617, in press (1961).

CHARLOTTE E. MOORE,
Atomic Physics Division

Stanford Radio Astronomy Institute, Stanford University, Stanford, California.

General. The Stanford 9.1-cm microwave spectroheliograph project began in 1955 and was completed in 1959. It has been extensively modified subsequently to enable observations of the moon and cosmic radio sources. The observing site (Heliopolis) is located at latitude $37^{\circ}23'9''N$, longitude $122^{\circ}11'3''W$, on the outskirts of the Stanford lands. The Radio Astronomy Institute forms part of the Radioscience Laboratory within which several other groups have contributed to astronomical studies, especially meteor, lunar, and solar radar. Thus, in 1959, Professor V. R. Eshleman and his associates made the first radar contact with the sun

and have recently completed new instruments for detailed radar examination of the outer solar corona.

Personnel. The microwave spectroheliograph was designed and constructed by Professor R. N. Bracewell, C. H. Sphar, and C. C. Lee, mechanical engineers, and D. D. Cudaback, R. S. Colvin, G. Swarup, and K. S. Yang, research assistants. C. H. Sphar has since transferred to another division of Stanford University. K. S. Yang obtained the degree of Engineer in the spring of 1961 and left to join the University of Illinois radio astronomy project. In March 1961, C. L. Seeger of the Leiden Observatory joined the Institute. In April 1961, G. Swarup was granted the Ph.D. degree and appointed Assistant Professor in the Department of Electrical Engineering. Z. Fazárcin, J. Picken and S. H. Zisk have joined the group as graduate students.

A. G. Little of Australia spent two years at Stanford working part of the time with Professor Heffner's group on the development of parametric amplifiers, and part of the time on observations of radio sources. He received the M.S. degree in April 1961 and returned to the University of Sydney. Prof. T. Kakinuma of Japan is visiting Assistant Professor for the period of October 1960 to September 1961.

Instruments. The spectroheliograph is described in references 1 and 16. A description of the wide-band parametric preamplifier is given in reference 12. Recently four 30-ft antennas have been placed in a north-south line with an over-all spacing of about 3800 wavelengths. They will be used to extend techniques of path-length measurement and of phase and gain control to very long transmission lines, and for studying particular aspects of Cygnus A (especially its scintillations) and other discrete sources.

Research. During the construction phase of the 32-element cross detailed investigations were made into the tolerance theory of large antennas,^{8,14} theory of radiometers,⁶ principles of interferometers,^{3,5,13} and related topics.^{2,4,7,21,24} A new phase-measurement technique was developed by Swarup and Yang which is well suited for accurate adjustment of the very large antennas now being constructed in various parts of the world.^{11,15}

During the last two years Swarup has obtained about 200 spectroheliograms at a wavelength of 9.1 cm. These are being published in the solar data bulletins of the National Bureau of Standards. This data has also been distributed to observatories as Stanford Radio Astronomy Institute Publications.¹⁷ Special maps have been furnished in connection with eclipse and rocket observations. Active regions can be detected shortly before the accompanying spot group comes around the photospheric limb. The spectroheliograms have enabled an accurate determination of the shape of the quiet sun at 9 cm¹⁸ and its variation with the solar cycle, and are also of significance in the derivation of models of the chromosphere and inner corona. Such characteristics of the sources of the slowly varying component as size,

brightness temperature, and height are being studied in order to form a model of the active regions.¹⁸ Kakinuma and Swarup studied the polarization of radio emission from active regions using a 1.5 minute-of-arc fan beam. Their observations show that, contrary to the theory of Piddington and Minnett, the radio emissions from the two portions of a bipolar group have opposite sense of polarization irrespective of which quadrant is occupied by the spot group.

Kakinuma has made a comparison of the characteristics of 10 active solar regions at many radio wavelengths and has concluded that the flux of the slowly varying component near 10 cm is higher than at shorter wavelengths. Detailed calculations support a hypothesis that the excess radio emission originates in the near-vicinity of sunspot groups and at harmonics of the gyrofrequency radiation.

D. Cudaback has obtained a series of accurate, high-resolution meridian and prime vertical scans across the moon using the two multielement arrays of the Stanford cross and the parametric wide-band receiver. These observations are being studied for evidence of monthly variations and limb darkening and for determining a model of the surface properties of the moon.

A. G. Little made observations of Vir A, Cen A, Cas A, Tau A, Sag A, Orion nebula and Omega nebula using the wide-band parametric receiver and a 2.3' fan beam. This is the highest direct resolving power yet brought to bear on these objects and, in practically every case, the source was partially resolved. The central part of Cen A was resolved into two parts separated by 4.8' east-west, and straddling the optical portion of NGC 5128.²²

R. N. BRACEWELL, *Director*

¹ Bracewell, R. N. 1957, "Antenna problems in radio astronomy," *IRE Natl. Convention Record* 5, Pt. 1, 68-71.

² Bracewell, R. N. 1957, "Application of communication theory to radio astronomy," *Proc. Symp. Communication Theory and Antenna Design*, Boston University, pp. H1-H26.

³ Bracewell, R. N. 1958, "Radio interferometry of discrete sources," *Proc. IRE*, 46, 97-105.

⁴ Bracewell, R. N. 1958, "Restoration in the presence of errors," *Proc. IRE* 46, 106-111.

⁵ Swarup, G. and Yang, K. S. 1959, "Interferometer phasing problems at microwave frequencies," *IRE Wescon Convention Record* 3, Pt. I, 17-24.

⁶ Colvin, R. S. 1959, "Faint signal limitations of radiometers," *ibid.* 3, Pt. 8, 52-58.

⁷ Bracewell, R. N. 1960, "Correcting noise maps for beam-width," *The Radio Noise Spectrum*, edited by D. H. Menzel (Harvard University Press, Cambridge, Massachusetts), pp. 141-150.

⁸ Bracewell, R. N. 1960, "Antenna tolerance theory," *Radio Wave Propagation, UCLA*, edited by W. C. Hoffman (Pergamon Press, London), pp. 179-183.

⁹ Cudaback, D. 1959, "Preliminary observations with Stanford microwave interferometer," *Publ. Astron. Soc. Pacific* 71, 537-539.

¹⁰ Bracewell, R. N. 1960, "Communications from superior galactic communities," *Nature* 186, 670-671.

¹¹ Swarup, G. and Yang, K. S. 1960, "Monitoring paraboloidal reflector antennas," *Proc. IRE* 48, 1918-19 November (also issued as *SRAP* No. 3).

¹² Little, A. G. 1961, "A wide band single diode parametric amplifier using filter techniques," *Proc. IRE* 49, 821-22 (also issued as *Stanford RAIP* No. 9).

¹² Bracewell, R. N. 1961, "Interferometry and the spectral sensitivity island diagram," *IRE Trans. on Antennas and Propagation* AP9, 59-67 (also issued as *Stanford RAIP* No. 4).

¹⁴ Bracewell, R. N. 1961, "Tolerance theory of large antennas," *IRE Trans. on Antennas and Propagation* AP9, 49-58 (also issued as *Stanford RAIP* No. 5).

¹⁵ Swarup, G. and Yang, K. S. 1961, "Phase adjustment of large antennas," *IRE Trans. on Antennas and Propagation* AP9, 75-81 (also issued as *Stanford RAIP* No. 6).

¹⁶ Bracewell, R. N. and Swarup, G. 1961, "The Stanford microwave spectroheliograph antenna, a micro-steradian pencil-beam interferometer," *IRE Trans. on Antennas and Propagation* AP9, 22-30 (also issued as *Stanford RAIP* No. 7).

¹⁷ Swarup, G. "Stanford microwave spectroheliograms for April, for May, for June," *Stanford Radio Astronomy Inst. Publ.* Nos. 8, 10, 11, December 1960, January 1961, and April 1961.

¹⁸ Swarup, G. 1961, "Studies of solar microwave emission using a highly directional antenna," *Sci. Rept.* No. 13, Stanford Electronics Laboratory.

¹⁹ Yang, K. S. 1961, "Transmission system of the Stanford microwave spectroheliograph," *Sci. Rept.* No. 14, Stanford Electronics Laboratory.

²⁰ Sturrock, P. A. 1961, "Irreversible processes in conservative dynamical system," *Stanford Radio Astronomy Inst. Publ.* No. 13.

²¹ Bracewell, R. N. 1961, "Proposal leading to future large radio telescopes," *Stanford Radio Astronomy Inst. Publ.* No. 14.

²² Bracewell, R. N. 1961, "Interferometry of Centaurus A," *Stanford Radio Astronomy Inst. Publ.* No. 15.

²³ Bracewell, R. N. and Swenson, G. W., Jr. 1961, "Some Russian radio telescopes," *Sky and Telescope* 22, 77-80.

²⁴ Bracewell, R. N. "Radio astronomy techniques," *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, in press), Vol. 54.

Yale Observatory, Yale University, New Haven, Connecticut. This report covers the two-year period, 1 July 1959 to 30 June 1961.

Personnel. The vacancy created by the departure of Dr. A. E. Lilley at the end of the academic year 1958-59 was not filled until a year later when Dr. J. N. Douglas was appointed instructor. Douglas had just received his Ph.D. degree in the department.

Dr. Gustav Land, a valued member of the Observatory staff since 1942, died on September 26, 1959. While at Yale he concentrated his activity primarily on investigations in photographic astrometry. He continued his work well beyond normal retiring age, and was active until a few months before his death.

During the year 1960-61, Assistant Professor Ludwig Oster was transferred from the Department of Physics to the Department of Astronomy.

Foreign Visitors. Dr. Gen-ichiro Hori, of the Department of Astronomy at the University of Tokyo, was research assistant in astronomy during the year 1959-60, research associate during the year 1960-61. Visitors for shorter periods were Dr. P. C. Choudhuri of the University of Lucknow, India, and Dr. Jean Meffroy, of the University of Montpellier, France. Both were at Yale during the spring term of the academic year 1959-60.

Zone Catalogues. During the two years covered by this report we have exerted our efforts toward accelerating the measurement and reduction of our unmeasured plate material on southern zones. Especially important was the renewed cooperation established with the Watson Scientific Computing Laboratory,

through its director, Dr. W. J. Eckert. The automatic measuring machine of this laboratory, after thorough overhauling, was employed for the measurement of plates in declinations -40° to -50° taken at Johannesburg in 1942. In the meantime, the measurement of plates in the zone -30° to -40° with the measuring machine in New Haven has been very nearly completed. This work, both in New Haven and New York, is being carried forward with the support of a contract with the Army Map Service.

Dr. Dorrit Hoffleit continues to be in charge of the program as a whole; her assistants in New Haven were Mrs. Mary E. Mullally, Miss Inge Schier, and Mrs. Iris Tsai. At the Watson Computing Laboratory Mrs. W. J. Eckert assumed the supervision of the work with Francisco Lopez and Mrs. Meredith Weddle as her assistants.

Parallax Program. Miss Louise Jenkins and D. S. Kimball continued their activity on this program. Since January 1960 W. G. Cleaver has participated in the work. The number of plates measured during the period was 758. Cleaver has concentrated his efforts mainly on parallax regions for which three epochs were observed with the object of searching for evidence of nonlinear proper motions. Miss Jenkins prepared a list of over a hundred measured Yale parallaxes for publication.

Celestial Mechanics. Research in celestial mechanics has continued, especially in directions stimulated by the current interest in artificial satellites. A general theory of drag-free motion of artificial satellites was published by D. Brouwer (*Astron. J.* 64, 378-397, 1959). This was followed by a joint paper by D. Brouwer and G. Hori on the theoretical evaluation of atmospheric drag in the motion of these bodies (*Astron. J.* 66, 193-225, 1961). In both investigations the principles of von Zeipel's method were used. The effectiveness of this method led to explorations by Brouwer and Hori, assisted by Mrs. Lois Frampton, on the use of von Zeipel's method for the purpose of extending and improving Delaunay's lunar theory. This work is being continued.

The increased demand for people trained in celestial mechanics caused the department to hold a four-week Summer Institute in Dynamical Astronomy at the Yale Observatory in the summer of 1959. This effort was supported jointly by the National Science Foundation and the National Aeronautics and Space Administration. This Summer Institute aimed to attract college teachers as well as workers in government agencies and in industry. Those in the first category were primarily college teachers who had little previous training in celestial mechanics, but desired to prepare themselves for teaching of an introductory college course in the subject. The first Summer Institute was successful enough to warrant a repetition in the summer of 1960, when a six-week Summer Institute was offered at Yale University, with National Science Foundation support.

A textbook *Methods of Celestial Mechanics*, by

D. Brouwer and G. M. Clemence, was published by the Academic Press in the spring of 1961.

Theoretical Astrophysics. Dr. Wildt has established a displacement theorem analogous to Wien's law for blackbody radiation, which holds for every pencil of the nonisotropic nonequilibrium radiation in a gray atmosphere in local thermodynamical equilibrium and describes rigorously the change in spectral intensity distribution resulting from a finite variation of the net flux traversing the atmosphere, or, in other words, of its effective temperature. This displacement theorem remains valid in a gray atmosphere, even in the absence of local thermodynamical equilibrium, provided that the variation of the source function with optical depth satisfies Planck's differential equation for the reversible adiabatic transformations of nonequilibrium radiation.

The inhomogeneous integral equation with Milne's kernel and an exponential inhomogeneous term expresses the energy balance in strict radiation equilibrium in a gray planetary atmosphere exposed to monodirectional external heating. An exact solution of this integral equation, due to Hopf, has been programmed for the Yale IBM 650 computer, and several dozen numerical solutions have been obtained by Wildt. They illustrate the temperatures in strict radiative equilibrium throughout a gray planetary atmosphere, as a function of the angle of incidence of solar radiation and of the optical depth below the surface of the (plane) atmosphere. These solutions, by a simple regraduation of the scales both of the independent and dependent variables, also can be made to represent a somewhat idealized greenhouse effect, such as would result if the ratio of the (gray) absorption coefficients for short-wave (solar) and long-wave (planetary) radiation were constant everywhere.

Dr. Oster completed a study of cyclotron radiation from relativistic particles, leading to expressions for the spectrum which include all relevant broadening mechanisms, such as collisions, relativistic Doppler effects and relativistic mass variability. Also completed is an investigation of the bremsstrahlung spectrum of non-relativistic electrons including quantum effects. An expansion of this program to nonequilibrium plasmas is underway in collaboration with several graduate students. An experimental search (in collaboration with B. Beaken of the Physics Department) for the non-relativistic cyclotron line from a gaseous discharge in air was successful. A comparison of the observed line contour showed excellent agreement with theoretical predictions. These research programs were supported by the Office of Naval Research and the National Aeronautics and Space Administration.

As a first application to solar physics, the thermal radio emission from the transition layer chromosphere-corona was computed with the improved absorption coefficient for bremsstrahlung and compared with recent observations. Very satisfactory agreement was found for the layer to which the underlying model of

the solar atmosphere is relevant (C. C. Brooks and L. Oster).

K. W. Philip and Oster considered and rejected the possibility of a large electric charge on the sun as suggested by V. A. Bailey. The results were published in *Nature*.

Radio Astronomy. J. N. Douglas and H. J. Smith continued and expanded their activities in planetary radio research in decametric wavelengths. Much of the effort during the two years was concerned with the improvement of instrumentation. The principal antenna projects included construction and mounting of a large crossed-Yagi polarimeter, installation of drives for the standard monitoring Yagis, erection of a pair of $110 \times 110 \times 200$ -ft fixed corner-reflectors at half-mile separation to serve as a broad-band interferometer, and installation of large Yagis at sites remote from the Observatory Bethany Station, specifically at Wesleyan University in Middletown and at Pomfret School in Pomfret, Connecticut. Appropriate electronics were developed for each system, including a complete polarization analyzer capable of high time-resolution, several multichannel spectrum-analyzers, and remote Jupiter radiometers. The principal observing projects were nearly continuous multichannel monitoring by six phase-switched interferometers covering the 13 to 23 Mc region, detailed spectrum analysis monitoring of a one-megacycle band, high-speed photographic recording of Jupiter and solar storms to bring out fine-structure events having durations down to the order of milliseconds, extended baseline monitoring combining the above techniques with the signals relayed in from the outlying stations to investigate the degree to which the ionosphere introduces the observed fine-structure, and the determination of the amount and sense of orientation of polarization of the elemental structure of the storms.

Results, many of which were developed in Douglas' thesis, include a new precise determination of the radio rotation period (presumably also the solid-body period) $9^h 55^m 29^s 37 \pm 0^s 13$ p.e. Frequency-drifting burst groups were detected. The conventional tendency for right-hand circular polarization was shown to be present also in each elemental burst with some anomalous left-hand events of uncertain origin. Strong correlation, over 100-km baseline, of events having durations as small as several hundredths of a second proves the presence of complex structure in at least some of the Jupiter radiation prior to the terrestrial ionosphere.

James P. Rodman and F. Arakel Bozayan provided a major share of the design and construction supervision for the polarimeter and corner reflectors; Michael M. Davis and Barry Lasker were instrumental in the electronic developments for these projects. Support for the work came from the Research Corporation, NSF and NASA grants, including NSF Summer Undergraduate Research Participation Programs.

Instrumental. A 20-inch reflecting telescope was con-

structed in the Observatory shop, with Rodman as designer and project director. The former mounting of the Loomis coelostat was adapted to serve as mounting for this instrument; optics were by Tinsley Laboratories, Berkeley, California. The instrument is nearly completed. It will be equipped with an automatic guiding system and photoelectric photometer. The construction of a photoelectric spectrophotometer for the instrument, designed by Rodman and Smith, is also approaching completion.

New plateholders were constructed for the Catalogue Camera. The Coleman digitizer, installed on the Gaertner measuring machine for 17×17 inch plates in 1959, has given good service. Work on a photoelectric setting device for this measuring machine is in progress. These improvements were made possible by a grant from the National Science Foundation.

For the Loomis telescope, A. A. Disco designed and constructed a semiautomatic plateholder with an electric clock system that closes the plateholder at a predetermined time, allowing the completion of the last full cycle.

Southern Astrograph. In July 1960, Yale University received a grant of \$750 000 from the Ford Foundation for the establishment of a 20-inch astrograph in the southern hemisphere. The instrument is primarily intended to extend to the south celestial pole the Lick Observatory's project of referring stellar proper motions to a background of distant galaxies.

The Rutherford Observatory of Columbia University will share with the Yale Observatory the responsibility for carrying out the observing program.

The optics for the new instrument have been ordered from the Perkin-Elmer Corporation. For the mounting we have decided on a fork-type design by B. G. Hooghoudt of Leiden, Netherlands. The mounting is being constructed by the firm of Rademakers, Rotterdam, Netherlands.

For the site of the new instrument an area in Western Argentina appears to be particularly attractive, but no definite decision on the choice of site has been reached.

Miscellaneous Research. Using the Butler photoelectric photometer Smith showed the star V703 Scorpii to be a dwarf Cepheid, with beat periods. Observations taken largely by J. McCullough and R. Wing in recent summers were particularly useful in this investigation. These observations were the fruit of a

program of training students in photometry with the Butler photoelectric photometer, and also of the NSF Summer Secondary School Teacher Research program.

An investigation of the unique radio object 3C-48 with the aid of plate material of the Harvard Observatory, was carried out by Smith and Miss Hoffleit, assisted by graduate students Brooks, Dirkswager, Gross, Marsden, and Rodman. The available information appears to be consistent with the hypothesis that the object is a type I supernova remnant, somewhat weaker than the Crab Nebula, roughly one-fifth the age and at five times the distance of the Crab Nebula.

Under the direction of D. S. Kimball, a detailed analysis was completed of the morphology, motions and magnetic disturbances related to the auroral display of 13-14 February 1958 over Alaska as recorded on five all-sky cameras. This study is to be published as a scientific report of the Geophysical Institute, University of Alaska, as a joint paper with Dr. T. Neil Davis.

Two further studies are in progress. First, a detailed analysis is being made of changes in auroral forms including velocities from all-sky photographs taken with U. S. cameras during the IGY in conjunction with the Auroral Data Center, Cornell University. Secondly, synoptic maps are being constructed of auroral activity over Antarctica during the IGY as recorded by five U. S. cameras. Data compiled from all Antarctic stations will be incorporated in the *Annals of the IGY* by F. Bond, Auroral Recorder, Melbourne, Australia. This U. S. part of the program is contracted through the Geophysical Institute.

University Computing Center. The Observatory has continued to derive substantial benefits from the services rendered by the University Computing Center, with Dr. Morris S. Davis as Director. A project for considerable expansion of the Computing Center, to be housed in a separate building is far advanced.

The Astronomical Journal. Beginning in January 1960, the American Institute of Physics assumed the responsibility for publishing the *Astronomical Journal* for the American Astronomical Society. The Editorial Office remained at the Yale Observatory. The change has had the desired effect of relieving the editors (Brouwer and Smith) and assistant editor (Mrs. Mary S. Albee) of much of the editorial and business routine.

DIRK BROUWER, *Director*

Author Index to Volume 66

(A) indicates Abstracts, (N) indicates Notices, (N&O) indicates Notes and Observations. It will be noted that there are no references to "see someone else" in the Author Index. Although it has meant a little duplication, there have been included under each author all articles in which he was author or co-author. All articles in which the author has participated are arranged in the chronological order of publication. A bibliography with author names arranged as in the original article is obtained by insertion of the author's name in the indicated (—) space under each indexed name.

- Abell, George O.** Evidence regarding second-order clustering of galaxies and interactions between clusters of galaxies. 66: 607—1961
- Abt, Helmut.** Frequency of binaries among A-type stars. 66: 277(A)—1961
- Adams, A. N. and S. M. Bestul.** Meridian observations of the moon, 1958–59. 66: 36(N&O)—1961
- Aikens, R., W. Powers, and J. A. Hynek.** High-contrast electronic image-orthicon techniques applied to the lunar surface. 66: 277(A)—1961
- Alexander, W. M., C. W. McCracken, and H. E. LaGow.** Dust particles of micron size associated with the Leonid meteor stream. 66: 277(A)—1961
- Aller, L. H. and D. J. Faulkner.** Spectrophotometry of gaseous nebulae in the Magellanic Clouds. 66: 37(A)—1961
- Ambartsumian, V. A.** Instability phenomena in systems of galaxies. 66: 536—1961
- Anders, Edward**
- Goles, Gordon G. and —.** Decay intervals of extinct radionuclides in meteorites and in the earth. 66: 44(A)—1961
- Aoki, Shinko.** Discussion on longitude differences of catalogues in view of determining ephemeris time. 66: 1—1961
- Athay, R. Grant**
- Further evidence of broadening of *CaII* line profiles in chromospheric spicules by magnetic fields. 66: 278(A)—1961
- House, Lewis L. and —.** Departures from thermodynamic equilibrium in chromospheric *MgI*, *CaI*, and *OI*. 66: 286(A)—1961
- Baade, Walter and Henrietta H. Swope.** The Draco system, a dwarf galaxy. 66: 278(A), 300—1961
- Bair, M. E.**
- Barrett, A. H., W. E. Howard, III, F. T. Haddock, —, J. J. Cook, and L. G. Cross.** Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. 66: 37(A)—1961
- Baker, N. and R. Kippenhahn.** The pulsations of models of δ Cephei stars. 66: 278(A)—1961
- Barnhart, Philip E., Walter E. Mitchell, Jr., Ronald E. Roll, and William H. Haynie.** A program of stellar photometry at 3 to 4 microns. 66: 37(A)—1961
- Barrar, Richard B.** Some remarks on the motion of a satellite of an oblate planet. 66: 11—1961
- Barrett, Alan H.**
- , W. E. Howard, III, F. T. Haddock, M. E. Bair, J. J. Cook, and L. G. Cross. Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. 66: 37(A)—1961
- Howard, William E., III, —, and Fred T. Haddock.** The measurement of microwave radiation from the planet Mercury. 66: 287(A)—1961
- Basinski, Jane M.**
- Bok, Bart J., Priscilla F. Bok, and —.** Color-magnitude arrays for two associations in the Large Magellanic Cloud. 66: 279(A)—1961
- Baum, W. A.**
- Ford, W. K., Jr., L. W. Fredrick, John S. Hall, T. E. Houck, and —.** Evaluation of mica-window image tubes. 66: 43(A)—1961
- Wilcock, W. L. and —.** Astronomical tests of an imaging photomultiplier. 66: 58(A)—1961
- Bell, Barbara.** A north-south asymmetry in location of solar sources of great geomagnetic storms. 66: 38(A)—1961
- Belserene, Emilia Pisani.** Changes in the periods of RR Lyrae stars in Omega Centauri. 66: 38(A)—1961
- Berbert, J. H.** Star catalogues on punched cards and magnetic tape. 66: 279(A)—1961
- Bestul, S. M.**
- Adams, A. N. and —.** Meridian observations of the moon, 1958–59. 66: 36(N&O)—1961
- Bianco, Lucilla**
- Giannuzzi, Maria A. and —.** Mean densities of visual binary stars. 66: 418—1961
- Binnendijk, L.** Light variation and orbital elements of AK Herculis. 66: 27, 38(A)—1961
- Blanco, V. M. and C. J. Lennon.** The ratio of total to selective absorption. 66: 524—1961
- Boggess, Albert, III.** Stellar flux measurements in the middle ultraviolet. 66: 279(A)—1961
- Bok, Bart J., Priscilla F. Bok, and Jane M. Basinski.** Color-magnitude arrays for two associations in the Large Magellanic Cloud. 66: 279(A)—1961
- Bok, Priscilla F.**
- Bok, Bart J., —, and Jane M. Basinski.** Color-magnitude arrays for two associations in the Large Magellanic Cloud. 66: 279(A)—1961
- Bookmyer, Beverly B.** Observations and times of minima for three W Ursae Majoris systems. 66: 24—1961
- Bracewell, R. N.**
- A radio telescope with a minute-of-arc beam. 66: 279(A)—1961
- Little, A. G. and —.** The central component of Centaurus A. 66: 290(A)—1961
- Brandt, John C.** Helium resonance radiation in the night and day sky. 66: 279(A)—1961
- Brouwer, Dirk**
- and Gen-ichiro Hori. Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. Part II. 66: 39(A)—1961
- and Gen-ichiro Hori. Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. 66: 193—1961
- and Gen-ichiro Hori. Appendix to theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. 66: 264—1961
- Bryant, R. W.** The effect of solar radiation pressure on the motion of an artificial satellite. 66: 430—1961
- Burbidge, E. M. and G. R. Burbidge.** Recent investigations of groups and clusters of galaxies. 66: 541—1961
- Burbidge, G. R.**
- Burbidge, E. M. and —.** Recent investigations of groups and clusters of galaxies. 66: 541—1961
- Burke, B. F. and J. W. Fidor.** 405-Mc observations of the galactic center. 66: 39(A)—1961

- Burke, J. Anthony
Ingrao, Hector C., Donald H. Menzel, and —. Study of infrared instrumentation for thermal photography of the moon. 66: 287(A)—1961
- Burland, Miriam S.
Millman, Peter M. and —. Meteor magnitude distributions. 66: 291(A)—1961
- Buscombe, William
— and Pamela M. Morris. Three southern spectroscopic binaries. 66: 39(A)—1961
— and Pamela M. Morris. Supergiant B stars in the Small Magellanic Cloud. 66: 279(A)—1961
- Callender, E. David. On the almost periodicity of satellite motion. 66: 134—1961
- Cameron, A. G. W. Anomalous abundances of atmospheric xenon isotopes. 66: 40(A)—1961
- Cameron, Robert C. and Charles Perry. Narrow-band photometry of Bahcock's magnetic star HD 215441. 66: 40(A)—1961
- Cameron, Winifred Sawtell and John A. O'Keefe. Evidence of isostasy and differentiation on the moon. 66: 280(A)—1961
- Carter, A. W. L.
Hindman, J. V., R. X. McGee, —, and F. J. Kerr. Neutral hydrogen in the Magellanic Clouds—a repeat survey using a multichannel receiver and digital recording. 66: 45(A); erratum: 275—1961
- Chandrasekhar, S.
Horak, Henry G. and —. Diffuse reflection by a semi-infinite atmosphere. 66: 286(A)—1961
- Chiu, Hong Yee
— Neutrino emission processes in old stars. 66: 40(A)—1961
— Model for pre-supernova stars. 66: 280(A)—1961
- Colgate, Stirling A., William H. Grasberger, and Richard H. White. Dynamics of supernova explosions. 66: 280(A)—1961
- Collins, George W. and C. M. Huffer. Use of a digital computer in the solution for the elements of eclipsing binary stars. 66: 41(A)—1961
- Collins, Oliver C. and Walter F. Hiltner. Determination of the velocity vector of a space vehicle from apparent changes in the angular separations of stars. 66: 41(A)—1961
- Cook, Allan F. and Fred A. Franklin. On the Adams prize problem for 1856. 66: 41(A)—1961
- Cook, J. J.
Barrett, A. H., W. E. Howard, III, F. T. Haddock, M. E. Bair, —, and L. G. Cross. Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. 66: 37(A)—1961
- Copeland, Jack
Tyler, Warren C. and —. Lunar temperature measurements at 8.6-mm wavelength. 66: 56(A)—1961
- Corliss, C. H. Experimental f values for 70 elements. 66: 281(A)—1961
- Courtes, G. and S. R. Pottasch. Motions in bright rim structures in diffuse nebulae. 66: 41(A)—1961
- Cox, John P. On second helium ionization as a cause of Cepheid instability. 66: 42(A)—1961
- Crawford, D. L.
— and J. C. Golson. $H\beta$ photometry of the Pleiades. 66: 42(A)—1961
— Photometry of G and K giants. 66: 281(A)—1961
- Cross, L. G.
Barrett, A. H., W. E. Howard, III, F. T. Haddock, M. E. Bair, J. J. Cook, and —. Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. 66: 37(A)—1961
- Cuffey, James. NGC 5466. 66: 71—1961
- Dahn, Conard C.
Page, Thornton, —, and F. Foster Morrison. Statistics of the double galaxies, and their formation. 66: 614—1961
- Danielson, Robert E. Penumbra filaments as convection rolls. 66: 42(A)—1961
- Detwiler, C. R., J. D. Purcell, and R. Tousey. The solar continuum in the extreme ultraviolet. 66: 281(A)—1961
- Deutsch, A. J., P. W. Merrill, and P. C. Keenan. Behavior of absorption features in the spectra of Mira variables. 66: 282(A)—1961
- de Vaucouleurs, G.
— Transit of Mercury observed at Austin, Texas. 66: 36(N&O)—1961
— Recent studies of clusters and superclusters. 66: 629—1961
- Diliberto, S. P., W. T. Kyner, and R. B. Freund. Application of periodic surface theory to the study of satellite orbits. 66: 118—1961
- Donn, Bertram. Coma formation and an artificial comet experiment. 66: 282(A)—1961
- Dugan, John F.
Spalding, John, —, and Curtis L. Hemenway. Experiments with high-sensitivity image orthicons I. 66: 296(A)—1961
- Duncombe, Juliana S. and Dorrit Hoffleit. Star catalogues on punched cards. 66: 283(A)—1961
- Erickson, William C. Crab nebula occultation measurements and the structure of the solar corona. 66: 283(A)—1961
- Evans, John W. and Horst Mauter. Local Doppler effects in the photosphere. 66: 42(A)—1961
- Faulkner, D. J.
Aller, L. H. and —. Spectrophotometry of gaseous nebulae in the Magellanic Clouds. 66: 37(A)—1961
- Field, George B. Can the Jupiter emission be cyclotron radiation? 66: 283(A)—1961
- Fireman, E. L.
Wright, F. W., P. W. Hodge, and —. A search for micrometeorites in the earth's atmosphere. 66: 298(A)—1961
- Firor, J. W.
Burke, B. F. and —. 405-Mc observations of the galactic center. 66: 39(A)—1961
- Fish, Robert A. Luminosity distribution of the spiral galaxy NGC 5055. 66: 283(A)—1961
- Fleischer, Robert and Masakazu Oshima. Positions of 517-megacycle solar bursts. 66: 43(A)—1961
- Ford, W. K., Jr., L. W. Fredrick, John S. Hall, T. E. Houck, and W. A. Baum. Evaluation of mica-window image tubes. 66: 43(A)—1961
- Fowler, William A., Jesse L. Greenstein, and Fred Hoyle. Nucleosynthesis during the early history of the solar system. 66: 284(A)—1961
- Franklin, Fred A.
Cook, Allan F. and —. On the Adams prize problem for 1856. 66: 41(A)—1961
- Franz, Otto G.
Lenham, Alan P. and —. Astrometric investigations of galactic clusters. I. NGC 129. 66: 16—1961

- redrick, L. W.
 Ford, W. K., Jr., —, John S. Hall, T. E. Houck, and W. A. Baum. Evaluation of mica-window image tubes. 66: 43(A)—1961
 — Spectra of planetary nebulae and stars in the near infrared. 66: 44(A)—1961
 Freund, R. B.
 Diliberto, S. P., W. T. Kyner, and —. Application of periodic surface theory to the study of satellite orbits. 66: 118—1961
 Iaposhchkin, Sergei. An eclipsing "Astronaut." 66: 284(A)—1961
 Jarstang, R. H. Atomic transition probability calculations 66: 284(A)—1961
 Jehrels, Thomas. Proposed observations of polarization and interstellar reddening from above the ozone with high-altitude balloons. 66: 44(A)—1961
 Giannuzzi, Maria A. and Lucilla Bianco. Mean densities of visual binary stars. 66: 418—1961
 Gingerich, Owen. A computer program for nongray stellar atmospheres. 66: 285(A)—1961
 Godfredsen, Eugene A. Remarks on the dynamical stability of galaxy clusters. 66: 285(A)—1961
 Goldberg, Leo. Project West Ford—properties and analyses. Introduction. 66: 105—1961
 Goldstein, S. J., Jr. Spectral indices of 60 discrete sources. 66: 285(A)—1961
 Goles, Gordon G. and Edward Anders. Decay intervals of extinct radionuclides in meteorites and in the earth. 66: 44(A)—1961
 Golson, J. C.
 Crawford, D. L. and —. $H\beta$ photometry of the Pleiades. 66: 42(A)—1961
 Grasberger, William H.
 Colgate, Stirling A., —, and Richard H. White. Dynamics of supernova explosions. 66: 280(A)—1961
 Greenstein, Jesse L.
 Fowler, William A., —, and Fred Hoyle. Nucleosynthesis during the early history of the solar system. 66: 284(A)—1961
 Haddock, Fred T.
 Barrett, A. H., W. E. Howard, III, —, M. E. Bair, J. J. Cook, and L. G. Cross. Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. 66: 37(A)—1961
 Howard, William E., III, Alan H. Barrett, and —. The measurement of microwave radiation from the planet Mercury. 66: 287(A)—1961
 Hagihara, Yusuke and Yoshihide Kozai. Critical inclination case of the satellite motion. 66: 45(A)—1961
 Hall, John S.
 Ford, W. K., Jr., L. W. Fredrick, —, T. E. Houck, and W. A. Baum. Evaluation of mica-window image tubes. 66: 43(A)—1961
 Härm, R. and M. Schwarzschild. Helium flash in Population II giants. 66: 45(A)—1961
 Harris, Daniel E. Radio spectrum of supernova remnants. 66: 45(A)—1961
 Haynie, William H.
 Barnhart, Philip E., Walter E. Mitchell, Jr., Ronald E. Roll, and —. A program of stellar photometry at 3 to 4 microns. 66: 37(A)—1961
 Heckmann, O. On the possible influence of a general rotation on the expansion of the universe. 66: 599—1961
 Helfer, H. L.
 Wallerstein, George and —. Metal-rich G-dwarf 20 Leo Minoris and two comparison stars. 66: 57(A)—1961
 — The kinematics of the local gas system. 66: 160—1961
 Hemenway, Curtis L.
 Spalding, John and —. Meteor observations with an image orthicon I. 66: 54(A)—1961
 Spalding, John, John F. Dugan, and —. Experiments with high-sensitivity image orthicons I. 66: 296(A)—1961
 Herget, Paul
 — The orbit of Comet Schwassmann-Wachmann 1. 66: 266—1961
 — Erratum: Elements and ephemeris of Comet Schwassmann-Wachmann 1925 II. 66: 275—1961
 — Errata: Elements and ephemeris of Comet Wirtanen 1948 b. 66: 275—1961
 Hiltner, W. A. and —. A low-background image tube for direct electronography. 66: 286(A)—1961
 Hiltner, Walter F.
 Collins, Oliver C. and —. Determination of the velocity vector of a space vehicle from apparent changes in the angular separations of stars. 66: 41(A)—1961
 Hindman, J. V., R. X. McGee, A. W. L. Carter, and F. J. Kerr. Neutral hydrogen in the Magellanic Clouds—a repeat survey using a multichannel receiver and digital recording. 66: 45(A); erratum: 275—1961
 Hobbs, R. W.
 Stephenson, C. B. and R. W. Hobbs. A list of new OB stars near the galactic nebula M17. 66: 186—1961
 — A study of the region of M17 at a wavelength of 3.75 cm. 66: 517—1961
 Hodge, Paul W.
 — Fornax dwarf galaxy. I. The globular clusters. 66: 83—1961
 — Fornax dwarf galaxy. II. The distribution of stars. 66: 249—1961
 — The distribution of stars in the Sculptor dwarf galaxy. 66: 384—1961
 — The distribution of stars in dwarf elliptical galaxies. 66: 286(A)—1961
 Wright, F. W., —, and E. L. Fireman. A search for micrometeorites in the earth's atmosphere. 66: 298(A)—1961
 Hoffleit, Dorrit
 — Request for errata in star catalogues of position. 66: 156(N)—1961
 — Forty-five variable stars in VSF 193. 66: 188—1961
 Duncombe, Julena S. and —. Star catalogues on punched cards. 66: 283(A)—1961
 Smith, Harlan J. and —. Photographic history and suggested nature of the radio source 3C-48. 66: 295(A)—1961
 Høg, Erik. Note to "Problems in meridian astronomy." 66: 531(N&O)—1961
 Holmberg, Erik. On the dynamics of the Virgo cluster. 66: 620—1961
 Horak, Henry G. and S. Chandrasekhar. Diffuse reflection by a semi-infinite atmosphere. 66: 286(A)—1961
 Hori, Gen-ichiro
 Brouwer, Dirk and —. Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. Part II. 66: 39(A)—1961
 — A short note on the two fixed center problem in three dimensions. 66: 46(A)—1961
 Brouwer, Dirk and —. Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. 66: 193—1961
 — The motion of a hyperbolic artificial satellite around the oblate earth. 66: 258—1961
 Brouwer, Dirk and —. Appendix to theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. 66: 264—1961

- A modification of von Zeipel's method for a choice of canonical variables especially suitable for orbits with small eccentricities. **66**: 286(A)—1961
- Houck, T. E.
 Ford, W. K., Jr., L. W. Fredrick, John S. Hall, —, and W. A. Baum. Evaluation of mica-window image tubes. **66**: 43(A)—1961
- House, Lewis L. and R. Grant Athay. Departures from thermodynamic equilibrium in chromospheric *MgI*, *CaI*, and *OI*. **66**: 286(A)—1961
- Howard, William E., III
 Barrett, A. H., —, F. T. Haddock, M. E. Bair, J. J. Cook, and L. G. Cross. Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. **66**: 37(A)—1961
 —, Alan H. Barrett, and Fred T. Haddock. The measurement of microwave radiation from the planet Mercury. **66**: 287(A)—1961
 — Effects of antenna scan rate and radiometer time constant on receiver output. **66**: 521—1961
- Hoyle, Fred
 Fowler, William A., Jesse L. Greenstein, and —. Nucleosynthesis during the early history of the solar system. **66**: 284(A)—1961
- Huang, Su-Shu
 — Observational evidence and proposed test for the mode of star formation. **66**: 46(A)—1961
 — Some dynamical properties of natural and artificial satellites. **66**: 157—1961
- Huffer, C. M.
 Collins, George W. and —. Use of a digital computer in the solution for the elements of eclipsing binary stars. **66**: 41(A)—1961
- Hynek, J. A.
 Aikens, R., W. Powers, and —. High-contrast electronic image-orthicon techniques applied to the lunar surface. **66**: 277(A)—1961
- Ingrao, Hector C., Donald H. Menzel, and J. Anthony Burke. Study of infrared instrumentation for thermal photography of the moon. **66**: 287(A)—1961
- Irwin, John B. Least-squares tables for visual binaries. **66**: 46(A)—1961
- Izsak, Imre G.
 — On satellite orbits with very small eccentricities. **66**: 129—1961
 — A determination of the ellipticity of the earth's equator from the motion of two satellites. **66**: 226—1961
- Jones, D. E.
 Sagan, Carl, K. M. Siegel, and —. On the origin of the Venus microwave emission. **66**: 52(A)—1961
- Kakinuma, T.
 — Radio observations of solar active regions at cm-dm wavelengths. **66**: 287(A)—1961
 — and G. Swarup. Polarization measurements of solar emission at a wavelength of 9.1 cm. **66**: 288(A)—1961
- Kalloghjan, A. On the dynamical instability of several groups of galaxies. **66**: 554—1961
- Kaplan, Lewis D. The structure and CO_2 content of Venus' atmosphere. **66**: 288(A)—1961
- Karmiöl, Edwin D. and John S. Youtcheff. Reliability approach to astronomical satellites. **66**: 47(A)—1961
- Keenan, P. C.
 Deutsch, A. J., P. W. Merrill, and —. Behavior of absorption features in the spectra of Mira variables. **66**: 282(A)—1961
- Kerr, F. J.
 Hindman, J. V., R. X. McGee, A. W. L. Carter, and —. Neutral hydrogen in the Magellanic Clouds—a repeat survey using a multichannel receiver and digital recording. **66**: 45(A); erratum: 275—1961
 — A new 21-cm interpretation of the spiral structure and motions of the Galaxy. **66**: 47(A)—1961
- Keivorkian, Jirair K. A uniformly valid asymptotic representation for all times of the motion of a satellite in the vicinity of the smaller body in the restricted three-body problem. **66**: 288(A)—1961
- King, Ivan
 — Star distribution in the globular cluster M15. **66**: 47(A)—1961
 — Shape of a rotating star cluster. **66**: 68—1961
- Kinman, T. D. Photoelectric observations of SX Phoenixis. **66**: 348—1961
- Kippenhahn, R.
 Baker, N. and —. The pulsations of models of δ Cephei stars. **66**: 278(A)—1961
- Koch, Robert H.
 — Times of minimum light for several eclipsing binaries. **66**: 35(N&O)—1961
 — Departures from the Russell model in TX Ursae Majoris. **66**: 230—1961
- Kozai, Yoshihide
 — Gravitational field of the earth derived from motions of three satellites. **66**: 8—1961
- Hagihara, Yusuke and —. Critical inclination case of the satellite motion. **66**: 45(A)—1961
 — Note on the motion of a close earth satellite with a small eccentricity. **66**: 132—1961
 — Tesseral harmonics of the gravitational potential of the earth as derived from satellite motions. **66**: 355—1961
- Kumar, S. S. A method for computing monochromatic fluxes in model stellar atmospheres. **66**: 288(A)—1961
- Kundu, M. R. Some studies on the occurrence of type IV solar bursts of continuum radiation. **66**: 289(A)—1961
- Kyner, W. T.
 Diliberto, S. P., —, and R. B. Freund. Application of periodic surface theory to the study of satellite orbits. **66**: 118—1961
- LaGow, H. E.
 Alexander, W. M., C. W. McCracken, and —. Dust particles of micron size associated with the Leonid meteor stream. **66**: 277(A)—1961
- Lancaster, E. R. The calculation of velocity from an ephemeris. **66**: 289(A)—1961
- Lassovszky, Karoly. On the accuracy of measurements made upon films photographed by Baker-Nunn satellite tracking cameras. **66**: 289(A)—1961
- Layzer, David. A remark on observational cosmology. **66**: 289(A)—1961
- Lemaître, G. Exchange of galaxies between clusters and field. **66**: 563—1961
- Lenham, Alan P.
 — and Otto G. Franz. Astrometric investigations of galactic clusters. I. NGC 129. **66**: 16—1961
- Strand, K. Aa., —, and T. Owen. Errata: A survey of stars with large proper motions in the region of the Orion constellation. **66**: 275—1961
- Lennon, C. J.
 Blanco, V. M. and —. The ratio of total to selective absorption. **66**: 524—1961
- Liller, William
 — Report on the effects of Project West Ford on optical astronomy. **66**: 114—1961

- Photoelectric photometry of comets. 66: 372—1961
- Tull, Robert G. and —.** Observed and computed white-dwarf energy curves. 66: 297(A)—1961
- Ulley, A. E.**
— Radio properties of an orbiting scattering medium. 66: 116—1961
— The temperature of Venus. 66: 290(A)—1961
- Ulmer, D. Nelson**
— Dynamics of the Pleiades cluster. 66: 48(A)—1961
— Virial theorem and the stability of clusters of galaxies. 66: 572—1961
- Lindsay, E. M.** A new catalogue of emission-line stars and planetary nebulae in the Small Magellanic Cloud. 66: 169—1961
- Lippincott, Sarah Lee.** Parallax and orbital motion of 42 Comae=ADS 8804 from plates taken with the 24-inch Sproul reflector. 66: 272—1961
- Little, A. G. and R. N. Bracewell.** The central component of Centaurus A. 66: 290(A)—1961
- MacLellan, D. C.**
— **Morrow, W. E., Jr., and —.** Properties of orbiting dipole belts. 66: 107—1961
- Markarian, B. E.** Physical chain of galaxies in the Virgo cluster and its dynamic instability. 66: 555—1961
- Marsden, B. G.** The orbit and ephemeris of periodic comet Oterma. 66: 246—1961
- Mauter, Horst**
— **Evans, John W. and —.** Local Doppler effects in the photosphere. 66: 42(A)—1961
- Mayall, N. U.** Observing time on Kitt Peak. 66: 274(N)—1961
- McCarthy, Martin F.** Fifty carbon stars in a region in Cygnus. 66: 291(A)—1961
- McCluskey, George E., Jr.**
— **Wood, Frank Bradshaw and —.** The eclipsing system V Tucanae. 66: 413—1961
- McCracken, C. W.**
— **Alexander, W. M., —, and H. E. LaGow.** Dust particles of micron size associated with the Leonid meteor stream. 66: 277(A)—1961
- McGee, R. X.**
— **Hindman, J. V., —, A. W. L. Carter, and F. J. Kerr.** Neutral hydrogen in the Magellanic Clouds—a repeat survey using a multichannel receiver and digital recording. 66: 45(A); erratum: 275—1961
— **and J. D. Murray.** Distribution of local hydrogen from a 21-cm multichannel survey of the sky. 66: 48(A); erratum: 275—1961
- McIntosh, Patrick S.**
— **Smith, Elske v. P. and —.** Spectrum of the flare of September 2, 1960. 66: 54(A)—1961
- McLaughlin, Dean B.**
— Latest cycle of the Be shell star β Mon. 66: 48(A)—1961
— An attempt to simulate supernova spectra. 66: 291(A)—1961
— Visual photometric observations of W Serpentis. 66: 351—1961
- McVittie, G. C.** Radio telescope of the University of Illinois. 66: 48(A)—1961
- Menzel, Donald H.**
— **Ingrao, Hector C., —, and J. Anthony Burke.** Study of infrared instrumentation for thermal photography of the moon. 66: 287(A)—1961
- Merrill, P. W.**
— **Deutsch, A. J., —, and P. C. Keenan.** Behavior of absorption features in the spectra of Mira variables. 66: 282(A)—1961
- Michie, Richard W.** Structure and evolution of globular clusters. 66: 49(A)—1961
- Milford, S. N.**
— X rays from collisions of stars. 66: 49(A)—1961
- Shen, S. P. and —.** On gamma rays from intergalactic space. 66: 295(A)—1961
- Milligan, J. E.**
— **Stecher, Theodore P. and —.** Stellar spectrophotometry below 3000 angstroms. 66: 296(A)—1961
- Millman, Peter M. and Miriam S. Burland.** Meteor magnitude distributions. 66: 291(A)—1961
- Minkowski, R.** NGC 6166 and the cluster Abell 2199. 66: 558—1961
- Mitchell, Walter E., Jr.**
— **Barnhart, Philip E., —, Ronald E. Roll, and William H. Haynie.** A program of stellar photometry at 3 to 4 microns. 66: 37(A)—1961
- Moffet, Alan T.** Structure of Virgo A. 66: 49(A)—1961
- Morris D.**
— **Radhakrishnan, V., —, and R. W. Wilson.** Hydrogen-line interferometer. 66: 51(A)—1961
- Morris, Pamela M.**
— **Buscombe, William and —.** Three southern spectroscopic binaries. 66: 39(A)—1961
— **Buscombe, William and —.** Supergiant B stars in the Small Magellanic Cloud. 66: 279(A)—1961
- Morrison, F. Foster**
— **Page, Thornton, Conard C. Dahn, and —.** Statistics of the double galaxies, and their formation. 66: 614—1961
- Morrison, John and Samuel Pines.** Reduction from geocentric to geodetic coordinates. 66: 15—1961
- Morrow, W. E., Jr., and D. C. MacLellan.** Properties of orbiting dipole belts. 66: 107—1961
- Muhleman, Duane O.** Early results of the 1961 JPL Venus radar experiment. 66: 292(A)—1961
- Murray, J. D.**
— **McGee, R. X. and —.** Distribution of local hydrogen from a 21-cm multichannel survey of the sky. 66: 48(A); erratum: 275—1961
- Neyman, Jerzy**
— **and Elizabeth L. Scott.** Estimation of the dispersion of redshift of field galaxies. 66: 50(A), 148—1961
— **and Elizabeth L. Scott.** Magnitude-redshift relation for galaxies in clusters in the presence of instability and absorption. 66: 581—1961
- Nigam, R. C.** A determination of the atmospheric oblateness from the motion of two low satellites. 66: 292(A)—1961
- Niklas, W. F.**
— **Hiltner, W. A. and —.** A low-background image tube for direct electronography. 66: 286(A)—1961
- Öpik, Ernst J.**
— Tidal deformations and the origin of the moon. 66: 50(A), 60—1961
— The survival of comets and cometary material. 66: 381—1961
- O'Keefe, John A.**
— **Cameron, Winifred Sawtell and —.** Evidence of isostasy and differentiation on the moon. 66: 280(A)—1961
— The cometary theory of tektite origin. 66: 292(A)—1961
- Orrall, Frank Q. and Henry J. Smith.** The passage of a flare spray through the solar emission corona. 66: 293(A)—1961
- Oshima, Masakazu**
— **Fleischer, Robert and —.** Positions of 517-megacycle solar bursts. 66: 43(A)—1961

- Oster, Ludwig. Free-free emission in the radio-frequency range. 66: 50(A)—1961
- Osterbrock, Donald E. Heating of the solar chromosphere, plages, and corona by magnetohydrodynamic waves. 66: 50(A)—1961
- Owen, T.
Strand, K. Aa., A. Lenham, and —. Errata: A survey of stars with large proper motions in the region of the Orion constellation. 66: 275—1961
- Page, Thornton
— Frequency of double galaxies in clusters. 66: 51(A)—1961
—, Conard C. Dahn, and F. Foster Morrison. Statistics of the double galaxies, and their formation. 66: 614—1961
- Payne-Gaposchkin, Cecilia. Variable stars in the Small Magellanic Cloud. 66: 293(A)—1961
- Penzias, A. A. Free hydrogen in the Pegasus I cluster of galaxies. 66: 293(A)—1961
- Perry, Charles
Cameron, Robert C. and —. Narrow-band photometry of Babcock's magnetic star HD 215441. 66: 40(A)—1961
- Pines, Samuel
— Variation of parameters for elliptic and near circular orbits. 66: 5—1961
Morrison, John and —. Reduction from geocentric to geodetic coordinates. 66: 15—1961
— Asymptotically singular differential correction matrices. 66: 293(A)—1961
- Porter, William S. The constitutions of Uranus and Neptune. 66: 243—1961
- Pottasch, Stuart R.
Courtes, G. and —. Motions in bright rim structures in diffuse nebulae. 66: 41(A)—1961
— Balmer line emission from a shell. 66: 294(A)—1961
- Powers, W.
Aikens, R., —, and J. A. Hynek. High-contrast electronic image-orthicon techniques applied to the lunar surface. 66: 277(A)—1961
- Purcell, J. D.
Detwiler, C. R., —, and R. Tousey. The solar continuum in the extreme ultraviolet. 66: 281(A)—1961
- Rabe, Eugene. Determination and survey of periodic Trojan orbits in the restricted problem of three bodies. 66: 500—1961
- Radhakrishnan, V., D. Morris, and R. W. Wilson. Hydrogen-line interferometer. 66: 51(A)—1961
- Roberts, Morton S. 21-cm maser observations of galaxies. 66: 294(A)—1961
- Roemer, Elizabeth. Astrometric observations and orbits of comets. 66: 368—1961
- Roll, Ronald E.
Barnhart, Philip E., Walter E. Mitchell, Jr., —, and William H. Haynie. A program of stellar photometry at 3 to 4 microns. 66: 37(A)—1961
- Rosen, B. New investigations on the C_2 spectrum at 4050 Å. 66: 383—1961
- Saari, J. M. and R. W. Shorthill. Infrared isothermal mapping of certain crater areas of the moon. 66: 52(A)—1961
- Sagan, Carl
— Abundance of water vapor on Mars. 66: 52(A)—1961
—, K. M. Siegel, and D. E. Jones. On the origin of the Venus microwave emission. 66: 52(A)—1961
- Sandage, Allan R. Ages of the open cluster NGC 188 and the globular clusters M3, M5, and M13 compared with the Hubble time. 66: 53(A)—1961
- Sanduleak, Nicholas. Hypersensitization gains in the near infrared. 66: 53(A), 526—1961
- Schwarzschild, M.
Härm, R. and —. Helium flash in Population II giants. 66: 45(A)—1961
- Scott, Elizabeth L.
Neyman, Jerzy and —. Estimation of the dispersion of redshift of field galaxies. 66: 50(A), 148—1961
Neyman, Jerzy and —. Magnitude-redshift relation for galaxies in clusters in the presence of instability and absorption. 66: 581—1961
- Seeger, C. L. and G. Westerhout. Observations of a plane-polarized component in the cosmic background radiation of 408 Mc. 66: 294(A)—1961
- Serkowski, K. Determination of differential limb darkening of eclipsing binaries from multicolor photometric observations. I. 66: 405—1961
- Sharov, A. S. On the photometric catalogue of stars in the region of the Orion nebula, compiled by P. P. Parenago. 66: 103(N&O)—1961
- Shen, S. P. and S. N. Milford. On gamma rays from intergalactic space. 66: 295(A)—1961
- Shorthill, R. W.
Saari, J. M. and —. Infrared isothermal mapping of certain crater areas of the moon. 66: 52(A)—1961
- Siegel, K. M.
Sagan, Carl, —, and D. E. Jones. On the origin of the Venus microwave emission. 66: 52(A)—1961
- Smith, Elske v. P. and Patrick S. McIntosh. Spectrum of the flare of September 2, 1960. 66: 54(A)—1961
- Smith, Harlan J. and Dorrit Hoffleit. Photographic history and suggested nature of the radio source 3C-48. 66: 295(A)—1961
- Smith, Henry J.
Orrall, Frank Q. and —. The passage of a flare spray through the solar emission corona. 66: 293(A)—1961
- Smith, O. K. Computation of coordinates from Brouwer's solution of the artificial satellite problem. 66: 359—1961
- Southworth, Richard B. Planetary perturbations and the Perseid meteor stream. 66: 295(A)—1961
- Spalding, John
— and Curtis L. Hemenway. Meteor observations with an image orthicon I. 66: 54(A)—1961
—, John F. Dugan, and Curtis L. Hemenway. Experiments with high-sensitivity image orthicons I. 66: 296(A)—1961
- Spradley, L. Harold, Jr. The new University of Georgia Observatory. 66: 54(A)—1961
- Stecker, Theodore P. and J. E. Milligan. Stellar spectrophotometry below 3000 angstroms. 66: 296(A)—1961
- Stephenson, C. B.
— Possible M-star members of NGC 6838. 66: 85—1961
— and R. W. Hobbs. A list of new OB stars near the galactic nebula M17. 66: 186—1961
- Stone, Yvonne H. and George Wallerstein. Spectrophotometric comparison of 7 Sextantis and Sirius. 66: 54(A)—1961
- Strand, K. Aa., A. Lenham, and T. Owen. Errata: A survey of stars with large proper motions in the region of the Orion constellation. 66: 275—1961
- Strong, John. Use of balloons and high-altitude aircraft for precursor satellite studies. 66: 54(A)—1961
- Struve, Otto. B8 component of β Lyrae. 66: 55(A)—1961
- Swarp, G.
Kakinuma, T. and —. Polarization measurements of solar emission at a wavelength of 9.1 cm. 66: 288(A)—1961
— Radio pictures of the sun at a wavelength of 9.1 cm. 66: 296(A)—1961

- Wihart, Thomas L.** Boundary conditions for stellar interior models. **66**: 297(A)—1961
- Wolpe, Henrietta H.**
- Baade, Walter and —.** The Draco system, a dwarf galaxy. **66**: 278(A), 300—1961
- Weske, R. G.** Numerical integrations for nonlinear compression waves and line profiles in Cepheid atmospheres. **66**: 55(A)—1961
- Wift, William Grant**
- Multicolor photoelectric photometry of bright galaxies. I. **66**: 390—1961
- The region of NGC 121 in the Small Magellanic Cloud. **66**: 297(A)—1961
- Wousey, R.**
- Detwiler, C. R., J. D. Purcell, and —.** The solar continuum in the extreme ultraviolet. **66**: 281(A)—1961
- Full, Robert G. and William Liller.** Observed and computed white-dwarf energy curves. **66**: 297(A)—1961
- Fyler, Warren C. and Jack Copeland.** Lunar temperature measurements at 8.6-mm wavelength. **66**: 56(A)—1961
- van Albada, C. B.** Evolution of clusters of galaxies under gravitational forces. **66**: 590—1961
- van Biesbroeck, G.**
- Comet observations. **66**: 96—1961
- Orbit of Comet 1948 I (Bester). **66**: 424—1961
- A search for stars of low luminosity. **66**: 528—1961
- van den Bergh, Sidney**
- Clustering of galaxies. **66**: 56(A)—1961
- Radio sources and clusters of galaxies. **66**: 297(A), 562—1961
- Stability of clusters of galaxies. **66**: 566—1961
- Vandervoort, Peter O.** Rayleigh-Taylor instability in compressible fluids. **66**: 56(A)—1961
- van der Waerden, B. L.** Secular terms and fluctuations in the motions of the sun and the moon. **66**: 138—1961
- van Herk, G. and A. J. J. van Woerkom.** Problems in meridian astronomy. **66**: 87—1961
- van Woerkom, A. J. J.**
- van Herk, G. and —.** Problems in meridian astronomy. **66**: 87—1961
- Vinti, John P.** Formulae for an accurate intermediary orbit of an artificial satellite. **66**: 514—1961
- Vorontsov-Velyaminov, B.** On the possibility of explosive formation of galaxy patterns. **66**: 551—1961
- Waak, John A.** An investigation of possible variations in the brightness temperature of the moon at 21 cm. **66**: 298(A)—1961
- Wallerstein, George**
- Stone, Yvonne H. and —.** Spectrophotometric comparison of 7 Sextantis and Sirius. **66**: 54(A)—1961
- and **H. L. Helfer.** Metal-rich G-dwarf 20 Leo Minoris and two comparison stars. **66**: 57(A)—1961
- Weinberg, J. L.** On the distribution of electron density inside the earth's orbit. **66**: 57(A)—1961
- Wentzel, Donat G.** Galactic motions in a large-scale magnetic field. **66**: 57(A)—1961
- Westerhout, G.**
- Seeger, C. L. and —.** Observations of a plane-polarized component in the cosmic background radiation of 408 Mc. **66**: 294(A)—1961
- Westerlund, Bengt**
- A heavily reddened cluster in Ara. **66**: 57(A)—1961
- Note on the nebulosity around RS Puppis. **66**: 57(A)—1961
- Population I in the Large Magellanic Cloud. **66**: 57(A)—1961
- Whipple, Fred L.** Problems of the cometary nucleus. **66**: 375—1961
- White, Richard H.**
- Colgate, Stirling A., William H. Grasberger, and —.** Dynamics of supernova explosions. **66**: 280(A)—1961
- Widing, Kenneth G.** The computed solar continuum in the ultraviolet. **66**: 298(A)—1961
- Wilcock, W. L. and W. A. Baum.** Astronomical tests of an imaging photomultiplier. **66**: 58(A)—1961
- Wildt, Rupert.** The gray planetary atmosphere. **66**: 298(A)—1961
- Wilson, R. W.**
- Radhakrishnan, V., D. Morris, and —.** Hydrogen-line interferometer. **66**: 51(A)—1961
- Wilson, Raymond H., Jr.** Bolometric solar constant related to radiation pressure effects on satellites. **66**: 58(A)—1961
- Wood, Frank Bradshaw and George E. McCluskey, Jr.** The eclipsing system V Tucanae. **66**: 413—1961
- Wright, F. W., P. W. Hodge, and E. L. Fireman.** A search for micrometeorites in the earth's atmosphere. **66**: 298(A)—1961
- Wurm, K.** Structure and development of the gas tails of comets. **66**: 362—1961
- Wyatt, Stanley P.** The diurnal atmospheric bulge and its effect on satellite motions. **66**: 299(A)—1961
- Young, Andrew T.** Stellar kinematics and spiral arms. **66**: 299(A)—1961
- Youtcheff, John S.**
- Karmiol, Edwin D. and —.** Reliability approach to astronomical satellites. **66**: 47(A)—1961
- Zadunaisky, Pedro E.** Atmospheric drag on artificial satellites. **66**: 59(A)—1961

Subject Index to Volume 66

Numerals in boldface type refer to volume numbers. (A) indicates abstract. (S) indicates symposium. (N&O) indicates notes and observations. (N) indicates notices.

Aberration

The determination of the velocity vector of a space vehicle from apparent changes in the angular separations of stars. Oliver C. Collins and Walter F. Hiltner. **66**: 41—1961(A)

American Astronomical Society

Abstracts, 107th Meeting. **66**: 37—1961
Abstracts, 108th Meeting. **66**: 277—1961
Symposium on Comets. **66**: 361—1961

Asteroids

Determination and survey of periodic Trojan orbits in the restricted problem of three bodies. Eugene Rabe. **66**: 500—1961

Astrometry

Astrometric observations and orbits of comets. Elizabeth Roemer. **66**: 368—1961(S)

Atmospheres, Planetary

The structure and CO_2 content of Venus' atmosphere. Lewis D. Kaplan. **66**: 288—1961(A)
The gray planetary atmosphere. Rupert Wildt. **66**: 298—1961(A)

Atmosphere, Terrestrial

Anomalous abundances of atmospheric xenon isotopes. A. G. W. Cameron. **66**: 40—1961(A)
Atmospheric drag on artificial satellites. Pedro E. Zadunaisky. **66**: 59—1961(A)
Helium resonance radiation in the night and day sky. John C. Brandt. **66**: 279—1961(A)
A determination of the atmospheric oblateness from the motion of two low satellites. R. C. Nigam. **66**: 292—1961(A)
The diurnal atmospheric bulge and its effect on satellite motions. Stanley P. Wyatt. **66**: 299—1961(A)

Atomic (and Molecular) Structure and Spectra

Experimental f values for 70 elements. C. H. Corliss. **66**: 281—1961(A)
Atomic transition probability calculations. R. H. Garstang. **66**: 284—1961(A)

Balloon Astronomy

Proposed observations of polarization and interstellar reddening from above the ozone with high-altitude balloons. Thomas Gehrels. **66**: 44—1961(A)
Use of balloons and high-altitude aircraft for precursor satellite studies. John Strong. **66**: 54—1961(A)

Celestial Mechanics

Discussion on longitude differences of catalogues in determining ephemeris time. Shinko Aoki. **66**: 1—1961
Variation of parameters for elliptic and near circular orbits. Samuel Pines. **66**: 5—1961
The gravitational field of the earth derived from motions of three satellites. Yoshihide Kozai. **66**: 8—1961
Some remarks on the motion of a satellite of an oblate planet. Richard B. Barrar. **66**: 11—1961
The reduction from geocentric to geodetic coordinates. John Morrison and Samuel Pines. **66**: 15—1961
Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite, Part II. Dirk Brouwer and Gen-ichiro Hori. **66**: 39—1961(A)
The critical inclination case of the satellite motion. Yusuke Hagihara and Yoshihide Kozai. **66**: 45—1961(A)
A short note on the two fixed center problem in three dimensions. Gen-ichiro Hori. **66**: 46—1961(A)

Application of periodic surface theory to the study of satellite orbits. S. P. Diliberto, W. T. Kyner, and R. P. Freund. **66**: 118—1961

On satellite orbits with very small eccentricities. Imre G. Izsak. **66**: 129—1961

Note on the motion of a close earth satellite with a small eccentricity. Yoshihide Kozai. **66**: 132—1961

On the almost periodicity of satellite motion. E. David Callender. **66**: 134—1961

Some dynamical properties of natural and artificial satellites. Su-Shu Huang. **66**: 157—1961

Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. Dirk Brouwer and Gen-ichiro Hori. **66**: 193—1961

A determination of the ellipticity of the earth's equator from the motion of two satellites. Imre G. Izsak. **66**: 226—1961

The motion of a hyperbolic artificial satellite around the oblate earth. Gen-ichiro Hori. **66**: 258—1961

Appendix to Theoretical Evaluation of Atmospheric Drag Effects in the Motion of an Artificial Satellite. Dirk Brouwer and Gen-ichiro Hori. **66**: 264—1961

A modification of von Zeipel's method for a choice of canonical variables especially suitable for orbits with small eccentricities. Gen-ichiro Hori. **66**: 286—1961(A)

A uniformly valid asymptotic representation for all time of the motion of a satellite in the vicinity of the smaller body in the restricted three-body problem. Jirair K. Kevorkian. **66**: 288—1961(A)

The calculation of velocity from an ephemeris. E. R. Lancaster. **66**: 289—1961(A)

A determination of the atmospheric oblateness from the motion of two low satellites. R. C. Nigam. **66**: 292—1961(A)

Asymptotically singular differential correction matrices. Samuel Pines. **66**: 293—1961(A)

Tesseral harmonics of the gravitational potential of the earth as derived from satellite motions. Yoshihide Kozai. **66**: 355—1961

Computation of coordinates from Brouwer's solution of the artificial satellite problem. O. K. Smith. **66**: 359—1961

The effect of solar radiation pressure on the motion of an artificial satellite. R. W. Bryant. **66**: 430—1961

Determination and survey of periodic Trojan orbits in the restricted problem of three bodies. Eugene Rabe. **66**: 500—1961

Formulae for an accurate intermediary orbit of an artificial satellite. John P. Vinti. **66**: 514—1961

Color-magnitude Relations

NGC 5466. James Cuffey. **66**: 71—1961

On the photometric catalogue of stars in the region of the Orion nebula, compiled by P. P. Parenago. A. S. Sharov. **66**: 103—1961(N&O)

Color-magnitude arrays for two associations in the Large Magellanic Cloud. Bart J. Bok, Priscilla F. Bok, and Jane M. Basinski. **66**: 279—1961(A)

Comets

The orbit and ephemeris of periodic comet Oterma. B. G. Marsden. **66**: 246—1961

The orbit of Comet Schwassmann-Wachmann 1. Paul Herget. **66**: 266—1961

Coma formation and an artificial comet experiment. Bertram Donn. **66**: 282—1961(A)

The cometary theory of tektite origin. John A. O'Keefe. **66**: 292—1961(A)

- Structure and development of the gas tails of comets. K. Würm. **66**: 362—1961(S)
- Astrometric observations and orbits of comets. Elizabeth Roemer. **66**: 368—1961(S)
- Photoelectric photometry of comets. William Liller. **66**: 372—1961(S)
- Problems of the cometary nucleus. Fred L. Whipple. **66**: 375—1961(S)
- The survival of comets and cometary material. Ernst J. Öpik. **66**: 381—1961(S)
- New investigations on the C_2 spectrum at 4050 Å. B. Rosen. **66**: 383—1961(S)
- Definitive orbit of Comet 1948 I (Bester). G. Van Biesbroeck. **66**: 424—1961
- Comets, Observations of**
- 956 *c* (Wirtanen)
Comet observations. G. Van Biesbroeck. **66**: 96—1961
- 957 *d* (Mrkos)
Comet observations. G. Van Biesbroeck. **66**: 97—1961
- 957 *g* (Periodic Harrington 1952 II)
Comet observations. G. Van Biesbroeck. **66**: 97—1961
- 958 *a* (Burnham)
Comet observations. G. Van Biesbroeck. **66**: 97—1961
- 958 *d* (Periodic Kopff 1906 IV)
Comet observations. G. Van Biesbroeck. **66**: 98—1961
- 958 *e* (Burnham-Slaughter)
Comet observations. G. Van Biesbroeck. **66**: 98—1961
- 959 *b* (Periodic Giacobini-Zinner)
Comet observations. G. Van Biesbroeck. **66**: 99—1961
- 959 *c* (Periodic Arend)
Comet observations. G. Van Biesbroeck. **66**: 99—1961
- 959 *e* (Alcock)
Comet observations. G. Van Biesbroeck. **66**: 99—1961
- 959 *f* (Alcock)
Comet observations. G. Van Biesbroeck. **66**: 99—1961
- 960 *a* (Burnham)
Comet observations. G. Van Biesbroeck. **66**: 100—1961
- 960 *b* (Wild)
Comet observations. G. Van Biesbroeck. **66**: 100—1961
- 942 VII (Periodic Oterma)
Comet observations. G. Van Biesbroeck. **66**: 100—1961
- 925 II (Periodic Schwassmann-Wachmann 1)
Comet observations. G. Van Biesbroeck. **66**: 100—1961
- Computation, Art of**
- The use of a digital computer in the solution for the elements of eclipsing binary stars. George W. Collins and C. M. Huffer. **66**: 41—1961(A)

Cosmology

- Nucleosynthesis during the early history of the solar system. William A. Fowler, Jesse L. Greenstein, and Fred Hoyle. **66**: 184—1961(A)
- A remark on observational cosmology. David Layzer. **66**: 289—1961(A)
- On the possible influence of a general rotation on the expansion of the universe. O. Heckmann. **66**: 599—1961

Earth

- The gravitational field of the earth derived from motions of three satellites. Yoshihide Kozai. **66**: 8—1961
- Decay intervals of extinct radionuclides in meteorites and in the earth. Gordon G. Goles and Edward Anders. **66**: 44—1961(A)
- A determination of the ellipticity of the earth's equator from the motion of two satellites. Imre G. Izsak. **66**: 226—1961
- Tesseral harmonics of the gravitational potential of the earth as derived from satellite motions. Yoshihide Kozai. **66**: 355—1961

Errata

- 66**: 275—1961

Galactic Structure

- 405 Mc observations of the galactic center. B. F. Burke and J. W. Firor. **66**: 39—1961(A)
- A new 21-cm interpretation of the spiral structure and motions of the galaxy. F. J. Kerr. **66**: 47—1961(A)
- The distribution of local hydrogen from a 21-cm multi-channel survey of the sky. R. X. McGee and J. D. Murray. **66**: 48—1961(A) [Note erratum, p. 275]
- Galactic motions in a large-scale magnetic field. Donat G. Wentzel. **66**: 57—1961(A)
- The kinematics of the local gas system. H. L. Helfer. **66**: 160—1961
- Fifty carbon stars in a region in Cygnus. Martin F. McCarthy S.J. **66**: 291—1961(A)
- Observations of a plane-polarized component in the cosmic background radiation of 408 Mc. C. L. Seeger and G. Westerhout. **66**: 294—1961(A)
- Stellar kinematics and spiral arms. Andrew T. Young. **66**: 299—1961(A)

Galaxies

- The structure of Virgo A. Alan T. Moffet. **66**: 49—1961(A)
- The frequency of double galaxies in clusters. Thornton Page. **66**: 51—1961(A)
- The clustering of galaxies. Sidney van den Bergh. **66**: 56—1961(A)
- The Fornax dwarf galaxy. I. The globular clusters. Paul W. Hodge. **66**: 83—1961
- The Fornax dwarf galaxy. II. The distribution of stars. Paul W. Hodge. **66**: 249—1961
- The Draco system, a dwarf galaxy. W. Baade and H. H. Swope. **66**: 278—1961(A)
- Luminosity distribution of the spiral galaxy NGC 5055. Robert A. Fish. **66**: 283—1961(A)
- Remarks on the dynamical stability of galaxy clusters. Eugene A. Godfredsen. **66**: 285—1961(A)
- The distribution of stars in dwarf elliptical galaxies. Paul W. Hodge. **66**: 286—1961(A)
- A remark on observational cosmology. David Layzer. **66**: 289—1961(A)
- The central component of Centaurus A. A. G. Little and R. N. Bracewell. **66**: 290—1961(A)
- Free hydrogen in the Pegasus I cluster of galaxies. A. A. Penzias. **66**: 293—1961(A)
- 21-cm maser observations of galaxies. Morton S. Roberts. **66**: 294—1961(A)
- Radio sources and clusters of galaxies. Sidney van den Bergh. **66**: 297—1961(A)
- The Draco system, a dwarf galaxy. Walter Baade and Henrietta H. Swope. **66**: 300—1961
- The distribution of stars in the Sculptor dwarf galaxy. Paul W. Hodge. **66**: 384—1961
- Multicolor photoelectric photometry of bright galaxies I. William Grant Tift. **66**: 390—1961
- Instability phenomena in systems of galaxies. V. A. Ambartsumian. **66**: 536—1961
- Recent investigations of groups and clusters of galaxies. E. M. Burbidge and G. R. Burbidge. **66**: 541—1961
- On the possibility of explosive formation of galaxy patterns. B. Vorontsov-Veyaminov. **66**: 551—1961
- On the dynamical instability of several groups of galaxies. A. Kalloshjan. **66**: 554—1961
- Physical chain of galaxies in the Virgo cluster and its dynamic instability. B. E. Markarian. **66**: 555—1961
- NGC 6166 and the cluster Abell 2199. R. Minkowski. **66**: 558—1961
- Radio sources and clusters of galaxies. Sidney van den Bergh. **66**: 562—1961
- The stability of clusters of galaxies. Sidney van den Bergh. **66**: 566—1961

Galaxies (Continued)

- The virial theorem and the stability of clusters of galaxies. D. Nelson Limber. **66**: 572—1961
- Magnitude-redshift relation for galaxies in clusters in the presence of instability and absorption. J. Neyman and E. L. Scott. **66**: 581—1961
- Evolution of clusters of galaxies under gravitational forces. G. B. van Albada. **66**: 590—1961
- The exchange of galaxies between clusters and field. G. Lemaitre. **66**: 603—1961
- Evidence regarding second-order clustering of galaxies and interactions between clusters of galaxies. George O. Abell. **66**: 607—1961
- Statistics of the double galaxies, and their formation. T. Page, C. C. Dahn, and F. F. Morrison. **66**: 614—1961
- On the dynamics of the Virgo cluster. Erik Holmberg. **66**: 620—1961
- Recent studies of clusters and superclusters. G. de Vaucouleurs. **66**: 629—1961

Galaxies, Redshifts

- Estimation of the dispersion of redshift for field galaxies. Jerzy Neyman and Elizabeth L. Scott. **66**: 50—1961(A)
- Estimation of the dispersion of the redshift of field galaxies. J. Neyman and E. L. Scott. **66**: 148—1961

Gamma Rays

- On gamma rays from intergalactic space. S. P. Shen and S. N. Milford. **66**: 295—1961(A)

Geomagnetism

- A north-south asymmetry in location of solar sources of great geomagnetic storms. Barbara Bell. **66**: 38—1961(A)

Hydrodynamics

- The Rayleigh-Taylor instability in compressible fluids. Peter O. Vandervoort. **66**: 56—1961(A)

Image Intensification

- Evaluation of mica-window image tubes. W. K. Ford, Jr., L. W. Fredrick, John S. Hall, T. E. Houck, and W. A. Baum. **66**: 43—1961(A)
- Meteor observations with an image orthicon. I. John Spalding and Curtis L. Hemenway. **66**: 54—1961(A)
- Astronomical tests of an imaging photomultiplier. W. L. Wilcock and W. A. Baum. **66**: 58—1961(A)
- High-contrast electronic image-orthicon techniques applied to the lunar surface. R. Aikens, W. Powers, and J. A. Hynek. **66**: 277—1961(A)
- A low-background image tube for direct electronography. W. A. Hiltner and W. F. Niklas. **66**: 286—1961(A)
- Experiments with high-sensitivity image orthicons. I. John Spalding, John F. Dugan, and Curtis L. Hemenway. **66**: 296—1961(A)

Instruments

- Evaluation of mica-window image tubes. W. K. Ford, Jr., L. W. Fredrick, John S. Hall, T. E. Houck, and W. A. Baum. **66**: 43—1961(A)
- The radio telescope of the University of Illinois. G. C. McVittie. **66**: 48—1961(A)
- Astronomical tests of an imaging photomultiplier. W. L. Wilcock and W. A. Baum. **66**: 58—1961(A)
- A radio telescope with a minute-of-arc beam. R. N. Bracewell. **66**: 279—1961(A)
- A low-background image tube for direct electronography. W. A. Hiltner and W. F. Niklas. **66**: 286—1961(A)
- Study of infrared instrumentation for thermal photography of the moon. Hector C. Ingrao, Donald H. Menzel, and J. Anthony Burke. **66**: 287—1961(A)

Intergalactic Matter

- Free hydrogen in the Pegasus I cluster of galaxies. A. A. Penzias. **66**: 293—1961(A)

- On gamma rays from intergalactic space. S. P. Shen and S. N. Milford. **66**: 295—1961(A)

Interplanetary Matter

- Helium resonance radiation in the night and day sky. John C. Brandt. **66**: 279—1961(A)

Interstellar Matter

- Proposed observations of polarization and interstellar reddening from above the ozone with high-altitude balloons. Thomas Gehrels. **66**: 44—1961(A)
- The radio spectrum of supernova remnants. Daniel E. Harris. **66**: 45—1961(A)
- Neutral hydrogen in the Magellanic Clouds—A repeat survey using a multichannel receiver and digital recording. J. V. Hindman, R. X. McGee, A. W. L. Carter, and F. J. Kerr. **66**: 45—1961(A) [Note erratum, p. 275]
- The distribution of local hydrogen from a 21-cm multichannel survey of the sky. R. X. McGee and J. D. Murray. **66**: 48—1961(A) [Note erratum, p. 275]
- A hydrogen-line interferometer. V. Radhakrishnan, D. Morris, and R. W. Wilson. **66**: 51—1961(A)
- The Rayleigh-Taylor instability in compressible fluids. Peter O. Vandervoort. **66**: 56—1961(A)
- On the distribution of electron density inside the earth's orbit. J. L. Weinberg. **66**: 57—1961(A)
- A heavily reddened cluster in Ara. Bengt Westerlund. **66**: 57—1961(A)
- The kinematics of the local gas system. H. L. Helfer. **66**: 160—1961
- 21-cm maser observations of galaxies. Morton S. Roberts. **66**: 294—1961(A)
- A study of the region of M17 at a wavelength of 3.75 cm. R. W. Hobbs. **66**: 517—1961
- The ratio of total to selective absorption. V. M. Blanco and C. J. Lennon. **66**: 524—1961

Jupiter

- Can the Jupiter emission be cyclotron radiation? George B. Field. **66**: 283—1961(A)

Magellanic Clouds

- Spectrophotometry of gaseous nebulae in the Magellanic Clouds. L. H. Aller and D. J. Faulkner. **66**: 37—1961(A)
- Neutral hydrogen in the Magellanic Clouds—A repeat survey using a multichannel receiver and digital recording. J. V. Hindman, R. X. McGee, A. W. L. Carter, and F. J. Kerr. **66**: 45—1961(A) [Note erratum, p. 275]
- Population I in the Large Magellanic Cloud. Bengt Westerlund. **66**: 57—1961(A)
- A new catalogue of emission-line stars and planetary nebulae in the Small Magellanic Cloud. E. M. Lindsay. **66**: 169—1961
- Color-magnitude arrays for two associations in the Large Magellanic Cloud. Bart J. Bok, Priscilla F. Bok, and Jane M. Basinski. **66**: 279—1961(A)
- Supergiant B stars in the Small Magellanic Cloud. William Buscombe and Pamela M. Morris. **66**: 279—1961(A)
- Variable stars in the Small Magellanic Cloud. Cecilia Payne-Gaposchkin. **66**: 293—1961(A)
- The region of NGC 121 in the Small Magellanic Cloud. William G. Tift. **66**: 297—1961(A)

Magnetic Fields

- Galactic motions in a large-scale magnetic field. Donat G. Wentzel. **66**: 57—1961(A)

Magnetohydrodynamics

- Heating of the solar atmosphere, plages, and corona by magnetohydrodynamic waves. Donald F. Osterbrock. **66**: 50—1961(A)

Mars

The abundance of water vapor on Mars. Carl Sagan. **66**: 52—1961(A)

Measurement Techniques

On the accuracy of measurements made upon films photographed by Baker-Nunn satellite tracking cameras. Karoly Lassovszky. **66**: 289—1961(A)

Mercury

The measurement of microwave radiation from the planet Mercury. William E. Howard, III, Alan H. Barrett, and Fred T. Haddock. **66**: 287—1961(A)

Mercury, Transits of

Transit of Mercury observed at Austin, Texas. G. de Vaucouleurs. **66**: 36—1961

Meridian Circle

Meridian observations of the moon, 1958–59. A. N. Adams and S. M. Bestul. **66**: 36—1961(N&O)

Problems in meridian astronomy. G. van Herk and A. J. J. van Woerkom. **66**: 87—1961

Problems in meridian astronomy. Erik Høg. **66**: 531—1961(N&O)

Meteorites

Decay intervals of extinct radionuclides in meteorites and in the earth. Gordon G. Goles and Edward Anders. **66**: 44—1961(A)

A search for micrometeorites in the earth's atmosphere. F. W. Wright, P. W. Hodge, and E. L. Fireman. **66**: 298—1961(A)

Meteors

Meteor observations with an image orthicon. I. John Spalding and Curtis L. Hemenway. **66**: 54—1961(A)

Dust particles of micron size associated with the Leonid meteor stream. W. M. Alexander, C. W. McCracken, and H. E. LaGow. **66**: 277—1961(A)

Meteor magnitude distributions. Peter M. Millman and Miriam S. Burland. **66**: 291—1961(A)

Planetary perturbations and the Perseid meteor stream. Richard B. Southworth. **66**: 295—1961(A)

The survival of comets and cometary material. Ernst J. Öpik. **66**: 381—1961(S)

Moon

Meridian observations of the moon, 1958–59. A. N. Adams and S. M. Bestul. **66**: 36—1961(N&O)

Tidal deformations and the origin of the moon. Ernst J. Öpik. **66**: 50—1961(A)

Infrared isothermal mapping of certain crater areas of the moon. J. M. Saari and R. W. Shorthill. **66**: 52—1961(A)

Lunar temperature measurements at 8.6-mm wavelength. Warren C. Tyler and Jack Copeland. **66**: 56—1961(A)

Tidal deformations and the origin of the moon. Ernst J. Öpik. **66**: 60—1961

Secular terms and fluctuations in the motions of the sun and the moon. B. L. van der Waerden. **66**: 138—1961

High-contrast electronic image-orthicon techniques applied to the lunar surface. R. Aikens, W. Powers, and J. A. Hynek. **66**: 277—1961(A)

Evidence of isostasy and differentiation on the moon. Winifred Sawtell Cameron and John A. O'Keefe. **66**: 280—1961(A)

Study of infrared instrumentation for thermal photography of the moon. Hector C. Ingrao, Donald H. Menzel, and J. Anthony Burke. **66**: 287—1961(A)

An investigation of possible variations in the brightness temperature of the moon at 21 cm. John A. Waak. **66**: 298—1961(A)

National Science Foundation

Conference on galaxies. **66**: 533—1961

Comets (Symposium). **66**: 361—1961

Navigation, Terrestrial and Interplanetary

The determination of the velocity vector of a space vehicle from apparent changes in the angular separations of stars. Oliver C. Collins and Walter F. Hiltner. **66**: 41—1961(A)

Nebulae

Spectrophotometry of gaseous nebulae in the Magellanic Clouds. L. H. Aller and D. J. Faulkner. **66**: 37—1961(A)

Measurement of microwave radiation at 3.45 cm from the planetary nebula NGC 6543. A. H. Barrett, W. E. Howard, III, F. T. Haddock, M. E. Bair, J. J. Cook, and L. G. Cross. **66**: 37—1961(A)

Motions in bright rim structures in diffuse nebulae. G. Courtes and S. R. Pottasch. **66**: 41—1961(A)

The spectra of planetary nebulae and stars in the near infrared. L. W. Fredrick. **66**: 44—1961(A)

Note on the nebosity around RS Puppis. Bengt Westerlund. **66**: 57—1961(A)

A new catalogue of emission-line stars and planetary nebulae in the Small Magellanic Cloud. E. M. Lindsay. **66**: 169—1961

A list of new OB stars near the galactic nebula M17. C. B. Stephenson and R. W. Hobbs. **66**: 186—1961

A study of the region of M17 at a wavelength of 3.75 cm. R. W. Hobbs. **66**: 517—1961

Neptune

The constitutions of Uranus and Neptune. William S. Porter. **66**: 243—1961

New Books Received

66: 104, 156, 191, 276—1961

Notices

Request for errata in star catalogues of position. **66**: 156—1961

Request for astronomical serials for Latin American libraries and observatories. **66**: 192—1961

Observing time on Kitt Peak. **66**: 272—1961

Resolution adopted by American Astronomical Society, June 20, 1961, at Nantucket, Massachusetts. **66**: 275—1961

Nuclear Reactions

Anomalous abundances of atmospheric xenon isotopes. A. G. W. Cameron. **66**: 40—1961(A)

Neutrino emission processes in old stars. Hong-Yee Chiu. **66**: 40—1961(A)

Decay intervals of extinct radionuclides in meteorites and in the earth. Gordon G. Goles and Edward Anders. **66**: 44—1961(A)

Model for pre-supernova stars. H. Y. Chiu. **66**: 280—1961(A)

The dynamics of supernova explosions. Stirling A. Colgate, William H. Grasberger, and Richard H. White. **66**: 280—1961(A)

Nucleosynthesis during the early history of the solar system. William A. Fowler, Jesse L. Greenstein, and Fred Hoyle. **66**: 284—1961(A)

Observatories

The new University of Georgia Observatory. L. Harold Spradley, Jr. **66**: 54—1961(A)

Reports of

Allegheny. **66**: 433—1961

Amherst. **66**: 433—1961

Chamberlin. **66**: 434—1961

Dearborn. **66**: 435—1961

Dyer. **66**: 436—1961

Florida. **66**: 439—1961

Flower and Cook. **66**: 637—1961

Georgetown. **66**: 440—1961

Observatories (Continued)

- Goethe Link. **66**: 441—1961
 Harvard. **66**: 443—1961
 Illinois. **66**: 447—1961
 Leander McCormick. **66**: 448—1961
 Lick. **66**: 450—1961
 Louisiana State. **66**: 455—1961
 Lowell. **66**: 455—1961
 Michigan. **66**: 458—1961
 National Aeronautics and Space Administration. **66**: 463—1961
 National Aeronautics and Space Administration, Addendum to. **66**: 638—1961
 National Bureau of Standards. **66**: 639—1961
 National Radio Astronomy. **66**: 465—1961
 Ohio State and Ohio Wesleyan. **66**: 470—1961
 Princeton. **66**: 473—1961
 Rochester. **66**: 475—1961
 Sagamore Hill. **66**: 476—1961
 Sproul. **66**: 478—1961
 Stanford. **66**: 640—1961
 United States Coast and Geodetic Survey. **66**: 479—1961
 United States Naval. **66**: 479—1961
 United States Naval Research. **66**: 483—1961
 Van Vleck. **66**: 489—1961
 Warner and Swasey. **66**: 490—1961
 Yale. **66**: 642—1961
 Yerkes and McDonald. **66**: 493—1961

Photography

- Hypersensitization gains in the near infrared. Nicholas Sanduleak. **66**: 53—1961(A)
 Hypersensitization gains in the near infrared. N. Sanduleak. **66**: 526—1961

Photometry

- A program of stellar photometry at 3 to 4 microns. Philip E. Barnhart, Walter E. Mitchell, Jr., Ronald E. Roll, and William H. Haynie. **66**: 37—1961(A)
 Narrow-band photometry of Babcock's magnetic star HD 215441. Robert C. Cameron and Charles Perry. **66**: 40—1961(A)
 H β photometry of the Pleiades. D. L. Crawford and J. C. Golson. **66**: 42—1961(A)
 Project West Ford—Report on the effects of Project West Ford on optical astronomy. William Liller. **66**: 114—1961
 Photographic history and suggested nature of the radio source 3C-48. Harlan J. Smith and Dorrit Hoffleit. **66**: 295—1961(A)
 The ratio of total to selective absorption. V. M. Blanco and C. J. Lennon. **66**: 524—1961

Planets

- On the Adams prize problem for 1856. Allan F. Cook and Fred A. Franklin. **66**: 41—1961(A)
 The abundance of water vapor on Mars. Carl Sagan. **66**: 52—1961(A)
 On the origin of the Venus microwave emission. Carl Sagan, K. M. Siegel, and D. E. Jones. **66**: 52—1961(A)
 The constitutions of Uranus and Neptune. William S. Porter. **66**: 243—1961
 Can the Jupiter emission be cyclotron radiation? George B. Field. **66**: 283—1961(A)
 The measurement of microwave radiation from the planet Mercury. William E. Howard, III, Alan H. Barrett, and Fred T. Haddock. **66**: 287—1961(A)
 The structure and CO₂ content of Venus' atmosphere. Lewis D. Kaplan. **66**: 288—1961(A)
 The temperature of Venus. A. E. Lilley. **66**: 290—1961(A)

Plasma Physics

- Free-free emission in the radio frequency range. Ludwig Oster. **66**: 50—1961(A)

Polarization

- Proposed observations of polarization and interstellar reddening from above the ozone with high-altitude balloons. Thomas Gehrels. **66**: 44—1961(A)
 Observations of a plane-polarized component in the cosmic background radiation of 408 Mc. C. L. Seeger and G. Westerhout. **66**: 294—1961(A)

Radiation Mechanisms

- Free-free emission in the radio-frequency range. Ludwig Oster. **66**: 50—1961(A)

Radiative Transfer

- Diffuse reflection by a semi-infinite atmosphere. Henry G. Horak and S. Chandrasekhar. **66**: 286—1961(A)
 The grey planetary atmosphere. Rupert Wildt. **66**: 298—1961(A)

Radio Astronomy

- Measurement of microwave radiation at $\lambda 3.45$ cm from the planetary nebula NGC 6543. A. H. Barrett, W. E. Howard, III, F. T. Haddock, M. E. Bair, J. J. Cook, and L. G. Cross. **66**: 37—1961(A)
 405 Mc observations of the galactic center. B. F. Burke and J. W. Fidor. **66**: 39—1961(A)
 Positions of 517-megacycle solar bursts. Robert Fleischer and Masakazu Oshima. **66**: 43—1961(A)
 The radio spectrum of supernova remnants. Daniel E. Harris. **66**: 45—1961(A)
 Neutral hydrogen in the Magellanic Clouds—A repeat survey using a multichannel receiver and digital recording. J. V. Hindman, R. X. McGee, A. W. L. Carter, and F. J. Kerr. **66**: 45—1961(A) [Note erratum, p. 275]
 A new 21-cm interpretation of the spiral structure and motions of the galaxy. F. J. Kerr. **66**: 47—1961(A)
 The distribution of local hydrogen from a 21-cm multichannel survey of the sky. R. X. McGee and J. D. Murray. **66**: 48—1961(A) [Note erratum, p. 275]
 The radio telescope of the University of Illinois. G. C. McVittie. **66**: 48—1961(A)
 The structure of Virgo A. Alan T. Moffet. **66**: 49—1961(A)
 Free-free emission in the radio-frequency range. Ludwig Oster. **66**: 50—1961(A)
 A hydrogen-line interferometer. V. Radhakrishnan, D. Morris, and R. W. Wilson. **66**: 51—1961(A)
 On the origin of the Venus microwave emission. Carl Sagan, K. M. Siegel, and D. E. Jones. **66**: 52—1961(A)
 Lunar temperature measurements at 8.6-mm wavelength. Warren C. Tyler and Jack Copeland. **66**: 56—1961(A)
 Project West Ford—Introduction. Leo Goldberg. **66**: 105—1961
 Project West Ford—Properties of orbiting dipole belts. W. E. Morrow, Jr. and D. C. MacLellan. **66**: 107—1961
 Project West Ford—Radio properties of an orbiting scattering medium. A. E. Lilley. **66**: 116—1961
 The kinematics of the local gas system. H. L. Helfer. **66**: 160—1961
 A radio telescope with a minute-of-arc beam. R. N. Bracewell. **66**: 279—1961(A)
 Crab nebula occultation measurements and the structure of the solar corona. William C. Erickson. **66**: 283—1961(A)
 Can the Jupiter emission be cyclotron radiation? George B. Field. **66**: 283—1961(A)
 Spectral indices of 60 discrete sources. S. J. Goldstein, Jr. **66**: 285—1961(A)
 The measurement of microwave radiation from the planet Mercury. William E. Howard, III, Alan H. Barrett, and Fred T. Haddock. **66**: 287—1961(A)
 Radio observations of solar active regions at cm-dm wavelengths. T. Kakinuma. **66**: 287—1961(A)

- Polarization measurements of solar emission at a wavelength of 9.1 cm. T. Kakinuma and G. Swarup. **66**: 288—1961(A)
- Some studies on the occurrence of type IV solar bursts of continuum radiation. M. R. Kundu. **66**: 289—1961(A)
- The temperature of Venus. A. E. Lilley. **66**: 290—1961(A)
- The central component of Centaurus A. A. G. Little and R. N. Bracewell. **66**: 290—1961
- Early results of the 1961 JPL Venus radar experiment. Duane O. Muhleman. **66**: 292—1961(A)
- Free hydrogen in the Pegasus I cluster of galaxies. A. A. Penzias. **66**: 293—1961(A)
- 21-cm maser observations of galaxies. Morton S. Roberts. **66**: 294—1961(A)
- Observations of a plane-polarized component in the cosmic background radiation of 408 Mc. C. L. Seeger and G. Westerhout. **66**: 294—1961(A)
- Photographic history and suggested nature of the radio source 3C-48. Harlan J. Smith and Dorrit Hoffleit. **66**: 295—1961(A)
- Radio pictures of the sun at a wavelength of 9.1 cm. G. Swarup. **66**: 296—1961(A)
- Radio sources and clusters of galaxies. Sidney van den Bergh. **66**: 297—1961(A)
- An investigation of possible variations in the brightness temperature of the moon at 21 cm. John A. Waak. **66**: 298—1961(A)
- A study of the region of M17 at a wavelength of 3.75 cm. R. W. Hobbs. **66**: 517—1961
- Effects of antenna scan rate and radiometer time constant on receiver output. William E. Howard, III. **66**: 521—1961
- Radio sources and clusters of galaxies. Sidney van den Bergh. **66**: 562—1961
- Redshifts, see Galaxies, Redshifts**
- Satellites, Artificial**
- The gravitational field of the earth derived from motions of three satellites. Yoshihide Kozai. **66**: 8—1961
- Some remarks on the motion of a satellite of an oblate planet. Richard B. Barrar. **66**: 11—1961
- The reduction from geocentric to geodetic coordinates. John Morrison and Samuel Pines. **66**: 15—1961
- The critical inclination case of the satellite motion. Yusuke Hagihara and Yoshihide Kozai. **66**: 45—1961(A)
- Reliability approach to astronomical satellites. Edwin D. Karmiol and John S. Youtcheff. **66**: 47—1961(A)
- The bolometric solar constant related to radiation pressure effects on satellites. Raymond H. Wilson, Jr. **66**: 58—1961(A)
- Atmospheric drag on artificial satellites. Pedro E. Zadunaisky. **66**: 59—1961(A)
- Project West Ford—Introduction. Leo Goldberg. **66**: 105—1961
- Project West Ford—Properties of orbiting dipole belts. W. E. Morrow, Jr., and D. C. MacLellan. **66**: 107—1961
- Project West Ford—Report of the effects of Project West Ford on optical astronomy. William Liller. **66**: 114—1961
- Project West Ford—Radio properties of an orbiting scattering medium. A. E. Lilley. **66**: 116—1961
- Application of periodic surface theory to the study of satellite orbits. S. P. Diliberto, W. T. Kyner, and R. B. Freund. **66**: 118—1961
- On satellite orbits with very small eccentricities. Imre G. Izsak. **66**: 129—1961
- Note on the motion of a close earth satellite with a small eccentricity. Yoshihide Kozai. **66**: 132—1961
- On the almost periodicity of satellite motion. E. David Callender. **66**: 134—1961
- Some dynamical properties of natural and artificial satellites. Su-Shu Huang. **66**: 157—1961
- Theoretical evaluation of atmospheric drag effects in the motion of an artificial satellite. Dirk Brouwer and Gen-ichiro Hori. **66**: 193—1961
- A determination of the ellipticity of the earth's equator from the motion of two satellites. Imre G. Izsak. **66**: 226—1961
- The motion of a hyperbolic artificial satellite around the oblate earth. Gen-ichiro Hori. **66**: 258—1961
- Appendix to Theoretical Evaluation of Atmospheric Drag Effects in the Motion of an Artificial Satellite. Dirk Brouwer and Gen-ichiro Hori. **66**: 264—1961
- Coma formation and an artificial comet experiment. Bertram Donn. **66**: 282—1961(A)
- A uniformly valid asymptotic representation for all times of the motion of a satellite in the vicinity of the smaller body in the restricted three-body problem. Jirair K. Kevorkian. **66**: 288—1961(A)
- On the accuracy of measurements made upon films photographed by Baker-Nunn satellite tracking cameras. Karoly Lassovszky. **66**: 289—1961(A)
- A determination of the atmospheric oblateness from the motion of two low satellites. R. C. Nigam. **66**: 292—1961(A)
- The diurnal atmospheric bulge and its effect on satellite motions. Stanley P. Wyatt. **66**: 299—1961(A)
- Tesseral harmonics of the gravitational potential of the earth as derived from satellite motions. Yoshihide Kozai. **66**: 355—1961
- Computation of coordinates from Brouwer's solution of the artificial satellite problem. O. K. Smith. **66**: 359—1961(A)
- The effect of solar radiation pressure on the motion of an artificial satellite. R. W. Bryant. **66**: 430—1961
- Formulae for an accurate intermediary orbit of an artificial satellite. John P. Vinti. **66**: 514—1961
- Saturn**
- On the Adams prize problem for 1856. Allan F. Cook and Fred A. Franklin. **66**: 41—1961(A)
- Solar Constant**
- The bolometric solar constant related to radiation pressure effects on satellites. Raymond H. Wilson, Jr. **66**: 58—1961(A)
- Spectrophotometry**
- Stellar flux measurements in the middle ultraviolet. Albert Boggess, III. **66**: 279—1961(A)
- Stellar spectrophotometry below 3000 angstroms. Theodore P. Stecher and J. E. Milligan. **66**: 296—1961(A)
- Observed and computed white-dwarf energy curves. Robert G. Tull and William Liller. **66**: 297—1961(A)
- Stars, Associations of**
- Population I in the Large Magellanic Cloud. Bengt Westerlund. **66**: 57—1961(A)
- Stars, Atmospheres of**
- A computer program for nongray stellar atmospheres. Owen Gingerich. **66**: 285—1961(A)
- A method for computing monochromatic fluxes in model stellar atmospheres. S. S. Kumar. **66**: 288—1961(A)
- Balmer line emission from a shell. Stuart R. Pottasch. **66**: 294—1961(A)
- Stars, Atmospheric**
- Numerical integrations for nonlinear compression waves and line profiles in Cepheid atmospheres. R. G. Teske. **66**: 55—1961(A)
- Stars, Catalogues of**
- Discussion on longitude differences of catalogues in determining ephemeris time. Shinko Aoki. **66**: 1—1961
- Problems in meridian astronomy. G. van Herk and A. J. J. van Woerkom. **66**: 87—1961

Stars, Catalogues of (Continued)

- Request for errata in star catalogues of position. **66**: 156—1961(N)
 Star catalogues on punched cards and magnetic tape. J. H. Berbert. **66**: 279—1961(A)
 Star catalogues on punched cards. Julena S. Duncombe and Dorrit Hoffleit. **66**: 283—1961(A)

Stars, Clusters of

- Astrometric investigations of galactic clusters. I. NGC 129. Alan P. Lenham and Otto G. Franz. **66**: 16—1961
 Changes in the periods of RR Lyrae stars in Omega Centauri. Emilia Pisani Belserene. **66**: 38—1961(A)
H β photometry of the Pleiades. D. L. Crawford and J. C. Golson. **66**: 42—1961(A)
 Star distribution in the globular cluster M15. Ivan King. **66**: 47—1961(A)
 The dynamics of the Pleiades cluster. D. Nelson Limber. **66**: 48—1961(A)
 Structure and evolution of globular clusters. Richard W. Michie. **66**: 49—1961(A)
 The ages of the open cluster NGC 188 and the globular clusters M3, M5, M13 compared with the Hubble time. Allan R. Sandage. **66**: 53—1961(A)
 A heavily reddened cluster in Ara. Bengt Westerlund. **66**: 57—1961(A)
 The shape of a rotating star cluster. Ivan King. **66**: 68—1961
 NGC 5466. James Cuffey. **66**: 71—1961
 The Fornax dwarf galaxy. I. The globular clusters. Paul W. Hodge. **66**: 83—1961
 Possible M-star members of NGC 6838. C. B. Stephenson. **66**: 85—1961

Stars, Collisions of

- X rays from collisions of stars. S. N. Milford. **66**: 49—1961(A)

Stars, Compositions of

- A spectrophotometric comparison of 7 Sextantis and Sirius. Yvonne H. Stone and George Wallerstein. **66**: 54—1961(A)
 The metal-rich G-dwarf 20 Leo Minoris and two comparison stars. George Wallerstein and H. L. Helfer. **66**: 57—1961(A)

Stars, Constitutions of

- Boundary conditions for stellar interior models. Thomas L. Swihart. **66**: 297—1961(A)

Stars, Double and Multiple

- Observations and times of minima for three W Ursae Majoris systems. Beverly B. Bookmyer. **66**: 24—1961
 The light variation and orbital elements of AK Herculis. L. Binnendijk. **66**: 27—1961
 Times of minimum light for several eclipsing binaries. Robert H. Koch. **66**: 35—1961(N&O)
 The light variations and orbital elements of AK Herculis. L. Binnendijk. **66**: 38—1961(A)
 Three southern spectroscopic binaries. William Buscombe and Pamela Morris. **66**: 39—1961(A)
 The use of a digital computer in the solution for the elements of eclipsing binary stars. George W. Collins and C. M. Huffer. **66**: 41—1961(A)
 Least-squares tables for visual binaries. John B. Irwin. **66**: 46—1961(A)
 The B8 component of β Lyrae. Otto Struve. **66**: 55—1961(A)
 Departures from the Russell model in TX Ursae Majoris. Robert H. Koch. **66**: 230—1961
 Parallax and orbital motion of 42 Comae=ADS 8804 from plates taken with the 24-inch Sproul refractor. Sarah Lee Lippincott. **66**: 272—1961
 The frequency of binaries among A-type stars. Helmut Abt. **66**: 277—1961(A)

- An eclipsing "Astronaut." Sergei Gaposchkin. **66**: 284—1961(A)
 Determination of differential limb darkening of eclipsing binaries from multicolor photometric observations. I. K. Serkowski. **66**: 405—1961
 The eclipsing system V Tucanae. Frank Bradshaw Wood and George E. McCluskey, Jr. **66**: 413—1961
 Mean densities of visual binary stars. Maria A. Giannuzzi and Lucilla Bianco. **66**: 418—1961

Stars, Dynamics of

- Star distribution in the globular cluster M15. Ivan King. **66**: 47—1961(A)
 The dynamics of the Pleiades cluster. D. Nelson Limber. **66**: 48—1961(A)
 Structure and evolution of globular clusters. Richard W. Michie. **66**: 49—1961(A)
 The shape of a rotating star cluster. Ivan King. **66**: 68—1961
 Stellar kinematics and spiral arms. Andrew T. Young. **66**: 299—1961(A)

Stars, Evolution of

- Neutrino emission processes in old stars. Hong-Yee Chiu. **66**: 40—1961(A)
 The helium flash in Population II giants. R. Härm and M. Schwarzschild. **66**: 45—1961(A)
 Observational evidence and proposed test for the mode of star formation. Su-Shu Huang. **66**: 46—1961(A)
 The ages of the open cluster NGC 188 and the globular clusters M3, M5, M13 compared with the Hubble time. Allan R. Sandage. **66**: 53—1961(A)
 Model for pre-supernova stars. H. Y. Chiu. **66**: 280—1961(A)
 The dynamics of supernova explosions. Stirling A. Colgate, William H. Grasberger, and Richard H. White. **66**: 280—1961(A)

Stars, Luminosity of

- A search for stars of low luminosity. G. Van Biesbroeck. **66**: 528—1961

Stars, Photometry of

- On the photometric catalogue of stars in the region of the Orion nebula, compiled by P. P. Parenago. A. S. Sharov. **66**: 103—1961(N&O)
 Photometry of G and K giants. D. L. Crawford. **66**: 281—1961(A)

Stars, Spectra of

- The spectra of planetary nebulae and stars in the near infrared. L. W. Fredrick. **66**: 44—1961(A)
 The latest cycle of the Be shell star β Mon. Dean B. McLaughlin. **66**: 48—1961(A)
 Possible M-star members of NGC 6838. C. B. Stephenson. **66**: 85—1961
 A new catalogue of emission-line stars and planetary nebulae in the Small Magellanic Cloud. E. M. Lindsay. **66**: 169—1961
 A list of new OB stars near the galactic nebula M17. C. B. Stephenson and R. W. Hobbs. **66**: 186—1961
 The frequency of binaries among A-type stars. Helmut Abt. **66**: 277—1961(A)
 Supergiant B stars in the Small Magellanic Cloud. William Buscombe and Pamela M. Morris. **66**: 279—1961(A)
 Behavior of absorption features in the spectra of Mira variables. A. J. Deutsch, P. W. Merrill, and P. C. Keenan. **66**: 282—1961(A)
 A method for computing monochromatic fluxes in model stellar atmospheres. S. S. Kumar. **66**: 288—1961(A)
 Fifty carbon stars in a region in Cygnus. Martin F. McCarthy S.J. **66**: 291—1961(A)
 Balmer line emission from a shell. Stuart R. Pottasch. **66**: 294—1961(A)

- Stellar spectrophotometry below 3000 angstroms. Theodore P. Stecher and J. E. Milligan. **66**: 296—1961(A)
- Observed and computed white-dwarf energy curves. Robert G. Tull and William Liller. **66**: 297—1961(A)
- Stars, Variable**
- Observations and times of minima for three W Ursae Majoris systems. Beverly B. Bookmyer. **66**: 24—1961
- The light variation and orbital elements of AK Herculis. L. Binnendijk. **66**: 27—1961
- Times of minimum light for several eclipsing binaries. Robert H. Koch. **66**: 35—1961(N&O)
- Changes in the periods of RR Lyrae stars in Omega Centauri. Emilia Pisani Belserene. **66**: 38—1961(A)
- The light variations and orbital elements of AK Herculis. L. Binnendijk. **66**: 38—1961(A)
- On second helium ionization as a cause of Cepheid instability. John P. Cox. **66**: 42—1961(A)
- Numerical integrations for nonlinear compression waves and line profiles in Cepheid atmospheres. R. G. Teske. **66**: 55—1961(A)
- Note on the nebulosity around RS Puppis. Bengt Westerlund. **66**: 57—1961(A)
- Forty-five variable stars in VSF 193. Dorrit Hoffleit. **66**: 188—1961
- Departures from the Russell model in TX Ursae Majoris. Robert H. Koch. **66**: 230—1961
- The Draco system, a dwarf galaxy. W. Baade and H. H. Swope. **66**: 278—1961(A)
- The pulsations of models of δ Cephei stars. N. Baker. **66**: 278—1961(A)
- Behavior of absorption features in the spectra of Mira variables. A. J. Deutsch, P. W. Merrill, and P. C. Keenan. **66**: 282—1961(A)
- An eclipsing "Astronaut." Sergei Gaposchkin. **66**: 284—1961(A)
- Variable stars in the Small Magellanic Cloud. Cecilia Payne-Gaposchkin. **66**: 293—1961(A)
- The Draco system, a dwarf galaxy. Walter Baade and Henrietta H. Swope. **66**: 300—1961
- Photoelectric observations of SX Phoenixis. T. D. Kinman. **66**: 348—1961
- Visual photometric observations of W Serpentis. Dean B. McLaughlin. **66**: 351—1961
- Determination of differential limb darkening of eclipsing binaries from multicolor photometric observations. I. K. Serkowski. **66**: 405—1961
- The eclipsing system V Tucanae. Frank Bradshaw Wood and George E. McCluskey, Jr. **66**: 413—1961
- Stars, Variable, Observations and Notes**
- KP Aql. **66**: 35—1961
- ZZ Cep. **66**: 35—1961
- β CMa. **66**: 297—1961
- RZ Com. **66**: 35—1961
- Z, RZ Dra. **66**: 35—1961
- AK Her. **66**: 24, 27, 38—1961
- SW Lac. **66**: 35—1961
- AM Leo. **66**: 26—1961
- SS Lib. **66**: 35—1961
- β Lyr. **66**: 55—1961
- V501, V508 Oph. **66**: 35—1961
- TY Peg. **66**: 35—1961
- SX Phe. **66**: 348—1961
- RS Pup. **66**: 57—1961
- GQ, GZ, HM, HN, HO, HQ, HT, HZ, IP, IY, KS, NP, NQ, NT, V939, V958 Sgr. **66**: 188—1961
- 28 new Hoffleit variables in Sgr. **66**: 188—1961
- W, AO Ser. **66**: 35—1961
- Y Sex. **66**: 35—1961
- RS Sct. **66**: 35—1961
- W, TX UMa. **66**: 25—1961
- AW Vir. **66**: 35—1961
- 32 variables in NGC 5466. **66**: 81—1961
- (12.7) 12^h12^m16^s 36°56' (1950). **66**: 284—1961
- 137 variables in Draco dwarf galaxy. **66**: 300—1961
- Sun**
- A north-south asymmetry in location of solar sources of great geomagnetic storms. Barbara Bell. **66**: 38—1961(A)
- The penumbral filaments as convection rolls. Robert E. Danielson. **66**: 42—1961(A)
- Local Doppler effects in the photosphere. John W. Evans and Horst Mauter. **66**: 42—1961(A)
- Secular terms and fluctuation in the motions of the sun and the moon. B. L. van der Waerden. **66**: 138—1961
- Further evidence of broadening of CaII line profiles in chromospheric spicules by magnetic field. R. Grant Athay. **66**: 278—1961(A)
- Sun, Atmosphere of**
- Heating of the solar chromosphere, plages, and corona by magnetohydrodynamic waves. Donald F. Osterbrock. **66**: 50—1961(A)
- Sun, Corona of**
- Crab nebula occultation measurements and the structure of the solar corona. William C. Erickson. **66**: 283—1961(A)
- Sun, Flares of**
- Spectrum of the flare of September 2, 1960. Elske v. P. Smith and Patrick S. McIntosh. **66**: 54—1961(A)
- The passage of a flare spray through the solar emission corona. Frank Q. Orrall and Henry J. Smith. **66**: 293—1961(A)
- Sun, Parallax of**
- Early results of the 1961 JPL Venus radar experiment. Duane O. Muhleman. **66**: 292—1961(A)
- Sun, Radio Observations of**
- Positions of 517-megacycle solar bursts. Robert Fleischer and Masakazu Oshima. **66**: 43—1961(A)
- Radio observations of solar active regions at cm-dm wavelengths. T. Kakinuma. **66**: 287—1961(A)
- Polarization measurements of solar emission at a wavelength of 9.1 cm. T. Kakinuma and G. Swarup. **66**: 288—1961(A)
- Some studies on the occurrence of type IV solar bursts of continuum radiation. M. R. Kundu. **66**: 289—1961(A)
- Radio pictures of the sun at a wavelength of 9.1 cm. G. Swarup. **66**: 296—1961(A)
- Sun, Spectrum of**
- The solar continuum in the extreme ultraviolet. C. R. Detwiler, J. D. Purcell, and R. Tousey. **66**: 281—1961(A)
- Departures from thermodynamic equilibrium in chromospheric MgI, CaI, and OI. Lewis L. House and R. Grant Athay. **66**: 286—1961(A)
- The computed solar continuum in the ultraviolet. Kenneth G. Widing. **66**: 298—1961(A)
- Supernovae**
- The radio spectrum of supernova remnants. Daniel E. Harris. **66**: 45—1961(A)
- Model for pre-supernova stars. H. Y. Chiu. **66**: 280—1961(A)
- The dynamics of supernova explosions. Stirling A. Colgate, William H. Grasberger, and Richard H. White. **66**: 280—1961(A)
- An attempt to stimulate supernova spectra. Dean B. McLaughlin. **66**: 291—1961(A)
- Photographic history and suggested nature of the radio source 3C-48. Harlan J. Smith and Dorrit Hoffleit. **66**: 295—1961(A)

Tables

Least-squares tables for visual binaries. John B. Irwin.
66: 46—1961(A)

Tektites

The cometary theory of tektite origin. John A. O'Keefe.
66: 292—1961(A)

Tidal Theory

Tidal deformations and the origin of the moon. Ernst J. Öpik. 66: 50—1961(A)

Tidal deformations and the origin of the moon. Ernst J. Öpik. 66: 60—1961

Secular terms and fluctuations in the motions of the sun and the moon. B. L. van der Waerden. 66: 138—1961

Time Determination

Discussion on longitude differences of catalogues in determining ephemeris time. Shinko Aoki. 66: 1—1961

Problems in meridian astronomy. Erik Høg. 66: 531—1961(N&O)

Uranus

The constitutions of Uranus and Neptune. William S. Porter. 66: 243—1961

Venus

On the origin of the Venus microwave emission. Carl Sagan, K. M. Siegel, and D. E. Jones. 66: 52—1961(A)

The structure and CO_2 content of Venus' atmosphere. Lewis D. Kaplan. 66: 288—1961(A)

The temperature of Venus. A. E. Lilley. 66: 290—1961(A)

X-Rays

X rays from collisions of stars. S. N. Milford. 66: 49—1961(A)